Reflections on working with *The Gang*: a journey towards computational fluency?

Motivations and Background

For many years, a question has persistently troubled me: to what extent can students who feel significantly challenged by working with number build genuine and sustained confidence in dealing effectively with calculations that come their way, both in lessons and examinations as well as in their lives more generally? By 'significantly challenged', I refer to the kind of student whose ability to deal with a problem or calculation in a secure way is, for example, undermined by uncertain knowledge of and weak fluency with number bonds, place value, multiplication tables or equivalences between fractions, decimals and percentages. My observations over many years in primary and secondary settings suggest that, as they become older, many students' confidence levels in this area of their learning may often stagnate and even appear to recede.

A supplementary question I would also wish to pose would be whether, for some learners, a sense of negative inevitability grows and belief in the prospect of progress fades. For me, the nature-nurture debate begins to emerge yet again. These questions have added resonance when put in the context of primary and secondary curricula which promote 'fluency' as a key principle. This is underscored still more strongly by the advent of SATs and GCSE examinations which place such heavy emphasis on the ability to calculate readily without the use of calculators. Anecdotal conversations with experienced teachers would seem to suggest that, as a result of insecurities in what many of us might term 'basic' skills and knowledge in arithmetic, the kind of 'significantly challenged' students I have in mind find the demands of the wider mathematics curricula increasingly difficult to meet. For example, we might, recognise the barriers presented by an insecure grasp of multiplication tables to exploring equivalent fractions, ratio or similarity.

The complexity of these questions intensifies further when one considers what we might mean by 'fluency' with number. In a persuasive and highly regarded article, Russell (2000) advances the view that *computational fluency* comprises three key elements:

Efficiency, Accuracy and Flexibility

Findings from her research in primary American schools highlight the errors and misconceptions that may inhibit achievement of any of these elements, when procedural approaches to calculation are not underpinned by understanding. Having analysed students' responses to a range of problems relating to addition, subtraction and multiplication, she suggests that errors, hesitancy and weak confidence levels may be attributed to an unhelpful 'orientation'. She writes:

The orientation of these students in mathematics class is to try to remember steps rather than to build on what they know in order to make sense of the problem.

She goes on to explore some interesting issues regarding the importance of *connectionist* thinking in securing confidence in carrying out calculations. Quoting from the *Principles and Standards for School Mathematics* (NCTM, 2000), she advances a view that,

Developing fluency requires a balance and connection between conceptual understanding and computational proficiency.

She urges us as teachers of mathematics to see procedures as *a web of connected ideas* and emphasises the importance of developing 'mathematical memory' over 'memorizing'. She concludes her paper by stating, *It* [computational fluency] *is much more than the learning of a skill; it is an integral part of learning with depth and rigor about number and operations.*

Askew (2010) and Thompson (1995) add to the *connectionist* picture when they explore the potential of learners deriving new facts from known facts. For example, if I know 7 x 6 = 42, surely that should support my ability to calculate 70 x 6, 14 x 6, 420 ÷ 7, 0.6 x 0.7 etc.? With all of this in mind and focussing on my initial questions of whether, as well as how, arithmetic fluency might be promoted in learners who feel 'significantly challenged' by number, I became personally motivated to explore the scope and limitations of these academic views on *connectionist* thinking. At the same time, it felt pertinent to probe widely held thought amongst academics [Mason *et al* (2009, 2012), Mason & Watson (2006), Lai & Murray (2012) and Marton *et al* (2003, 2006, 2013)] that learners might benefit by developing an ability to 'notice' and 'see' connections, patterns and structures in mathematics.

Strategic Approach

Essentially, my approach was rooted in informal action research at one of our partnership secondary schools, Belper School and Sixth Form Centre, with a group of four Year 8 students, whom I and the head of department grew fondly to refer to as *The Gang*. From a cohort of around 190 students, the four participants were identified systematically with the support of the head of department. The students had persistently exhibited insecure numeracy skills and were positioned close to the bottom of rankings based on ongoing departmental assessments. Additionally, all four had performed well below expected levels in Key Stage 2 SATs at the end of their primary schooling.

Throughout, I intended the research to enable participants to offer honest perceptions of their abilities to deal with number and computation mentally. The initial individual interviews challenged the participants to evaluate their own current confidence levels in dealing mentally with problems involving integers within the context of each individual operation as well as thinking mentally about fractions, decimals and percentages. The conversations also provided me with the opportunity to share the principles of my approach with them as well as the dispositions I was looking for in them. The following points were made clear:

- They needed to try and believe they were capable of becoming more confident and competent in their use, understanding and recall of number. The importance of this would be supported by Dweck (2000) and her ideas on *Growth Mindset*.
- Consistent engagement and honest reflection had to be givens. They would be expected to keep their school planners and the record sheet (Figure 1) up-to-date:

Figure 1

- It was important to persevere when the going got tough and to try not to give up. Bandura's (1993) thinking on the importance of self-efficacy has particular resonance here.
- Supported by the resource packs provided, I expected them to commit themselves to independent and consistent practice at home. One asked whether they would be punished with a detention if they failed to comply! I explained how that would defeat the object……
- They needed to look after their resource packs carefully and bring them to every session. Packs consisted of multiplication table practice cards, double-sided complements to 100 cards (e.g. 62 backed with 38), a weblink card (Figure 2), a Connection Cloud mini-whiteboard and pen, a set of fraction-decimal-percentage matching cards and a Gattegno (place value) grid with transparent counter (Figure 3).

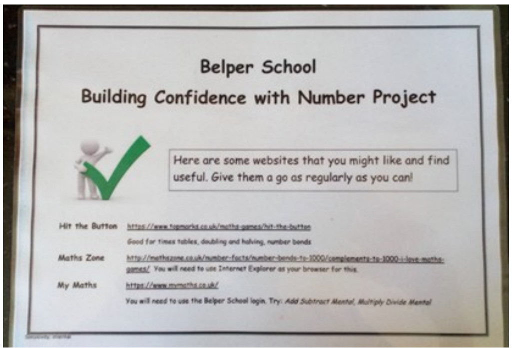


Figure 2

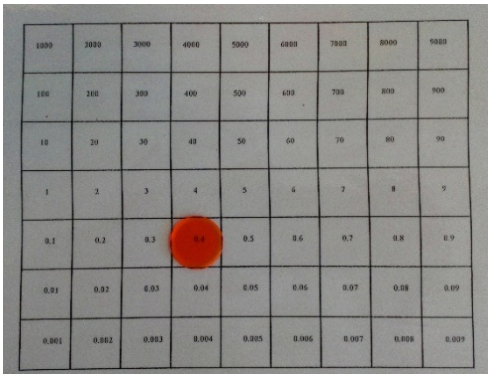


Figure 3

- Although the project was a joint effort between me and them, they crucially needed to accept their own responsibility for learning. They would be expected to try and develop proactive independence and autonomy.
- They needed to see themselves as 'mathematical detectives' who were looking for connections between numbers and operations.

The enquiry was structured into the following one-hour sessions:

Session	Target Students	Focus
1	2 individually	Initial assessments and interviews
2	2 individually	Initial assessments and interviews
3	All	Noticing and exploiting connections: addition and subtraction
4	All	Noticing and exploiting connections: multiplication and division
5	All	Noticing and exploiting connections: fractions, decimals and percentages1
6	All	Noticing and exploiting connections: fractions, decimals and percentages 2

7	All individually	Post-project assessments

Reflective Narratives on *The Gang*

Student C: Luke

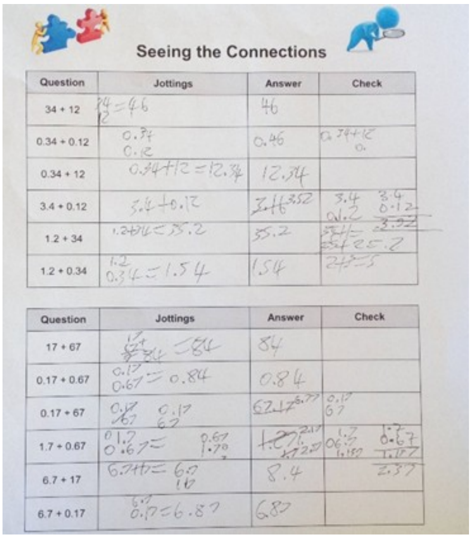
Success rate at bringing resource pack: 100% (1 absence)

Success rate at engaging with independent work between sessions: 66%

At the outset, Student A assessed himself as under-confident in all aspects of mental number work except addition. He commented,

I get really nervous about fractions.

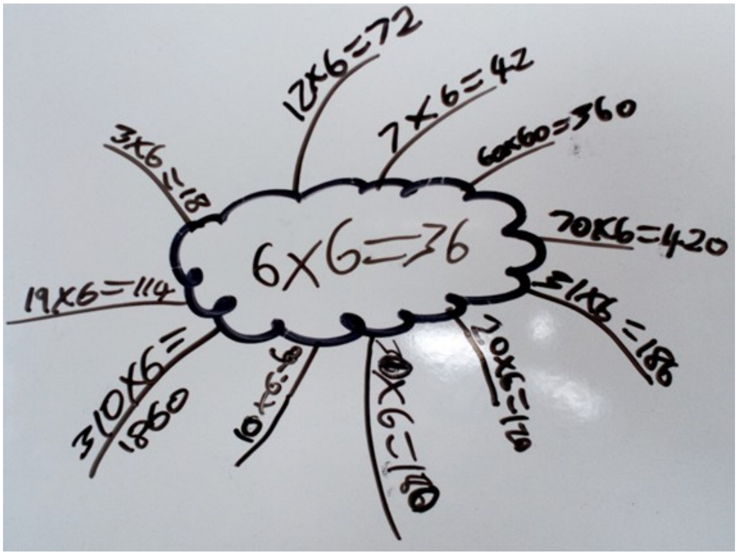
Outcomes of my initial assessments appeared to confirm his relative confidence in his addition skills. He had instant recall of bonds to 10 and, after a few initial errors, was able to deal with complements to 100 quite fluently and with no support. Additionally, his final assessment confirmed that he had generated an ability to find integer complements to 100 even more quickly (17 correct out of 18 in two minutes). He attributed this to consistent practice with the double-sided complements cards. His final assessment also suggested a growing confidence to bond single place decimals to 10. He was one of the two participants who secured this development in learning.



Student A's responses to the upper section above suggest, even with integer addition, a predilection for using a columnar method. Interestingly, he appears, with 34 + 12, to 'line 'em up' for column addition but then realises he can actually deal with this in his head! The order in which he adds the tens and units digits is not obvious, however. With 0.34 + 0.12, columns appear to support him well. Interestingly, he maintains a horizontal layout for 0.34 + 12, suggesting he has spotted key clues and that he feels his place value skills can cope without the need for a columnar structure. He clearly makes a place value error with 3.4 + 0.12 but manages to correct himself with a check using a column method. Similar evidence of strengths and uncertainties in this student's thinking emerge over the rest of this exercise.

Stronger commitment to independent practice at home would probably have enhanced his progress. Verbally hesitant to contribute, initially he grew to offer responses and thinking in a more confident way. As confirmed by Figure 4 and judging by outcomes from my final assessment, Student A's responses to activities confirmed a growing readiness, where integers were involved, to look for and find connections between calculations and their answers.

Figure 4



Decimals continued to pose significant challenges in this respect, however.

Student D: Sasha

Success rate at bringing resource pack: 33% (1 absence)

Success rate at engaging with independent work between sessions: 66%

My initial conversation with and assessment of Student B revealed a significant lack in self-confidence to deal with most aspects of number work mentally. Open and honest in her judgements, she explained that, although she did make some progress in primary and secondary school, thinking about and dealing with numbers mentally still posed issues and unsettled her. She disclosed, for example,

I have never understood how to halve numbers.

Initial assessments confirmed an inability to recall complements to 10 without the use of fingers and also major uncertainties about bonding given numbers to 100. For instance, she could offer no answer for the complement of 63 and could succeed in working out complements to 100 of 89, 74 and 36 only by using a 100 square and with my support to avoid errors. However, the introduction of an empty number line on a mini-whiteboard with some modelling of 'bridging through multiples of 10' from me appeared to make a difference to her confidence levels and success rate. Her recall of the 5 times table was fluent and she confirmed an ability to explain the inherent patterns. Her recall of the 6 and 7 times tables was unreliable, particularly of the higher multiples.

This student's commitment to independent practice at home proved to be less strong than others in the group. This appeared to have a detrimental effect on her rate of progress over time. For example, in her final assessment, she managed only 5 complements to 100 in two minutes. Complements to using one decimal place numbers proved too challenging. Her engagement in sessions was very positive, however, and responses confirmed a growing ability to deal with decimals in other ways. For example, after being exposed to some questions and answers supported by a Gattegno (place value) grid and a transparent counter (Figure 3), she was able to respond more confidently to questions involving multiplication and division of one place decimals without resorting to the addition or subtraction of zeros! Use of the Gattegno grid had enabled her to become more aware of and ‘notice’ the movement of non-zero digits.

Once again, this student’s ability to spot and develop connections between calculations involving integers grew well over time. Her work below (Figure 5) offers sound testimony for this. Interestingly, she decided to introduce 2 x 6 = 12 for herself. From that, she was able to produce answers to 20 x 6 and thence 19 x 6. The latter proved challenging only from the point of view of subtracting 6 from 120……. Responses to a *Connection Cloud* exercise involving decimals proved less confident (Figure 6). Although accurate, the responses evidenced here required much support from me.

Figure 5

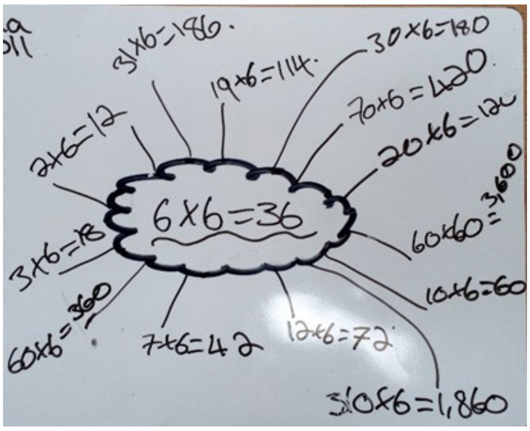
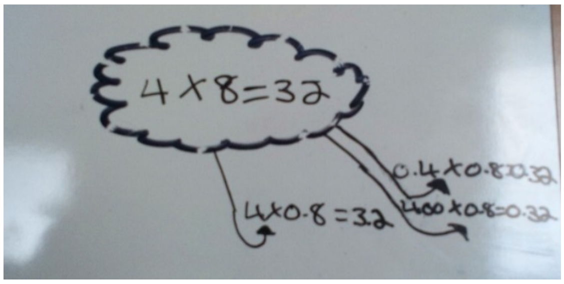


Figure 6



Student A: Ellie

Success rate at bringing resource pack: 100%

Success rate at engaging with independent work between sessions: 75%

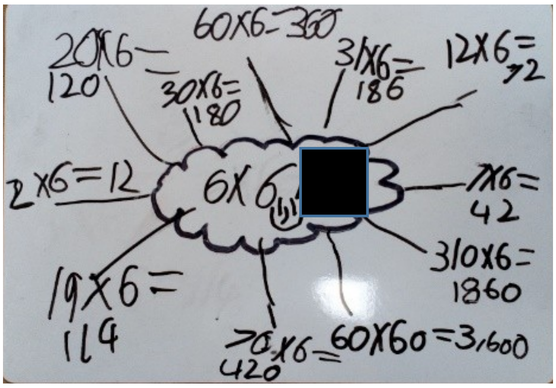
My initial interview with Student C produced many significant findings and admissions. The following points emerged:

- She felt confident with column arithmetic for addition, subtraction and multiplication, but still needed to use fingers to support.
- She disclosed she could rarely deal with any calculation mentally, no matter what the operation.
- Division posed particular problems for her in all respects ‘unless the numbers fit within the times tables I know’.
- ‘I struggle with fractions and decimals and do not get percentages at all.’

The initial assessment of her ability to bond integers confirmed the validity of her self-analysis. For example, she was unable to bond 6 or 2 to 10 without the use of fingers. Unsurprisingly, complements to 100 generated even more uncertainty. Even with the support of a 100-square, these proved challenging. For instance, counting on in 10s proved unreliable and the response of 39 as a complement to 100 for 71 was very typical. Hesitancy and uncertainty pervaded all responses. All of that produced some interesting starting points! On the positive side, knowledge and understanding of the 5 times table was fluent and convincing, as was recall of the lower multiples in the 6 and 7 times tables.

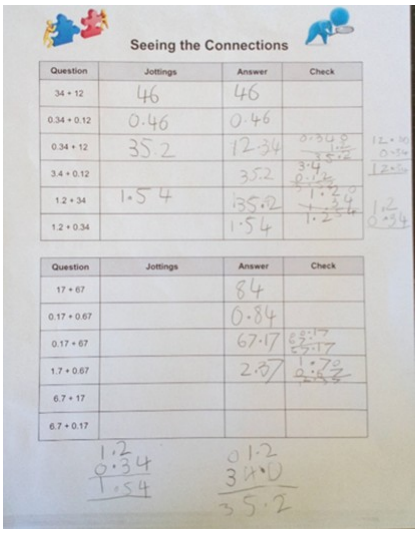
Over the course of the project and despite such obvious gaps in her knowledge and understanding, Student C offered willing, if at times inaccurate verbal contributions to our discussions. She also displayed a genuine desire to make progress. Some clear evidence emerged that she had succeeded. For example, her work (see Figure 7) testified her ability to use the potential of *connectionist* thinking to solve problems. Neither 60 x 6 nor 60 x 60 posed any issue. Verbally, she also confirmed that she knew 3 x 6 = 18. From that, she was able to deal with a chain of problems: 30 x 6, 31 x 6 and 310 x 6.

Figure 7



Handling decimals mentally with any kind of confidence persisted for this student, however. Evidence from her work in Figure 8 shows how, particularly in the first section of questions, she resorts perhaps too easily to a columnar method, making clear errors with little appreciation of place value. That said, the second section provides some more hopeful evidence of progress with her response to the question 67 + 0.17. Here, observation confirmed she generated the correct answer mentally before checking her answer using an accurate columnar method.

Figure 8



Ultimately, this student's final individual interview testified that a commitment to independent practice at home had generated noticeable improvements in her ability to recall randomly asked questions about the 6, 7 and 8 times tables fluently as well as complements to 100.

Student B: Will

Success rate at bringing resource pack: 100%

Success rate at engaging with independent work between sessions: 100%

In his initial interview, Student D assessed himself as being confident with mental addition, subtraction and multiplication of integers but under-confident with division, fraction decimals and percentages. His initial assessment confirmed his instant recall of complements to 10. Significantly, however, when challenged to find complements to 100, he struggled when the given number was not a multiple of 10. For instance, when given 33, he responded with 77 and took 15 seconds to correct himself. Further examples confirmed he was dealing with the tens digit first and was harbouring the misconception that the tens digits needed to add to ten. Once I had shared my observation with him, however, he began to offer correct responses, sometimes supporting his thinking with the use of a mini-whiteboard. His initial assessment also confirmed he was fluent in his recall and understanding of the 5 times table but very hesitant about the 6 times table. The outcomes raised questions about the accuracy of his self-assessments offered at this stage.

Over the course of the project, however, Student D displayed consistent commitment. Of the four, for instance, evidence suggests that he was the most committed to autonomous development of fluent recall of basic number facts, making comments such as,

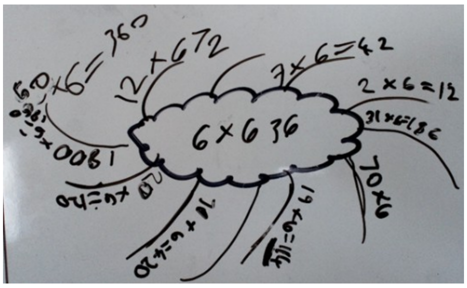
I have done a lot. I had time in the holidays.

and,

I have been working hard on my 6s, 7s and 8s but still find the 7s hard.

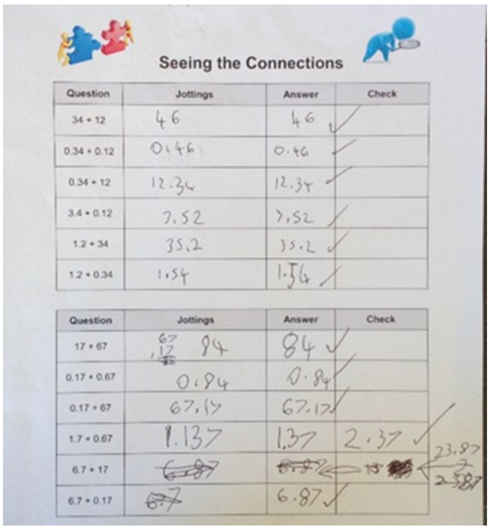
Interestingly, although evidence from an early session suggested he was limited in his ability to see the relationship between calculations in a consistent way, the *Connection Cloud* in Figure 9 for 6 x 6 = 36, suggests that he was able to make clear and convincing progress.

Figure 9



Note how he has recognised the potential of doubling to solve 12 x 6 (he did not know this as a fact) and also his ability to use other known facts to help solve related problems. For instance, he uses 6 x 6 = 36 to find 7 x 6 = 42 by adding 6 on (confirmed by his verbal commentary) and thence to generate 70 x 6 = 420. He explained that this has to be true since 70 was ten times larger than 7.

Over time, Student D displayed an ever-increasing ability to see the relationship between questions and strengthening insight into how this may be advantageous in solving more complex problems. Although less secure in this respect when decimals became involved, Figures 10 and 11 highlight how, even when faced with more significant place value issues, he began to apply his skills and knowledge more confidently. Although this obviously generated potential scope for error, he maintained an interesting determination to deal mentally with the challenges. His response to 1.7 + 0.67 illustrates this was not always plain sailing for him!



Seeing the Connections

Question	Jottings	Answer	Check
34 + 12	46	46	
0.34 + 0.12	0.46	0.46	
0.34 + 12	12.34	12.34	
3.4 + 0.12	3.52	3.52	
1.2 + 34	35.2	35.2	
1.2 + 0.34	1.54	1.54	

Question	Jottings	Answer	Check
17 + 67	84	84	
0.17 + 0.67	0.84	0.84	
0.17 + 67	67.17	67.17	
1.7 + 0.67	1.137	1.37	2.37
6.7 + 17	~~6.87~~		
6.7 + 0.17	~~6.7~~	6.87	

Figure 10

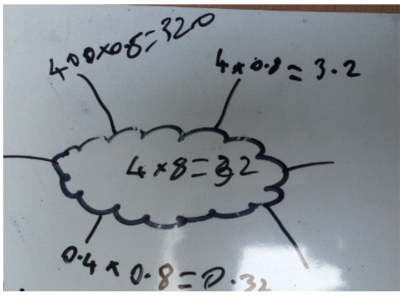


Figure 11

Implications and next steps

Obviously, given the small sample size, and the relatively short duration of the project (approximately 4 months), analysis of outcomes and drawing of any conclusions need to be tentative. I dare, nonetheless, to share the following thoughts!

Probably my most significant observation about these ‘significantly challenged learners’ within the field of mental number work would be the vital importance of creating positive dispositions to learning and the belief, both on the part of the student and the teacher, that progress can be made. A readiness to persist when the going got tough also proved to be key in securing progress with in-school activities. Resilience did not appear to be an issue for these learners and responses from all four to in-class activities consistently suggested that they were prepared to be tenacious even when faced with the challenges of dealing with calculations involving decimals.

However, experience of working with *The Gang* testified that progress was consistently rooted in a readiness not just to engage positively in class but also to find the motivation to extend their efforts autonomously beyond the classroom. Interestingly, the student who claimed to have worked most regularly and consistently at home, Student D, made most progress. In his final assessment interview he managed, independently, to complete 22 complements to 100 when, in his initial interview, he had struggled to succeed with any until I supported. He also demonstrated a fluent ability to bond numbers with one decimal place to 10 in a way that, at earlier stages in the project, he simply could not. Conversely, Student B, who probably worked least reliably between sessions (she also temporarily lost her resource pack!) made least progress. That said, her final interview assessment, confirmed she had learned to become very efficient with *connectionist* thinking. Clearly, responsibilities for generating positive attitudes to learning rest with both the learner and the teacher. However, the project did not consider the potentially key role of parental support in any focussed way, and I sense this element of responsibility within the learning process warrants more thought and exploration. After all, without support, autonomy and resilience are characteristics which may be difficult for any learner to develop and maintain at the best of times.

All four students confirmed that they had found the pack of resources supportive. They commented on the engaging nature of the IT resources and on how these had encouraged them in particular to practise multiplication tables as well as to think more confidently about decimals. The *Connection Cloud* board also became a firm favourite. Arguably, this proved to be the key resource for scaffolding learning. Consistent evidence emerged that this resource enabled students not just to ‘see’ [(Mason (2006)] the connections between numbers and calculations but also, crucially, to take control of creating connections in order to solve problems. Final assessments confirmed that all four had become much more aware of the potential of using known facts to generate further facts using the *Connection Cloud* boards to support. From my perspective, the Gattegno (place value) grids contributed significantly to the students’ appreciation of the impact of multiplication or division of decimals by 10 or 100. In a very simple way, it enabled me to focus the students’ attention on the movement of non-zero digits rather than on the appearance or disappearance of zeros.

On reflection, my experiences of working with *The Gang* allow me to offer a hopeful response to my original questions about students who feel ‘significantly challenged’ in their relationship with number. Within one narrow focus of Russell’s interpretation of *computational fluency* i.e. efficiency, accuracy and flexibility in approach to mental calculation, these students can unquestionably secure a stronger fundamental knowledge base and, crucially, build higher levels of self-confidence. That said, my question also includes the word ‘sustained’. That goes beyond the remit of this project and remains to be seen! Without doubt, my experiences imply there is no inevitability to the prospect of progress fading for these students but, as argued before, the key dispositions of commitment and resilience appear vital.

From the perspective of a secondary school my observations appear to argue a strong case for a targeted and systematic approach over time if progress is to be sustained. A cursory glance at the content of current GCSE papers would confirm the importance and advantages of strong numeracy skills for students of all abilities. Implicitly, this point would also seem to substantiate the view that positive and effective collaboration between Learning Support and mathematics teams is crucial. Bearing in mind that developing computational fluency for all its students, let alone for those who feel 'significantly challenged', is but one small aspect of a mathematics team's work, the key collaborative role of a school's Learning Support team cannot be under-estimated.

A final thought from one of *The Gang*. On being challenged to calculate 41 x 7 from 40 x 7 = 280, a fact she had generated from being given 4 x 7 = 28, she proceeded to use short multiplication to calculate the correct answer of 287. On completion of the written calculation, she looked at me, smiled and commented, 'I have just noticed. I could have just added 7.' Indeed, there is hope!

References

Askew M. (2010) *A response to Girling and Zarzycki*. Accessed 12.30 28/06/08 at www.atm.org.uk

Bandura, A. (1993) Perceived Self-Efficacy in Cognitive Development and Functioning, *Educational Psychologist*, 28(2), pp117-148

Dweck, C.S. (2000) Self-theories. Their Role in Motivation, Personality and Development. Abingdon: Routledge

Lai, M.Y., Murray, S. (2012) *Teaching with Procedural Variation: A Chinese Way of Promoting Deep Understanding of Mathematics.* Accessed 14.15, 09/07/18 at http://www.cimt.org.uk/journal/lai.pdf

Marton, F., Runesson, U., & Tsui, A. (2003). *The space for learning*. In F. Marton & A. Tsui (Eds.), Classroom discourse and the space for learning (pp3-40). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Mason, J, Stephens, M.and Watson, A. (2009) Appreciating Mathematical Structure for All, *Mathematics Education Research Journal 2009, Vol. 21, No. 2, pp.10-32*

Mason, J., Nilsson P., Ryve, A. (2012) Establishing mathematics for teaching within classroom interactions in teacher education, *Educational Studies in Mathematics, 9/1/2012, Vol. 81, Issue 1, pp.1-14*

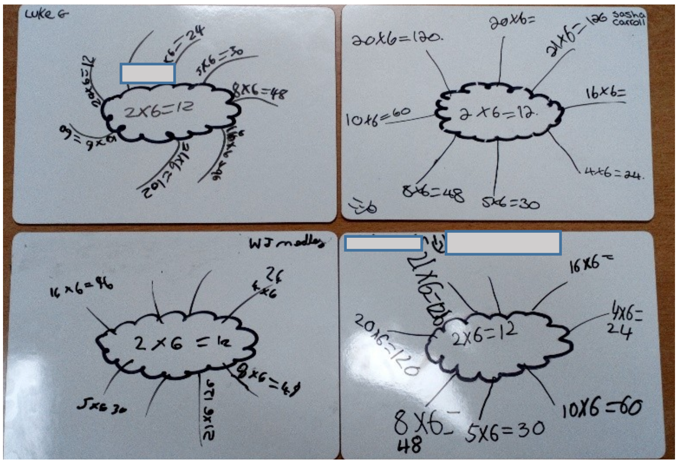
Marton, F., Runesson, U., & Tsui, A. (2003). The space for learning. In F. Marton & A. Tsui (Eds.), *Classroom discourse and the space for learning* (pp 3-40). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

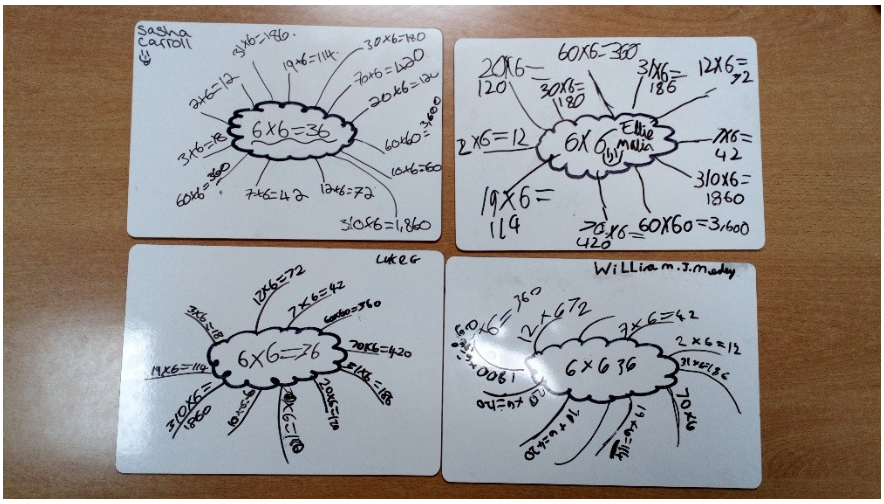
Marton, F., Pang, M.F. (2006) On Some Necessary Conditions of Learning, *Journal of the Learning Sciences (Vol. 15, Issue 2, pp. 193-220)*

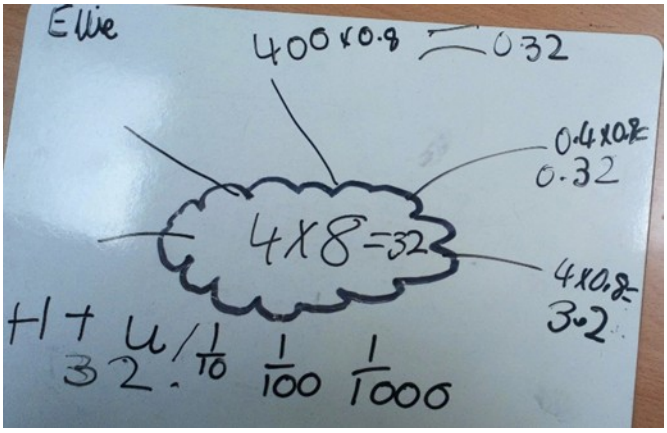
Marton, F., Pang, M.F. (2013) Meanings are acquired from experiencing differences against a background of sameness, rather than from experiencing sameness against a background of difference: Putting a conjecture to the test by embedding it in a pedagogical tool, *Frontline Learning Research 1 (2013) 24 - 41*

Russell, S. (2000) Developing computational fluency with whole numbers. In: *Teaching Children Mathematics. Nov 2000, Vol. 7 Issue 3, 154; National Council of Teachers of Mathematics, Inc. , Database: Academic OneFile*

Watson, Anne and Mason, John (2006). Seeing an exercise as a single mathematical object: using variation to structure sense-making. *Mathematics thinking and learning, 8(2), pp. 91–111.*


Luke G
2x6=12
5x6=30
8x6=48
Sasha Carroll
20x6=120
2x6=12
10x6=60
16x6=
4x6=24
5x6=30
WJ medley
2x6=12
16x6=96
5x6 30
2x6=12
20x6=120
16x6=
4x6=24
10x6=60
5x6=30
8x6=48


Sasha Carroll
6x6=36
2x6=12
3x6=18
7x6=42
12x6=72
19x6=114
31x6=1,860
Ellie
6x6
60x6=360
2x6=12
19x6=114
7x6=42
12x6=72
310x6=1860
60x60=3,600
Luke G
6x6=36
12x6=72
7x6=42
3x6=18
19x6=114
70x6=420
William.J.medley
6x6 36
2x6=12
7x6=42


Ellie
4x8=32
0.32
0.4x0.8
0.32
4x0.8
3.2
H T U / 1/10 1/100 1/1000
32.