OPTIMUM DESIGN OF REINFORCED CONCRETE SKELETAL SYSTEMS USING NON-LINEAR PROGRAMMING TECHNIQUES

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A thesis submitted in partial fulfilment of the requirements of the University of Derby for the degree of Doctor of Philosophy

by

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TO

MY MOTHER

To the World you might be just one person,

but to me you might just be the World.

AND FAMILY

For their love, encouragement,

sacrifice and understanding.

As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.

Albert Einstein (1879-1955)

Abstract

The last 30 years has seen a rapid development in a wide variety of different techniques and approaches applied to solve optimisation problems in structural design. However, recent surveys have shown insufficient penetration of optimisation methods in structural design practice, especially in the field of optimum design of realistic reinforced concrete structural systems.

This research investigates new approaches in the use of cost-efficient optimisation and applies these to the multi-level design of skeletal systems. It has produced implementation theory and general computer programs for the automatic optimum (improved) design of realistic, rigidly jointed reinforced concrete structures based on the ultimate limit state theory, as embodied in BS 8110. Guided by the adopted problem-seeks-optimum design approach and conscious of its practical application, this research provided novel approaches to the development of realistic cost objective function formulations, to design constraint handling, to the assessment of multiple and worst-scenario loading arrangements and to the development of a model that groups structural elements in a manner that both improves efficiency of the optimisation algorithm and mirrors design office practice.

The validity of structural optimisation has been established, dependent directly on the balance between the mathematical model of the objective function and the design constraints, the algorithm that is applied, and the physical reality of the structural problem and its practical application. Hence, taking into consideration the main components of skeletal system superstructures and substructures, suitable optimisation algorithms were implemented, and their performance reported to the structural problem formulation and the limitations encountered. The research has shown that optimising structural problems with single load case does not give a realistic minimum cost of a structure, and that frames consisting of multiple beam and column groups in general produce a more cost efficient design. Further work is suggested both for improving structural problem formulations and for implementing appropriate optimisation techniques.

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Related Publications

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Ceranic, B. and Fryer, C., (1999), 'Sensitivity Analysis and Optimum Design Curves for the Minimum Cost Design of Singly and Doubly Reinforced Concrete Beams', accepted to *Structural Optimisation*

Ceranic, B. and Fryer, C., (1999), 'An Application of Simulated Annealing to the Optimum Design of Reinforced Concrete Retaining Structures', *Optimisation and Control in Civil and Structural Engineering*, Civil-Comp Press, ISBN 0-948749-62-8, pp.47-53

Ceranic, B. and Fryer, C., (1999), 'An Application of A Real Number Micro-GA's to the Shape Optimisation of In-Plane Stressed Plate Elements', accepted to *2nd International Conf. On Engineering Computational Technology*, Belgium

Ceranic, B. and Fryer, C., (1999), 'A Constrained Simulated Annealing Approach to the Minimum Cost Design of Reinforced Concrete Cantilever Retaining Walls', *Proc.of 3rd World Congress on Structural and MultiDisciplinary Optimisation*, New York, USA, pp.593-595

Ceranic, B. and Fryer, C., (1998), 'A Genetic Algorithm Approach to the Minimum Cost Design of Reinforced Concrete Flanged Beams under Multiple Loading Conditions', *Proc.of International Australasian Conference on Structural Optimisation*, Sydney, Australia, pp. 71-78

Fryer, C., Ceranic B., (1997), 'Optimum Design of Reinforced Concrete Skeletal Structures: A Comparative Study between Volume and Cost Optimisation', *Proc.of 2nd World Congress on Structural and MultiDisciplinary Optimisation*, Zakopane, Poland, Vol. 1, pp. 731-737

Ceranic, B., Fryer, C., Baines, R.W., (1999), 'An Application of Simulated Annealing to the Optimum Design of Reinforced Concrete Retaining Structures', submitted to *Computers&Structures*

Ceranic, B. and Fryer, C., (2000), 'Minimum Cost Design and Sensitivity Analysis of the Reinforced Concrete Slabs with Different Supporting Conditions using Genetic Algorithms', to be submitted to *International Journal of Structural Engineering - ASCE*

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Notations

a	Height of the retaining wall heel beam
A_c	Area of concrete
A_s	Area of tension reinforcement
A_s'	Area of compression reinforcement
A_{sc}	Area of column reinforcement
b	Breadth of the beam section
b^0	Breadth of beam evaluated at the initial design point
bc	Breadth of the column section
bc^0	Breadth of column evaluated at the initial design point
bc_{opt}	Optimum breadth of the column section
b_f	Effective width of flange
b_{max}	Upper bound on section breadth
b_{min}	Lower bound on section breadth
b_{opt}	Optimum breadth of the beam section
b_w	Breadth of the web
c	Cover to reinforcement
C_c	Unit costs of concrete
C_{jk}	Relation coefficient for the main reinforcement between non-critical (k-th) and critical beam in j-th beam group
C_s	Unit costs of steel
d	Effective depth of the beam section
d'	Depth from the top of compression face to centroid of compression reinforcement
d_1	Depth of the retaining wall base
$Df(x_o)$	Function gradient vector evaluated at the initial design point
d_{min}	Minimum effective depth
d_{opt}	Optimum effective depth
d_w	Height of the retaining wall stem
E_{si}	Number of expected copies for i-th population member
$f(x_o)$	Function evaluated at the initial design point
f_1	Stress in the column top reinforcement set
f_2	Stress in the column bottom reinforcement set
f_{cu}	Characteristic concrete strength
f_{ub}	Upper bound bending stress
f_y	Characteristic strength of steel
$G_b\{M\}$	Bending constraints for beams
$G_c\{C\}$	Stress constraints for columns
G_k	Characteristic dead load
$g_k(x_i)$	Equality constraints
$G_s\{M\}$	Shear constraints for beams

H	Height of the column
h	Overall depth of the beam section
h^0	Depth of beam evaluated at the initial design point
hc	Depth of the column section
hc^0	Depth of column evaluated at the initial design point
hc_{opt}	Optimum depth of the column section
h_f	Height of the flange
$h_j(x_j)$	Inequality constraints
h_{max}	Upper bound on section depth
h_{min}	Lower bound on section depth
h_{opt}	Optimum depth of the beam section
k_{1i}, k_{2i}	Moving limits for design variables
L	Length of the beam
$\mathrm{L}(x_i, \lambda_j)$	Lagrange multipliers
l_1	Length of the retaining wall toe
l_2	Length of the retaining wall heel
L_x	Shorter slab span (x-direction)
L_y	Longer slab span (y-direction)
M	Ultimate design bending moment
N	Population size
N	Ultimate design axial force
NBBG	Number of beams in beam group
NBG	Total number of beam groups
NCCG	Number of columns in column group
NCG	Total number of column groups
NCRG	Number of columns in column reinforcement ratio group
NRG	Total number of column reinforcement ratio groups
p_c	Probability of the crossover
p_m	Probability of the mutation
p_s	Probability of the selection
p_{si}	Probability of selection of i-th population member
q	Material cost ratio (C_s / C_c)
Q_k	Characteristic imposed load
r	Ratio of reinforcement cover to effective depth d
r_j	Rank of j-th population member
r_a	Gradient of the area column reinforcement ratio contours
r_g	Global value of r_a
r_p	Value of r_a at the intersection of K = 1 and zero reinforcement contour
T	Current system temperature
T_o	Initial system temperature
V	Ultimate design shear force
V	Volume objective function
v_{max}	Upper bound shear stress
w_1	Width of the retaining wall stem at the top

w_2	Width of the retaining wall stem at the bottom
w_b	Width of the retaining wall heel beam
x	Depth of the neutral axis
x_i^o	Initial design point
z	Lever arm
Z	Cost objective function
$Z(x_o)$	Cost function evaluated at the initial design point
Z_c	Total cost of concreting
Z_f	Total cost of formworking
Z_s	Total cost of reinforcing
β	BS8110 moment coefficient for slabs
α	Cooling rate
ρ	Reinforcement ratio for tension steel (A_s / bd)
ρ'	Reinforcement ratio for compression steel (A_s' / bd)
ρ_{bound}	Boundary reinforcement ratio
ρ_c	Column reinforcement ratio
γ_m	Partial safety factor for material strength
ρ_{max}	Maximum reinforcement ratio
ρ_{min}	Minimum reinforcement ratio
α_{mn}	Stress factorised column reinforcement ratio ($\rho_c f_y / f_{cu}$)
ρ_{opt}	Optimum reinforcement ratio
$\nabla Z(x_o)$	Cost function gradient vector evaluated at the initial design point
$[D]$	Displacement transformation matrix
$[k]$	Unassembled element stiffness matrix
$[K]$	Overall stiffness matrix
$\{W\}$	Vector of external joint loads
$\{X\}$	Vector of joint displacements
$\{x_i\}$	Vector of the new unknown cross-section variables
$\{x_o\}$	Vector of known cross-section variables evaluated at the initial design point

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1.

Introduction

This chapter provides an overview of the research aims and objectives, it outlines the scope of the work and structure of the thesis. It also addresses the importance of optimum or improved design concepts for realistic reinforced concrete structures. The reasons for the lack of penetration of structural optimisation into practical design of concrete structures are highlighted.

1.1 Overview

Computer technology has made it possible for engineers to design complex reinforced concrete structures without direct recourse to cumbersome mathematics. This has led to a growth in commercially available design programs. However, their use relies either on the individual designer's intuition, or in a few cases, on some form of heuristic or knowledge based expert system as a decision-making support tool. As part of the complex analysis and design process, the designer is required to make certain assumptions about the cross sectional properties of the structural members, and then carry out an analysis to check if the structure satisfies the design requirements. If not, a new assumption is made based on the previous analysis, and the whole process is repeated until the design converges to an acceptable solution. Such an approach is best described as a repeated *check analysis* or conventional heuristic approach, in which the most important part of selecting the sections depends upon the designer's intuition and experience. Every trial involves considerable computational effort, and often it is not clear which direction will lead the designer towards a more economical structure.

In structural optimisation, this conventional heuristic approach is replaced by a systematic, goal-oriented design process. The nature and ways of arriving at the optimum solution differs according to the type of the optimisation algorithm employed. However, the common feature for each algorithm is that the computer becomes central for searching and sorting through the similar design concepts to achieve the most economical design. In general, optimisation arrives at a design that the engineer could, equally well, have obtained if he/she were prepared to invest the time and money to search among all of the design alternatives. The principal advantage of structural optimisation should, therefore, be its saving in design time and cost. By combining structural analysis and design into a single process it is possible to eliminate much of the computer input/output and the costly data handling, whilst producing an economical structure that satisfies the design requirements. Furthermore, at the preliminary design stage, the designer could more easily investigate different types of structures, taking into account economical design as an objective to be achieved.

Considering all these facts, it would seem logical to assume that structural optimisation should have a substantial part to play in everyday design practice, and become a standard design tool.

1.2 Optimisation of Reinforced Concrete Skeletal Structures

With the development of ultimate limit state theory in the late 1960's and introduction of the corresponding standards, reinforced concrete structures underwent a radical change in design philosophy. This was done by applying partial factors of safety, both to the loads and to the material strength, and allowing them to be varied so that they may be used either with plastic conditions in the ultimate limit state or within the elastic stress range at service loads. Subsequently, researchers have concentrated almost entirely on investigating the strength of structural components, or proposing more accurate methods of analysis. In comparison with steel structures, work on the optimal design of reinforced concrete structures can only be found in isolated papers and publications. Steel is by far the most popular material in structural optimisation studies, due to the fact that its material properties and homogeneity are easier to model. According to the extensive structural optimisation survey by Cohn (1995), reinforced concrete publications represent only some 4% of the overall reviewed work. This may be due in part to the more complex nature of concrete design, reflecting the difficulties associated with a composite material. Furthermore, minimum material volume (minimum weight) optimisation gives a reliable indication of minimum costs for steel structures. This is not necessarily the case for reinforced concrete structures, as minimising the volume of concrete does not take into account the difference between the unit volume costs of reinforcement and concrete.

Cohn and Dinovitzer (1994) conclude that 'It seems reasonable to assume that optimisation could become more attractive to practising designers if more examples of its application to reinforced concrete were available, especially for realistic structures, loading conditions, and limit states'.

1.2.1 *Identification of Structural Forms and Design Constraints*

To study the advantages and limitations of structural optimisation for reinforced concrete skeletal structures, it is essential to categorise the basic structural elements and forms encountered in civil engineering. In this way, a clearer picture of the suitability of different non-linear programming techniques for identified structural problems can be obtained, and how these may be incorporated into an optimisation system. The basic structural elements may be identified as beams, columns, slabs and walls either as independent structural members or as part of a rigidly jointed frame structure.

1.2.2 *The Structural Design Process*

The aim of structural design is to achieve structures that satisfy the requirements of the client at an acceptable cost whilst ensuring safety of the structure under the worst loading conditions. Furthermore, under normal working conditions the designer must ensure that the deformation of a structure does not affect its appearance, durability or performance. Despite the difficulty in assessing the precise loading and variations in the strength of concrete and steel, these requirements have to be met.

For reinforced concrete structures, the current British Standards (BS8110 1985) and European Standards (EC2 1992) are based on the *limit state* design method whereby the probability of the structure becoming unfit for its intended use should be acceptably low. Limits states are divided into two classes; *ultimate limit states* are those which, when exceeded, result in partial or total collapse of the structure, and *serviceability limit states* are those which, when exceeded, do not cause collapse, but leave the structure in an unserviceable condition.

The overall design process usually consists of three main stages; planning and conceptual structural design, structural analysis, and design and detailing of the members.

1.2.3 *Methods of Structural Analysis*

Structural optimisation depends on the accuracy of structural analysis. The method of *subframes* explained in BS8110 offers a simplified approach to the analysis of frame structures, suitable for *hand* calculations but at the expense of accuracy. For the analysis of complete structures, there are two well established methods when formulating the optimum design problem. The matrix *force* (or *flexibility*) method involves the concept of redundancies, consequently it is not equally efficient for statically determinate and indeterminate structures. Although it can involve the solution of a smaller number of equations than the *stiffness* method, it is not conducive to computer programming because the choice of redundants is not unique. The *stiffness* or *joint displacement* method however, expresses the internal forces in terms of the joint displacements. Once the analytical model of a structure has been defined, no further engineering decisions are required. This is now the primary method used in matrix analysis of structures due to the fact that it does not involve the concept of redundancies and can be easily automated. For these reasons the *stiffness* method is employed in this research.

1.2.4 *Choice of the Optimisation Method*

Reinforced concrete structures can be described as a set of quantities defining topology, material properties, configuration, loading conditions, cross-sectional dimensions and percentages of reinforcement. In the optimum design method topology, material properties, configuration and loading conditions are pre-assigned, whilst member sizes and areas of reinforcement are treated as continuos design variables. The design objective is to minimise a corresponding structural volume or cost function whilst satisfying constraints imposed on these variables, defined by the following set of design requirements:-

i) The *stress* constraints, which ensure that the stresses in each member do not exceed the values calculated with respect to the ultimate limit state theory, as embodied in British Standards *BS 8110* and European Code *EC2*.

ii) The *deflection* constraints, which keep the joint deflections below their specified allowable values.

iii) Upper and lower bound *dimension* constraints, which keep the section dimensions between specified boundary values.

iv) Upper and lower bound *reinforcement ratio* constraints, which keep the reinforcement percentages between specified boundary values.

v) The *stiffness* constraints which ensure the basic equilibrium of the structure itself.

vi) Additional constraints which could be considered in specific structural cases. For example *cracking*, *buckling*, *instability*, *fatigue*, etc.

Once the structural problem and the design constraints have been identified, the question of the optimisation method has to be addressed. Research into suitable optimisation techniques that take into account the complex nature of reinforced concrete, and can be cognisant of practical design methodology and its implementation is required.

1.3 Aims, Objectives and Scope of the Research

The aim of this research is to adopt a *problem-seeks-optimum design* approach for skeletal reinforced concrete structures to produce a researcher-developed, but practice-orientated, robust, reliable and efficient optimisation code. The research will use the design procedures and analytical methods that are familiar to the designer, when applied to realistic reinforced concrete skeletal structures. In this way, the designer is more likely to recognise the significant improvements that may be achieved by the systematic and goal-orientated design method offered by structural optimisation.

This research considers a representative selection of reinforced concrete structural forms and design conditions to test the proposed optimisation methods under practical circumstances. It is not the intention of this research work to produce commercial optimisation software, but to investigate the application of structural optimisation through a number of graded representative problems. However, the structure of the computer software developed in this research has been designed to allow for modification of objective functions and inclusion of other design constraints, if so required.

1.4 Research Methodology

To achieve the stated aims, the research methodology undertaken included:-

(i) Identify design constraints associated with realistic reinforced concrete skeletal structures, loading conditions and limit states using relevant structural design theory and codes of practice (BS8110).

(ii) Incorporate these constraints into the global minimum volume design of 2D skeletal structures using non-linear optimisation techniques and novel approaches to the multi-level optimisation.

(iii) Investigate methods for simplifying the optimisation process by developing an implementation theory for retaining the critical constraints only, and undertake the testing and sensitivity analysis to evaluate its performance.

(iv) Derive novel objective functions and implement suitable optimisation methods for the minimum cost design of reinforced concrete beams, columns, slabs and retaining walls. Investigate how to incorporate these within an elemental optimisation procedure as part of a multi-level optimisation process.

(v) Critically assess the advantages and limitations of the implemented optimisation methods by evaluating both their suitability and performance with regard to the developed objective functions and problem formulations.

(vi) Design a suite of computer programs and algorithms that combine analysis and optimum design with the practical requirements of economy and buildability. Validate by comparison with the exhaustive search of standard design solutions and with commercial software packages where appropriate.

1.5 The Structure of Thesis

The thesis is divided into eight chapters and a brief description of each is given below

Chapter 1 places the research in context, states the aims, objectives, the research methodology and outlines the structure of the thesis.

Chapter 2 reports on a study of structural optimisation literature with emphasis on the application in the domain of realistic reinforced concrete skeletal structural systems.

Chapter 3 introduces an approach, developed in the research, to the application of the Lagrangian Multiplier Method to the minimum material cost design of singly and doubly reinforced concrete beams. Derivation of the stress constraints and cost objective functions for both cases are given, together with an outline of how the approach can be implemented.

Chapter 4 develops a methodology to formulate a volume optimisation approach for reinforced concrete skeletal structural systems. The modified sequential linear programming method is implemented and includes constraint handling based on the principle of retaining only the critical constraints. The methodology is developed to formulate the structural optimisation problem and to derive stress constraints according to British Standards BS 8110. Sensitivity analysis and detailed testing are performed and results reported. The method is applied to a number of structural examples and its performance discussed.

Chapter 5 extends the methodology to include the minimum cost design of the aforementioned structural systems under multiple loading conditions and limit states. The research reported in Chapter 4 provides the basis for investigating minimum cost design, incorporating many of the algorithms and techniques that were developed and tested in Chapter 3 and 4. A novel approach is developed for formulating the objective function and for the grouping of structural members within the multi-level optimisation process. A suite of computer programs is developed, extensively tested and cost sensitivity analysis is performed. The method is compared with standard design and genetic algorithm solutions obtained using commercial software, and results are extensively tested and analysed. Comparison between volume and cost optimisation is carried out and the cost sensitive parameters in volume optimisation algorithm are identified.

Chapter 6 describes the further developments of the cost objective function and problem formulation, investigating the implementation of genetic algorithm (GA) code to overcome the limitations encountered using traditional mathematical programming approaches. A new approach to the formulation of objective function is introduced, incorporating the additional costs associated with labour and formwork. Multiple loading conditions are also considered and implemented into the optimum design. A fast re-analysis approach of the population members for frame structures is developed. Different GA operators and parameters are proposed and these are tested on a range of structural problems. The results of detailed testing and cost sensitivity analysis are reported.

Chapter 7 describes the minimum cost design of reinforced concrete cantilever walls as constituent parts of skeletal system substructures, investigating the application of the simulated annealing (SA) algorithm. The proposed implementation of the algorithm is a highly practical approach to the design process, incorporating realistic loading conditions and limit states, together with material and labour costs associated with concreting, reinforcing and formworking. A new probabilistic weight estimate (PWE) approach to the constraint handling is introduced and its performance compared with existing approaches. Testing of simulated annealing control parameters and cost sensitivity analysis is performed, reporting on the suitable configurations for these types of structural systems.

Chapter 8 gives conclusions on both the developments of the structural problem formulations, and the application of suitable techniques for the optimum or improved design of realistic reinforced concrete skeletal structural systems. Recommendations for further research are included, with possible directions for future development.

2.

Literature Survey

This chapter provides an overview of research in the field of structural optimisation, paying particular attention to reinforced concrete structures. An evaluation of different structural optimisation approaches is conducted, identifying their main advantages and limitations. A 'problem-seeks-optimum design' approach is discussed and its significance to this research is highlighted.

2.1 Current Structural Optimisation Research

The last 30 years of modern mathematical optimisation has seen developments in a wide variety of different techniques and approaches applied to structural design. However, recent surveys have shown a disappointing penetration of optimisation methods in the field of optimum design of realistic 2D/3D reinforced concrete structural systems. Structural optimisation can be defined as the development and application of practice-orientated, interactive and automated computer techniques and software for improving designs within defined costs and constraints. Such designs consider a structure at the lowest cost, with the objective of fulfilling a specific purpose. They must also consider safety, service life, maintenance, aesthetic requirements and future adaptability. In essence, structural optimisation is a design concept that replaces a conventional *trial-and-error* approach by a systematic, goal-orientated design process.

2.1.1 *Overview*

Modern structural optimisation (SO) began in early 1960's when Schmit (1960), first combined finite element analysis with a non-linear numerical optimisation method to create what he called 'Structural Synthesis', *see* Figure 2.1.

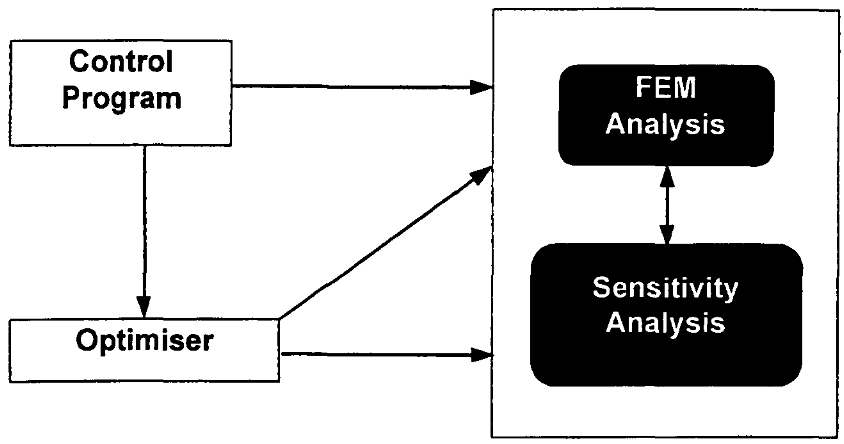


Figure 2.1 *Pre- 1970's SO methods*

During the 1960's and 1970's, the dominant research focused on optimisation algorithms and techniques. Aspects relating to selection, convergence and economy were investigated, with the majority of problems being theoretically and dimensionally small. Research was also concerned with improved algorithms to decrease computation time. A wide gap existed between numerical and analytical optimisation. Post-1974 methods (*see* Figure 2.2), as classified by Vanderplaats (1993), introduced structural analysis approximations and constraint screening to create an optimum design method comparable to the conventional heuristic approach, but using mathematical techniques to obtain an optimum solution. He states that 'to make best use of the most advanced approximation methods, it is necessary to create combined analysis/optimisation software from the beginning to be a fully integrated capability, rather than adding optimisation to an existing program'.

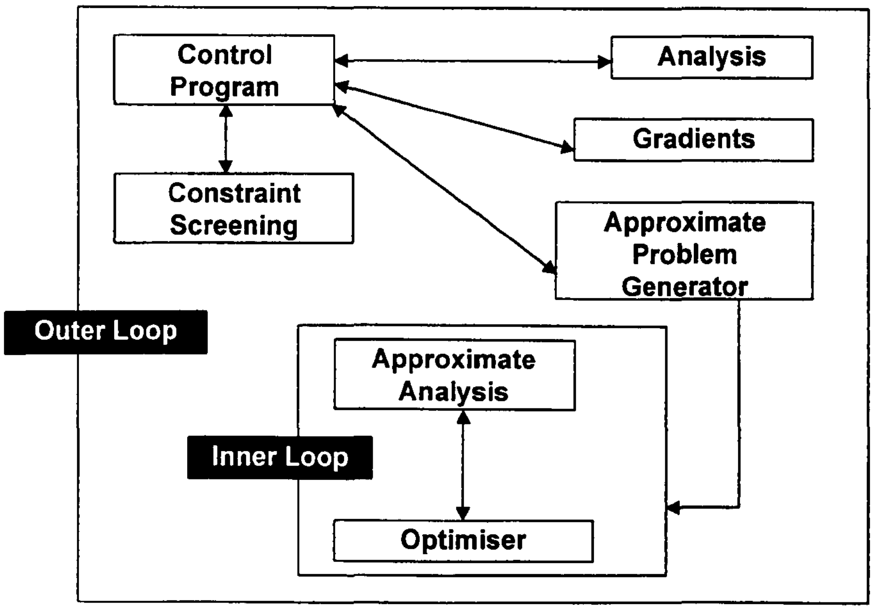


Figure 2.2 *Post-1974 SO methods*

In a recent extensive survey, Cohn (1995) noted that the vast majority of the published work dealt with the mathematical *solution-seeks-problem* rather than the engineering *problem-seeks-optimum design* aspect of optimisation of structures (*see* Figures 2.3 and 2.4). In this context, the following relevant findings of Cohn's (1995) survey are highlighted:

1. Reinforced/prestressed concrete and composite structures are mostly found in isolated papers, representing only some 4% of the reviewed structural optimisation examples from the catalogue of over 500 selected entries.
2. With few exceptions, multiple and worst-scenario loading arrangements are yet to find their way into structural optimisation.
3. Structural optimisation could become more attractive to practising engineers if its application to an increasing number of reinforced concrete examples were investigated, especially for realistic structures.
4. Recognised need for research-developed but practice orientated, user-friendly and automated structural optimisation code, addressing the designer's tendency to favour the *problem-seeks-optimum design* approach.

These findings are representative of current research in the field of reinforced concrete structural optimisation, expressing some of the possible causes of a disappointing penetration in professional design practice.

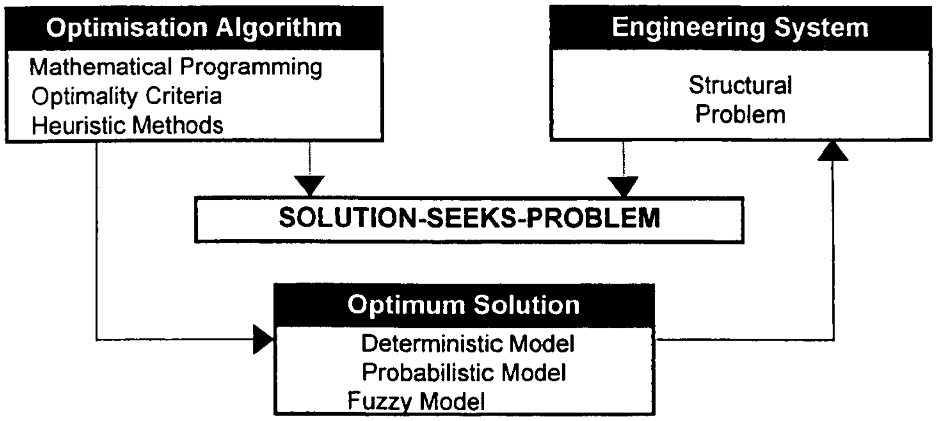


Figure 2.3 *Mathematical Optimisation*

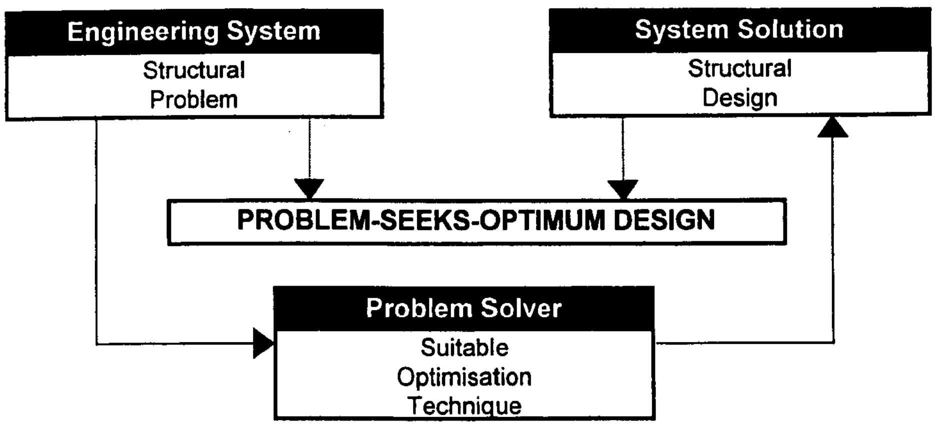


Figure 2.4 *Engineering Optimisation*

To address these issues it will require a short and long-term strategy. The former requires the development of practice-orientated computer software for the optimum design of specific reinforced concrete structural systems, following the ***problem-seeks-optimum design*** approach presented in Figure 2.4. The latter requires the further education of structural engineers to familiarise themselves with optimisation techniques and concepts.

2.1.2 *Literature Survey Approach*

Research into the structural optimisation of realistic reinforced concrete structures is reported sporadically and in mainly isolated papers. Publications differ in the type and size of structural problems considered and the optimisation methods employed. This literature survey is specifically on the formulation of the objective function, the identification of the design constraints, the assemblage of the programming problem and the selection of suitable optimisation algorithms. It is conducted in cognisance to a practical design methodology and its implementation in the design office. The key factors in formulating the structural optimisation problem, identified from the literature survey are as follows:

(i) Type of optimisation and uncertainty level, *i.e.* deterministic or probabilistic

(ii) Identification and formulation of the structural problem

(iii) Consideration of single and multiple loading conditions

(iv) Identification of design variables and assemblage of objective function

(v) Consideration of serviceability limit state (SLS) and ultimate limit state (ULS) design

(vi) Formulation and handling of design constraints

(vii) Selection of suitable optimisation technique

The inter-relationships between these key factors are presented in Table 2.1

Uncertainty Level	Variables and Parameters			Formulation of Structural Optimisation Problem		Optimisation Techniques
	Geometry	Loading	Material	Objective	Constraints	
Deterministic	Section Member Structure Structural Layout Complex Systems	Single Load Case Multiple Load Case	Reinforced Concrete	Single Multiple	SLS Stress Deflection Cracking Fatigue Buckling Local Damage	Mathematical Programming Optimality Criteria
Probabilistic		Static *Dead* *Imposed* *Wind* Dynamic	Elasto-Plastic	Economy *Minimum Weight* *Minimum Costs*	ULS Collapse Instability	Heuristic Methods

Table 2.1 *Factors in RC Structural Problem Identification*

The formulation of the structural optimisation problem and the choice of the variables and parameters are problem dependent and specific to the type of reinforced concrete structure being investigated. The selection of a suitable optimisation method is also influenced by the specific nature of the underlying problem. Hence, this literature survey includes papers on a range of optimisation techniques, fully exploring their advantages and limitations. Although most of these techniques have been applied to steel structures, the concepts behind their implementation can be translated to the optimum design of reinforced concrete structural systems. This literature survey therefore concentrates on the application of optimisation methods to both steel and reinforced concrete structures. The majority of the published work falls into two main categories, generic and problem specific. The former is mainly concerned with the mathematical aspects of those methods, investigating the techniques for improved efficiency and robustness. The latter however, investigates the implementation of those methods to specific structural optimisation problems, analysing different formulations and their effectiveness. These classifications are important when investigating the suitability of optimisation methods, and hence the literature survey reports on both.

Optimisation methods can be classified into mathematical programming, optimality criteria or modern heuristic methods. Implementation of these methods for different types of structural problems requires additional decisions to be made, so that they may have different performances in practice. Moreover, the combinations of these fundamental strategies as reported in the literature, make the classification of non-linear programming techniques even more difficult. Hence, the selection of suitable optimisation methods is problem dependent and therefore requires a thorough examination of existing non-linear programming techniques.

2.2 Mathematical Programming Methods

Published literature on structural optimisation overwhelmingly reports on the application of traditional non-linear optimisation techniques based on mathematical programming. One of the primary tasks associated with this approach is the determination of the gradients of both the objective function and the design constraints. In general, the objective function and the constraints are non-linearly dependent on the design variables, and in some cases this relationship may be highly non-linear. Once the gradients for a particular design stage are known, a number of optimisation methods may be selected. Reviews such as Cohn (1994), classed mathematical programming methods as 'extensively surveyed and thought to be the most general and powerful optimisation approaches', based on explicit formulations of design objectives and constraints. However, these methods can have serious limitations, mostly related to convergence and computational efficiency for large-scale problems.

Mathematical programming methods surveyed in this research can be divided into the following sub-classes:

(i) sequential linear programming (SLP) methods
(ii) sequential quadratic programming (SQP) methods
(iii) penalty methods (PM)
(iv) multiplier methods (MM)
(v) geometric programming (GP)
(vi) generalised reduced gradient methods (GRGM)

2.2.1 *Sequential Linear Programming (SLP)*

Research on the application of SLP, although mostly published on steel structures, has given a valuable insight into the implementation of the method. Erbatur and Al-Hussainy (1992) reported on the simple application of SLP for weight optimisation of 2D steel structures, avoiding structural analysis and allowing for the automatic selection of steel sections. Akbora et al. (1993) studied the optimisation of steel structural frames with elastic and plastic constraints, formulating the problem to be solved in direct linear form. Other researchers reported on the generic issues of the SLP method, such as Chen (1993), who investigated the calculation of move limits for SLP, essential for the performance and reliability of the algorithm. Mulkay and Rao (1998) investigated fuzzy heuristics for SLP, implementing a fuzzy logic to improve its behaviour based on the current iterative values of the design constraints and changes in search direction. Kirsh (1997), on the other hand, was concerned with investigating the effectiveness of angle and error move limits for approximate structural optimisation, which could be useful to obtain improved approximation of SLP algorithms.

Applications of the SLP are also reported in the field of reinforced concrete structures, mainly applied to simplified problem formulations. For example, Chung and Sun (1994) investigated the weight optimisation of RC beams using SLP for modifying the design variables, and the gradient projection method for calculating search directions. Fryer and Ceranic (1997) reported on the application of SLP using dynamic move limits to the minimum volume and material cost design of reinforced concrete skeletal structures.

SLP techniques are reported to be quite powerful and reliable due to the special problem structure, and in particular due to numerical limitations that prevent the usage of higher order methods in some cases. The idea is to approximate the non-linear programming problem by a linear one to obtain a new iterate. The principal advantage is that the linearised problem can then be solved by any standard linear programming software. The method is reasonably straightforward to implement, providing that this linear programming sub-problem is available. It can be applied to solve small, medium and large structural design problems successfully. The limitations are generally due to linearisation errors, convergence to local optima and other standard problems of non-convergence related to mathematical programming techniques.

2.2.2 *Sequential Quadratic Programming (SQP)*

Researchers are primarily concerned with algorithm efficiency and the potential for automation rather than problem specific applications. Lassen (1993) reported on the application of SQP for the sub-optimisation of member groups when applied to large-scale steel frameworks. Huang and Arora (1996) investigated the self-scaling, implicit SQP approach, where the major drawback of calculation and storage of large matrices is avoided, resulting in improved efficiency and reliability. Zhang and Fleury (1997) discussed the application of SQP and its performance when compared to convex approximation methods. Mahmoud et al. (1994) proposed the combination of a commercial finite element package, a quasi-analytical method and SQP with active set strategy for optimisation. To demonstrate the feasibility of the proposed methodology, the design optimisation of a unit injector Focker arm was presented. Abramson and Chrissis (1998) investigated the integration of SQP with an existing automated structural optimisation system, testing the results on three large-scale optimisation problems.

Sequential quadratic programming methods extend the idea of SLP by approximating also the second-order information to obtain a fast final convergence speed. Due to the increased calculations they are less suitable for large size structures. However, for small and medium size structures they are shown to be reliable and efficient techniques.

2.2.3 *Penalty Methods (PM)*

Research in the application of PM mainly concentrates on modifications and improvements, and is mostly applied to structural steel examples. Haridas and Rule (1997) investigated the application of a modified interior penalty algorithm for the minimum weight of steel structures fabricated from non-prismatic beams and subject to multiple load cases. Khot et al. (1995) discussed the application of a modified barrier penalty function and Newton method for unconstrained optimisation. Bental and Zibulevsky (1997) introduced a new type of barrier penalty function to solve convex programming problems. Snyman et al. (1994) reported on the application of a dynamic penalty-function method for the determination of minimum weight of steel trusses and frames.

Some applications of the penalty based methods have been used for reinforced concrete structures, such as Zielinski et al. (1995), who investigated the use of internal penalty functions through two sets of iterations to short-tied reinforced concrete columns. Hannan et al. (1993) presented an application of the conjugate gradient algorithm with parabolic exterior penalty function to the minimum volume design of reinforced concrete footings subjected to wind loading.

Penalty methods belong to the first attempts to solve constrained optimisation problems successfully. A sequence of unconstrained optimisation problems are constructed and solved, so that the minimiser of each unconstrained problem converges to the solution of the constrained one. The resulting non-linear programming problem can be solved by any standard minimisation technique, *e.g.* quasi-Newton search direction combined with a line search. The main disadvantage is that large penalties may lead to ill-conditioned unconstrained problems. Furthermore, the line search must be performed quite accurately due to the steep and narrow valleys in the feasible region created by the penalty terms. Penalty methods are often combined with augmented Lagrangian multiplier methods to solve these problems.

2.2.4 *Lagrangian Multiplier Methods (LMM)*

Also referred to as augmented Lagrangian methods (since the objective function is augmented by a term including information about the Lagrangian function). Reviews such as that by Arora *et al.* (1995), reported on the successful applications of LMMs in engineering optimisation, especially when constrained problems are considered. Adamu *et al.* (1994) described an application of the continuum-type optimality criteria (COC) method to the design of reinforced concrete beams where the conditions of minimality are derived using the augmented Lagrangian method. The costs that are minimised include those of concrete, reinforcement and formwork with active constraints on maximum deflection, bending and shear strength. In their further work, Adamu and Karihaloo (1994) outlined the procedure for the application of the discretised continuum-type optimality criteria (DCOC) method, theoretically established by Zhou and Rozvany (1993), to reinforced concrete beams with similar optimum design problem formulation. Kuhn-Tucker necessary conditions were used to obtain an explicit mathematical derivation of optimality criteria, followed by an iterative procedure for designs that consider both the depth and reinforcement ratio or depth alone as design variables. This algorithm was further modified and applied to multispan beam structures (Adamu and Karihaloo 1994), with each span assumed to have a uniform section and varying reinforcement ratio along its length. Han *et al.* (1996) described a successful application of the DCOC method to multispan partially prestressed concrete beams both for rectangular and T- section, modifying the cost function and design constraints to suit the considered structural system.

The application of this technique combined with genetic algorithms for automating the constraint's penalty handling is described by Adeli and Cheng (1994). Bental and Zibulevsky (1997) applied a non-quadratic augmented Lagrangian for which the penalty parameters are a function of the multipliers. Other authors investigated augmented methods based on the approximation concepts to improve the performance of the algorithm. Coster and Stander (1996) explained the application of the augmented Lagrangian method to steel space structures, with approximation using a partitioned secant matrix updating technique to achieve higher efficiency of the algorithm. Singh and Yadav (1993) investigated approximation concepts to the augmented Lagrangian method for the minimum weight design of a wing box element. Boffey and Yates (1997) described a simplex based Lagrangian scheme for the solution of weight minimisation of structural steel trusses.

The application of the LMM in its primary form to the optimisation of concrete structural elements has been reported. For example, Cohn and Lounis (1992) used a projected Lagrangian algorithm for the optimum design of prestressed concrete beams. Al-Salloum and Siddigi (1994) described a successful application of LMM, but only for singly reinforced concrete beams, not considering the region of the feasible design space where the optimum solution is that of a doubly reinforced section. The research presented in this thesis reports on the application of the LMM to the minimum cost design of both singly and doubly reinforced concrete beams of rectangular section. As reported by Ceranic and Fryer (1997,1999), this design approach has been successfully employed for estimating the upper-bound reinforcement ratios for skeletal structural members, giving comparable results to those obtained using genetic algorithms and an improved approximation method based on sequential linear programming.

Multiplier methods try to avoid the disadvantages of penalty algorithms where large penalties may lead to ill-conditioned unconstrained sub-problems. The LMM's perform a direct transformation of a constrained problem to an unconstrained one, achieving a final solution through a series of successive unconstrained optimisation subproblems. However, in their extensive survey Schittkowski *et al.* (1994) concluded that the solution of these successive unconstrained optimisation problems is likely to require a large number of function and gradient evaluations, hence affecting the efficiency of the algorithm. To overcome this problem, the LMM is often combined with other optimisation approaches.

2.2.5 *Geometric Programming Methods (GP)*

Researchers have been mostly concerned with the generic problems of GP methods, such as Chen (1992) who investigated the application to steel plane trusses, approximating the constraints by single-term polynomials. Two approaches were proposed; first to transform the GP problem directly into a standard linear formulation, and second to transform the same problem into the dual form. Research on the application of GP to RC structures has concentrated on simple structural systems or failure modes, which are easily identified and expressed as linear constraints. Ramsay and Johnson (1998) applied GP to the optimisation of fracture patterns using yield-line analysis for different slab configurations. Chakrabarty (1992) investigated the application of geometric programming to the least-cost design of RC beams, considering the cost of materials and shuttering.

These methods optimise a non-linear objective function in the form of a polynomial, while satisfying a set of constraints which are also polynomials. GP methods have shown exceptional efficiency for small structural problem formulations with limited number of the variables. However, this efficiency deteriorates when the size of the structural problem is increased, still requiring some standard optimisation technique to be employed for the corresponding sub-problem.

2.2.6 *Generalised Reduced Gradients Methods (GRG)*

Schittkowski et al. (1994) reported that these methods are very reliable, particularly when some features similar to those of sequential quadratic programming are implemented. An additional significant advantage of these methods is that whenever an iteration is stopped, the final design is feasible. A limitation of the GRG methods is that extensive function evaluations are required, as it is necessary to project the new iterate back to the feasible region every time a constraint is violated. Furthermore, the search has to start from a feasible iterate, which is not easily recognised for some structural optimisation problems.

To overcome some of these limitations, GRG methods have been combined with other approaches. For example, Parkinson and Wilson (1986) investigated a development of the hybrid algorithm between sequential quadratic and the generalised reduced gradient method for constrained non-linear programming problems, reporting on the improved efficiency and reliability.

These methods convert the original non-linear problem into a problem with non-linear equality constraints by introducing artificial slack variables. Additional lower bound constraints are also imposed for the slack variables. Step search is then employed starting from a feasible iterate, by for example conjugate gradient or quasi-Newton method. If the new iterate violates constraints then it will be projected on the feasible domain by a Newton type technique.

2.3 Optimality Programming Methods

This second class of non-linear programming techniques, as classified by Schittkowski et al. (1994), are problem dependent and often presented in optimisation literature side-by-side with mathematical programming techniques, generally as conflicting methods.

They generally consist of two subclasses; optimality criteria (OC) and convex approximation (CA) methods. The basic concept behind these methods is rejection of the generality of mathematical programming and utilisation of physical characteristics of the structural optimisation problems.

2.3.1 *Optimality Criteria (OC) Methods*

Salajegheh (1997) reported on the efficiency of the OC method applied to a variety of structural problems with approximated member forces and nodal displacements. The main objective of the work was to reduce the number of required static and dynamic analyses within the algorithm, and hence improve computer efficiency. Zhou and Haftka (1995) investigated the derivation of discrete continuum type optimality criteria (DCOC) methods directly from the traditional OC methods. They have further developed a derivative-based version of the DCOC method hoping to help researches in the understanding of the method. Patnaik *et al.* (1995) performed a detailed study of the merits and limitations of the OC methods for the minimum weight design of steel structures, subjected to the multiple load conditions under stress, displacement and frequency constraints.
Applications of the OC methods have also been used for RC structures. Fadaee and Grierson (1996,1998) presented an optimality criteria method for the optimum design of 3D reinforced concrete structures having beams, columns and shear walls. Moharrami and Grierson (1993) investigated the effectiveness of an iterative optimisation strategy offered by the optimality criteria method on the convergence to the optimum design of RC frameworks. Adamu and Karihaloo (1994, 1995) reported on the application of a discretised continuum-type optimality criteria to the optimum design of reinforced concrete beams and frames, respectively.

These are problem dependent methods that focus on known or assumed features of the optimum, searching for a solution in its vicinity. Optimality criteria methods have a somewhat limited applicability, although they provide for considerably improved computer efficiency.

2.3.2 *Convex Approximation Methods (CA)*

Research is mainly concerned with the mathematical aspects of CA methods, investigating the techniques to overcome its limitations. Svanberg (1987) controlled the degree of convexification by introducing so-called *moving asymptotes*, resulting in greater flexibility and better convergence of the algorithm (method of moving asymptotes-MMA). To overcome the drawback of the dependency on the initial starting point, Zillober (1993) added a line search procedure to a standard convex approximation method, similar to the approach used in sequential quadratic programming. Zhang and Fleury (1997) reported on the modification of convex approximation methods for structural optimisation, proposing the so-called *fitting scheme* to overcome practical difficulties relating to the evaluation of second-order derivates. Kegl and Oblak (1997) presented an improved approximation technique for gradient based methods by adding an appropriate convex term to each conventional approximating function.
Applications of this method to RC structures are scarce and mainly in combination with other methods. Min and Kikuchi (1997) applied a sequential convex approximation method together with a dual method for optimal reinforcement design of structures under buckling load.

These methods are often referred to as sequential convex programming (SCP) methods. The main concept is to use a convex approximation of the original problem instead of a linear or quadratic one, and then to solve the resulting non-linear sub-problem by a specifically designed algorithm that takes advantage of the simplified problem structure. Consequently, convex approximation methods are only useful in cases where the evaluation of the function and gradient values are much more expensive than the internal computations to solve the resulting sub-problem. The advantage of these methods is their respectable degree of efficiency, exploiting the special features of the underlying design problem. However, they are limited by a lower degree of reliability, since starting the algorithm from an inappropriate initial design point may result in non-convergence.

2.4 Heuristic Methods

The modern heuristic methods have shown extraordinary promise in both conceptual simplicity and computational efficiency. Furthermore, their ability to overcome the problems associated with the traditional mathematical programming techniques have made them particularly attractive, especially when realistic structures, loading conditions and limit states are considered. These methods can be generally classified as:

(i) genetic algorithms (GA) and evolution strategies (ES)
(ii) simulated annealing (SA) methods
(iii) neural networks (NN) methods
(iv) tabu search (TS) methods
(v) other methods, such as biological growth techniques, *etc.*

2.4.1 *Genetic Algorithms (GA)*

It was Holland (1962), who first established genetic algorithms on a sound theoretical basis, clearly recognising the analogy between the principle of natural selection and the general optimisation in the artificial setting. The important theory of schemata was also developed by Holland (1975), providing a mathematical tool for explaining the similarity templates for given string classes.

Over the past decade, the application of genetic algorithms has been investigated by many authors, showing an impressive flexibility and diversity in the type of problems solved. In the field of structural optimisation many successful applications have been reported, ranging from general strategies to specific solutions. Some authors, such as Adeli and Cheng (1993) and Jenkins (1992), reported on general applications of genetic algorithms to structural design and optimisation. Others investigated different variants and strategies to improve the performance of genetic algorithms when applied to specific structural optimisation problems, such as Le Riche and Haftka (1992), Soh and Yang (1996) and Coello et al. (1997). Leite and Topping (1998) reported on improved genetic operators designed to ensure a balance between effective exploration and selective pressure. Adeli and Cheng (1994a) investigated constrained genetic algorithm optimisation, introducing an augmented Lagrangian Multiplier Method to determine the minimum weight design of high-rise steel structures and space frames. This approach avoided extensive numerical testing to find a suitable value for the penalty function coefficients. In latter work, Adeli and Cheng (1994b) extended their previous work by presenting two augmented Lagrangian algorithms utilising the multiprocessing capabilities of high-performance computers. Other authors investigated more specific approaches considering certain features of GA control parameters or variable strings to improve the algorithm. Lu et al. (1996) presented an improved strategy for GA's in structural optimisation, introducing feasible and infeasible individual strings, and related space for the individual string. They also adopted the use of structural approximation analysis by artificial neural networks. Rajeev and Krishnamoorthy (1997) reported on genetic algorithms based methodologies for design optimisation of steel trusses, describing the improvements found in their two-phase variable string length genetic algorithms. Jenkins (1997a) discussed the general issues related to the application of natural algorithms to structural design optimisation. He investigated the development of space condensation approaches that lead to a more economical application of the algorithm. The approach to adaptivity of controls and type of penalty function used for constrained optimisation is further explained. Some authors investigated various approaches in handling multiobjective optimisation problems, such as Dhingra and Lee (1994) who investigated the application of GA's to single and multiobjective structural optimisation with discrete-continuous variables. They proposed a co-operative game theoretic approach to model the multiple objective functions. Extensive work on the application of GAs for conceptual design is reported, for example in the work of Parmee (1995), Maher and Poon (1995), Matthews and Rafiq (1995) and Grierson (1996, 1999). In particular, Parmee has been an advocate of using GAs not merely as optimisation engines but also as artificially intelligent search tools.

The application of GAs in the field of realistic reinforced concrete structural systems is not widespread, although a number of solutions to specific problem formulations is reported. Coello et al. (1997) investigated the application of GA's to the optimum design of reinforced concrete beams, arguing that more realistic designs are obtained than those based on the mathematical programming techniques. Rafiq and Southcombe (1998), investigated the optimum design and detailing of reinforced concrete biaxial columns, searching for the optimum reinforcement of a given set of section sizes and loading. Kocer and Arora (1996) discussed two approaches in the optimisation of prestressed concrete transmission poles. In the first approach, they used the branch and bound algorithm for discrete variables, enumeration method for integer variables and sequential quadratic programming for the continuous variables. The second approach, however, used genetic algorithms for all variables. Ceranic and Fryer (1998, 2000) reported on the application of GA's to the minimum cost design of reinforced concrete skeletal systems that are composed of beams and columns cast in situ with slabs to form an integral structure.

Genetic algorithms are stochastic global search and optimisation methods based on the mechanics of natural selection and genetic processes of biological organisms. They systematically modify tentative solutions of a design problem scanning through the feasible population, and producing new offspring generations of improved fitness with respect to the preceding parental population. The advantages are that the problem of feasibility of the optimum design becomes "insignificant", as stopping the algorithm short of reaching a real optimum still ensures a possible near-optimum solution.

The "blindness" of genetic algorithms to the nature of applied structural problems, their ability to avoid gradients and linearisation errors, and their efficiency in dealing with discontinuous design equations makes them particularly attractive when compared with traditional mathematical programming methods. The main limitation of genetic algorithms is that they require an extensive sensitivity analysis for the control parameters, which are dependent on the class and type of the structural problem. Furthermore, genetic algorithms are unconstrained problem solvers, and therefore suitable techniques for constraint handling have to be introduced when constrained optimisation problems are considered.

Broadly speaking, genetic algorithms are part of the larger class of evolutionary algorithms (EAs), which also includes evolutionary programming (EP), evolutionary strategies (ESs) and genetic programming (GP).

2.4.2 *Evolutionary Structural Optimisation Method (ESO)*

Over the last decade, a number of applications of the ESO method are reported, such as the research by Steven et al. (1993, 1997), Hinton et al. (1995) and Querin et al. (1996). They demonstrated that the method is capable of generating an optimum structure, at times with better results than some classical examples. Although the method has exhibited a respectable performance when applied to structural optimisation problems, there has been no formal mathematical verification that ESO is a valid optimisation method that follows definite standard robust optimisation techniques. It has been argued that because the fundamental principle of ESO is intuitive and logically simple, that is a statement of fact and therefore must be true on its own accord. However, Querin (1997) stated that this is a weak justification of ESO, because as the method is applied to more complex problems, what started as an intuitive method needs to have a more mathematical theoretical basis.

This method presents an application of evolutionary strategy to problems in structural optimisation. It can be best described as an approach that exhibits the characteristics of both heuristic and gradient based optimisation methods. ESO performs a search through the structural domain on the gradient-based principle locating both local and global optimums. However, it does not stop when an apparent minimum has been located, instead the evolution process continues to evolve the structure in a search for a better one.

2.4.3 *Simulated Annealing (SA) Algorithms*

The algorithm was first proposed by Kirkpatrick et al. (1983) and, independently, by Cerny (1985). In a past decade researchers have shown a growing interest in the application of SA to structural optimisation. Hence, some of the research is concerned with comparison of SA methods to other heuristic search techniques. In their survey, Thanedar and Vanderplaats (1995) reviewed available methods for discrete variable structural optimisation, comparing simulated annealing and genetic algorithms to the branch and bound based techniques. Similarly, Huang and Arora (1997) described the comparison of SA algorithms, GA's and branch and bound techniques when applied to the optimum design of steel structures using standard sections. Bennage and Dhingra (1995) investigated the application of SA to single and multiobjective structural optimisation problems. Their results indicate that, in several instances, simulated annealing out performs gradient-based and discrete optimisation techniques used in the comparison. Other authors are more concerned with specific aspects of the application of simulated annealing to certain structural problems. Tzan and Pantelides (1996) reported on the annealing strategy for obtaining the optimal design of structural systems. Two features of a method were described; an automatic reduction of the search range and sensitivity analysis for the design variables. The same authors (1997), investigated dynamic or time-varying constraints for structures, proposing a modified iterative simulated annealing (MISA) method for optimum design of structural systems with those types of constraints.

Attiquallah and Rao (1995) reported on the parallel processing application of the simulated annealing algorithm, investigating different parallelisation techniques. The results from testing indicated that large structural designs can be optimised in significantly shorter times even on relatively small parallel processing configurations. Shim and Manoochehri (1997) presented a combinatorial optimisation procedure based on simulated annealing for generating the optimal configuration of structural members. Design examples were given and the effects of changing the SA parameters on the final configuration were examined.

Simulated annealing is a stochastic relaxation technique which is based on the analogy to the physical process of annealing a metal. The solution to a general optimisation process can be associated with this system states behaviour. The cost of a structure corresponds to the concept of energy and moving to any new set of design variables corresponds to a change of state. Simulated annealing randomly generates new configurations by sampling from the probability distribution of the system. It employs a random search which not only accepts changes that decrease the objective function, but also changes that increase it with a certain probability. This paradigm is the major advantage of simulated annealing algorithms, making them less susceptible to pitfalls of convergence to a local optimum. The limitations of SA algorithms are similar to those reported for genetic algorithms.

2.4.4 *Neural Networks (NN) Algorithms*

The beneficial use of NN for approximate analysis in the structural optimisation was recognised. Jenkins (1997b) described the application of neural networks to the approximate analysis of right-grillage structures, offering observations on accuracy, network topology and training. He further argued the significance of approximate analysis for inclusion in the optimisation process, testing a variety of conceptual designs. Papadrakakis et al. (1998) reported on the application of evolution strategies (ES) on the shape and sizing optimisation. A neural network model was used to replace the structural analysis phase and to compute the necessary data for the optimisation procedure. A back propagation algorithm was implemented for training the NN, which is then used to predict the values of the objective function and the constraints. They reported on improved efficiency for large-scale problems, since the use of neural networks avoided time consuming repeated analyses. Some authors were more concerned with generic issues of NN, examining the performance of the application. Kodiyalam and Gurumoorthy (1996) investigated the use of neural networks utilising modified back propagation learning. They reported on faster convergence of the learning process, applying the model to aerospace composite materials. Other authors reported on more specific applications, such as Adeli and Karim (1997) that discussed the application of NN for optimisation of cold-formed steel beams. The computational model has been applied to three different commonly used types of cross-sectional shapes; hat-, I- and Z-shapes.

Neural networks can be defined as an interconnected assembly of simple processing elements, units or nodes, whose functionality is loosely based on a neuron. They present a statistical model of a real world system built by tuning a set of connection strength parameters, or weights, obtained by a process of adaptation to, or learning from, a set of training parameters. The advantage of neural networks is the capability of providing satisfactory approximations to systems that can not be solved by traditional methods. Furthermore, they are able to learn from data and generalise using non-visible rules, coping well with the introduced errors. As limitations, it is recognised that they lack precision and the formality of traditional computing applications. The application of neural networks in structural optimisation is still quite novel, but the potential is recognised, especially as a hybrid with other techniques.

2.4.5 *Tabu Search (TS) Algorithms*

Dhingra and Bennage (1995) reported on the application of tabu search to discrete and continuous variable structural optimisation. They discussed an application of the complementary mechanisms in tabu search; tabu restrictions and aspiration criteria, for guiding the search in the design space. Their results indicated that in several instances tabu search outperforms some traditional gradient-based and discrete optimisation techniques. Bland (1998) investigated the use of tabu search to structural design optimisation with reliability constraints. Weight minimisation of steel space trusses was tested, reporting on the effectiveness of TS when dealing with stochastic variables and effects of displacement and buckling (*Euler*) constraints.

Tabu search is a local search method that starts from a feasible design and iteratively improves upon it. To avoid possible cycles in the search path, a history of prohibited moves is kept in a *taboo* list. Since the tabu list is not unbounded, an *escape* mechanism is provided as a second means of breaking out from the possible cyclic path. An *aspiration* criteria can also be incorporated, defining the rules which prefer certain moves to others, thus avoiding restrictivness of the tabu rules. This rule-based search can be very effective, and researches in the field claim that it often outperforms other known search based techniques, both in accuracy and efficiency. The disadvantage of this technique is that it requires considerable expertise and experimentation to construct the rules for correct control of its dynamic nature. Furthermore, tabu search has to start from a feasible random design. Finally, a clear shortcoming of tabu search, as noted by Glover and Taillard (1993), is its theoretical incompleteness with an inability to prove its search success and convergence behaviour.

Applications of other search strategies for RC structures are reported, such as mixed-discrete fuzzy optimisation applied to the reinforced concrete beams, investigated by Shih and Lai (1995). Samman and Erbatur (1995) reported on the application of direct search techniques to determine steel ratios for cost optimum design of RC beams. Burns and Balling (1991) investigated the utilisation of a variety of search strategies including continuous and discrete optimisation, optimality criteria and some heuristic search methods.

2.5 Conclusions

The literature survey was conducted with specific reference to the optimisation of reinforced concrete structures, and with regard to identified factors in structural problem formulation outlined in Section 2.1.1. It was found that the majority of the published work is on the optimisation of steel structures, with applications to realistic RC structures being reported sporadically and in mainly isolated publications. The literature survey also revealed that the selection of a suitable optimisation method is influenced by the specific nature of the underlying structural problem, its formulation, and by the choice of the objective function and corresponding design variables. The survey concentrated on the application of optimisation methods to both steel and reinforced concrete structures, since the concepts behind their implementation for steel structures can be translated to the optimum design of reinforced concrete structural problems. The optimisation methods, broadly classified as mathematical programming, optimality criteria and modern heuristic methods, have been surveyed with regard to both their potential suitability and applicability when realistic structural systems and loading conditions are considered. The survey was performed continually throughout the research, constantly reassessing the methodology with reference to the new findings of the published work. It was concluded that traditional MP optimisation techniques were most widely researched and shown to be important when investigating both the behaviour and different objective functions for RC structural systems subjected to critical design constraints. However, it was also observed that to be implemented they require a significant simplification of the underlying problem, and hence the optimum may not be representative of the real structure. The application of heuristic optimisation techniques as viable alternatives to traditional optimisation methods was suggested in the literature, having the ability to model more complex and realistic structural problem formulations. In this context, their considerable potential is recognised and is being researched.

For these reasons, the adopted research methodology proposed the application of both mathematical programming and heuristic methods, with the intention to critically assess and validate their advantages and limitations when applied to identified RC structural systems with realistic objective functions and design constraints.

3.

Minimum Cost Design of Beams

This chapter investigates the use of the Lagrangian Multiplier Method for the minimum cost design of reinforced concrete rectangular beams under limit state design conditions. Cost objective functions and stress constraints are derived for singly and doubly reinforced beams and implemented within an optimisation method. Cost sensitivity analysis, detailed testing and comparisons with conventional design office methods are performed and the results reported.

3.1 Introduction

The material costs of reinforced concrete beams are dependent on their dimensions, reinforcement ratios and the unit costs of concrete and steel reinforcement. Whilst trying to optimise the cost of a beam, certain conditions have to be met so that the equilibrium of the section is maintained and the requirements of relevant standards are satisfied. Although considered as simple structural elements, the minimum cost of beams is difficult to achieve using conventional office design methods, as theoretically speaking, there are an infinite number of alternative beam dimensions and reinforcement ratios that can yield a similar moment of resistance. These elements are often the major components in reinforced concrete skeletal structures, and hence an investigation of their minimum cost design is considered an important factor in this research.

The work undertaken reports on the application of the Lagrangian Multiplier Method (LMM) to the minimum cost design of both singly and doubly reinforced concrete beams of rectangular section. Formulations of the optimisation problems are expressed in closed forms that are particularly suited to solution using the Lagrangian Multiplier Method, as previously discussed in the Section 2.2.4. Al-Salloum and Siddigi (1994) describe a successful application of LMM, but only for singly reinforced concrete beams, not considering the region of the feasible design space where the optimum solution is that of a doubly reinforced section. This research shows that explicit optimum design equations for a doubly reinforced section can be derived, providing the designer with a practical and intuitive solution without the need for iterative trials.

3.2 Lagrange Multiplier Method

The Lagrangian Multiplier Method applies to the optimisation of a multivariate objective function expressed as

$$y = f(x_1, x_2, \ldots x_n) \tag{3.1}$$

subject to equality constraints of the form

$$g_i(x_1, x_2, \ldots x_n) = 0 \qquad i = 1, 2, \ldots m \tag{3.2}$$

where n is the number of independent variables and m is the number of constraints; m must be less than n by definition of the problem.

The procedure is to construct the unconstrained Lagrangian function L of the form

$$L(x_1, x_2, \ldots, x_n, \lambda_1, \lambda_2, \ldots, \lambda_m) = f(x_1, x_2, \ldots, x_n) + \sum_{i=1}^{m} \lambda_i g_i(x_1, x_2, \ldots, x_n) \tag{3.3}$$

where the unspecified constants λ_i are the Lagrange multipliers determined in the course of the extremisation.

The necessary conditions for L to posses an extreme (*stationary point*) are

$$\frac{\partial L}{\partial x_k} = \frac{\partial f}{\partial x_k} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial x_k} = 0 \qquad k = 1,2,\ldots,n \tag{3.4}$$

and

$$\frac{\partial L}{\partial \lambda_i} = g_i = 0 \qquad i = 1,2,\ldots,m \tag{3.5}$$

Equation 3.5 simply restates the original constraints acting on the solution space of the objective function $y = f(x_1, x_2, \ldots x_n)$. Equations 3.4 and 3.5 are a system of $n+m$ equalities with $n+m$ unknowns. Hence, their solution will yield stationary values for $x_1, x_2, \ldots, x_n$ and $\lambda_1, \lambda_2, \ldots, \lambda_m$ from which the optimum solution can be obtained.

3.3 Implementation of the Lagrangian Multiplier Method

Reinforced concrete beams of rectangular section are primarily designed to resist the action of flexural bending and are classified in BS8110 as either singly or doubly reinforced. In the case of the former, reinforcement is provided to resist the tensile forces, whilst for the latter, reinforcement is designed to resist both the tensile and compressive forces in the beam. Bending equilibrium equations for singly and doubly reinforced beams can be derived from BS8110, relating the applied bending moment to the beam geometry, material strengths and area of reinforcement. The total material cost of a beam per unit length is a function of the material costs, beam geometry and area(s) of reinforcement, the latter being dependent on the classification of the beam.

3.3.1 *Singly Reinforced Beam*

In this research, the material cost of a singly reinforced beam per unit length is given by

$$C = C_s A_s + C_c A_c = C_s \rho bd + C_c(d+c)b \tag{3.6}$$

where C_s and C_c are the costs of steel and concrete per unit volume respectively, A_s is the area of tension reinforcement, A_c is the area of concrete, ρ is the reinforcement ratio (A_s/bd), b and d are the breadth and effective depth of the section respectively (*see* Figure 3.1), c is the cover to the tension reinforcement and r is the ratio of reinforcement cover to effective depth d.

Setting the ratio of the material costs to $q = C_s/C_c$, the objective function is re-written as

$$C = C_c b[q\rho d + (1+r)d] \tag{3.7}$$

If the breadth of the section is considered fixed, and it is assumed that the ratio r and ultimate design moment M remain constant, equation (3.7) is reduced to

$$C' = q\rho d + (1+r)d \tag{3.8}$$

since $C_c b$ is a constant.

The geometry of a rectangular beam is shown in Figure 3.1 together with the simplified rectangular stress block as given in BS 8110. The procedure for derivation of the stress equilibrium constraint developed in this research is as follows

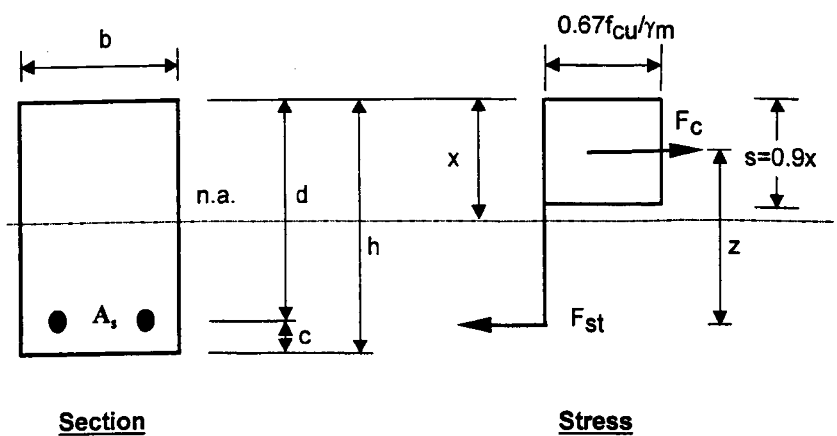


Figure 3.1 *Singly reinforced section with simplified rectangular stress block*

Taking moments about the centroid of the compression block gives

$$M = 0.87 f_y A_s z \tag{3.9}$$

where M is the ultimate design moment, f_y is the characteristic strength of steel and z is the lever arm.

Taking moments about the centroid of the tension reinforcement gives

$$M = 0.402 f_{cu} xbz \tag{3.10}$$

where x is the depth of the neutral axis and f_{cu} is the characteristic concrete strength.

Using equations (3.9) and (3.10) and noting from the stress diagram that $z = d - 0.45x$, the following equation is derived for z

$$z = d\left(1 - 0.98\frac{f_y}{f_{cu}}\frac{A_s}{bd}\right) \tag{3.11}$$

Substituting equation (3.11) into equation (3.9) and dividing through by bd^2 gives the following equilibrium constraint

$$\frac{M}{bd^2} = 0.87 f_y \rho\left(1 - 0.98\frac{f_y}{f_{cu}}\rho\right) \tag{3.12}$$

To achieve the most economical design, it is required to minimise the objective function given by equation (3.8) subject to the equilibrium constraint given by equation (3.12). Combining these equations in a format suitable for implementation by the Lagrangian Multiplier Method, the following expression was obtained

$$\theta = (\rho q d + a_3 d) + \lambda\left[a_1 \rho d^2 (1 - a_2 \rho) - M\right] \tag{3.13}$$

where

$$a_1 = 0.87 f_y b; \quad a_2 = 0.98 f_y / f_{cu}; \quad a_3 = 1 + r \tag{3.14}$$

The partial derivates of the Lagrangian function are given by

$$\frac{\partial\theta}{\partial\rho} = q + \lambda\left[a_1 d(1 - 2a_2\rho)\right] = 0 \tag{3.15a}$$

$$\frac{\partial\theta}{\partial d} = q\rho + a_3 + 2\lambda\rho\left[a_1 d\left(1 - a_2\rho\right)\right] = 0 \tag{3.15b}$$

$$\frac{\partial\theta}{\partial\lambda} = a_1\rho d^2\left(1 - a_2\rho\right) - M = 0 \tag{3.15c}$$

Using this system of equations, the optimum reinforcement ratio ρ_{opt} is derived as

$$\rho_{opt} = \frac{1}{\dfrac{q}{1+r} + 1.96\dfrac{f_y}{f_{cu}}} \tag{3.16}$$

The corresponding optimum effective depth d_{opt} is obtained by back substituting equation (3.16) into equation (3.15c), giving

$$d_{opt} = \sqrt{\frac{M}{0.87 f_y \rho_{opt} b\left(1 - 0.98\rho_{opt} f_y / f_{cu}\right)}} \tag{3.17}$$

The research concluded that equation (3.16) is only valid for singly reinforced beams and it is therefore necessary to determine the upper bound value of ρ_{opt} beyond which the optimum solution will be a doubly reinforced section. The maximum moment of resistance of a singly reinforced section is given by

$$M = 0.156 f_{cu} b d^2 \tag{3.18}$$

Equating this with equation (3.9) and setting the lever arm $z = 0.775d$ as specified in BS 8110, the boundary reinforcement ratio ρ_{bound} between a singly and doubly reinforced section is derived as

$$\rho_{bound} = 0.2314\frac{f_{cu}}{f_y} \tag{3.19}$$

Figure 3.2 is a graphical representation of the optimum reinforcement ratio given by equation (3.16), and shows the family of q-lines for a typical fixed value of $r = 0.15$. The values of the optimum reinforcement ratio are constrained between the maximum and minimum reinforcement ratios, as specified in BS 8110. Although a series of similar graphs can be plotted depending on the assumed value of the ratio r, it has been found within the research that the minimum cost is not significantly sensitive to changes in this ratio, which in itself has tightly banded values.

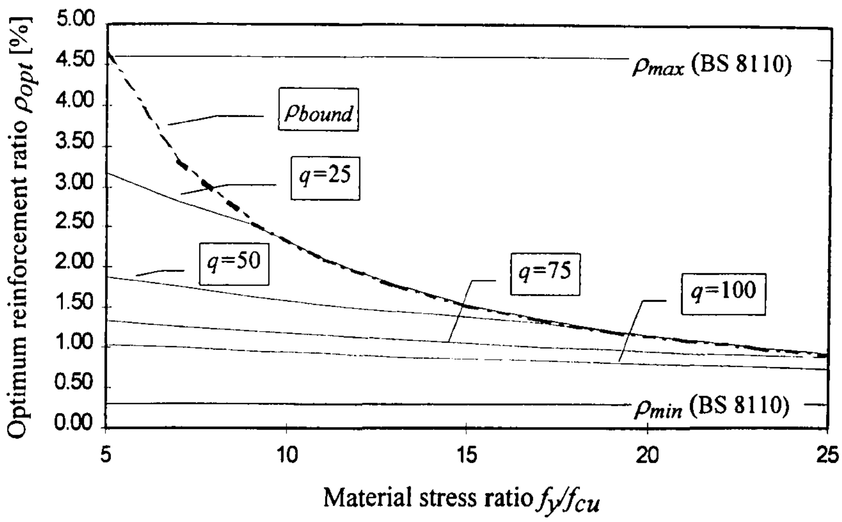


Figure 3.2 *Optimum reinforcement ratio versus stress ratio f_y/f_{cu} for singly reinforced beams*

Figure 3.2 shows that for an increase in the material cost ratio q, the optimum solution requires a corresponding reduction in the reinforcement ratio ρ_{opt}. Under identical loading conditions, this reduction is compensated by an increase in the effective depth of the section d, as obtained from equation (3.17). The q-lines are valid until they intersect the boundary reinforcement ratio curve. Above this line it is concluded that the optimum solution is given by a doubly reinforced section, and hence the research extended the implementation to consider its optimum design.

3.3.2 *Doubly reinforced concrete beam*

In this research, the material cost of a doubly reinforced beam per unit length is given by

$$C = C_s\left(A_s + A_s^{'}\right) + C_c A_c = C_c b\left[q\left(\rho + \rho^{'}\right)d + \left(1 + r\right)d\right] \tag{3.20}$$

The following relationship between the reinforcement ratio for tension steel ρ and for compression steel ρ' is given by

$$\rho' = \rho - 0.2314\, f_{cu}/f_y \tag{3.21}$$

assuming that the stress in the compression reinforcement has reached yield stress and the ratio $d'/d \leq 0.215$, where d' is the depth from the top of the compression face to the centroid of the compression reinforcement (*see* Figure 3.3).

Substituting equation (3.21) into equation (3.20) gives the final form of cost objective function as

$$C = C_c b\,[\,q(2\rho\text{-}0.231 f_{cu}/f_y)d + (1+r)d\,] \tag{3.22}$$

Figure 3.3 shows the geometry of the rectangular beam section and the simplified rectangular stress block for a doubly reinforced beam. When the ultimate design moment M exceeds the moment of resistance of a singly reinforced section ($0.156 f_{cu} b d^2$), compression reinforcement is required. For this condition, the depth of the neutral axis is specified in BS8110 as $x=0.5d$, to ensure a tension failure with a ductile section. Within this research the following procedure for obtaining the stress equilibrium constraint was developed

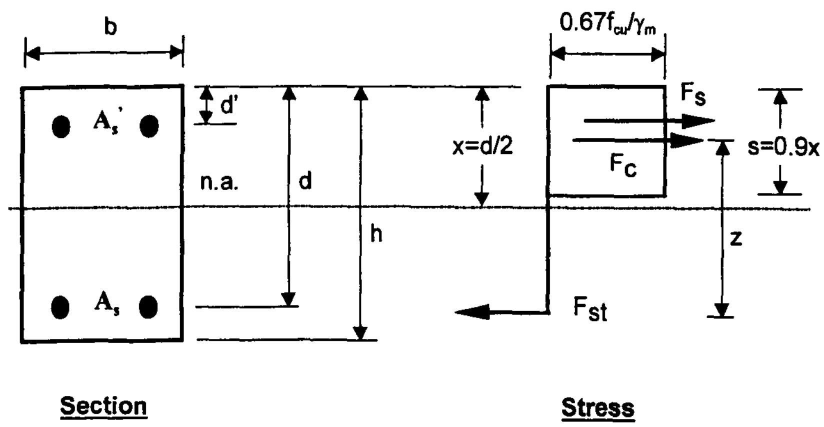


Figure 3.3 *Doubly reinforced section with simplified rectangular stress block*

Considering the equilibrium of the horizontal forces and assuming that the tension steel is at yield, gives

$$f_s'A_s' = 0.87f_yA_s - 0.2f_{cu}bd \tag{3.23}$$

Taking moments about the centroid of the tension reinforcement gives

$$M = 0.156f_{cu}bd^2 + f_s'A_s'(d-d') \tag{3.24}$$

Eliminating $f_s'A_s'$ from equation (3.24) yields

$$M = 0.156f_{cu}bd^2 + [0.87f_yA_s - 0.2f_{cu}bd](d-d') \tag{3.25}$$

Re-writing equation (3.25) in a form similar to equation (3.12) gives following equilibrium constraint

$$\frac{M}{bd^2} = 0.156f_{cu} + f_{cu}\left[0.87\frac{f_y}{f_{cu}}\rho - 0.2\right]\left(1-\frac{d'}{d}\right) \tag{3.26}$$

Using equations (3.22) and (3.25), and formulating the problem for solution by the Lagrangian Multiplier Method, the optimum reinforcement ratio for the tension steel is derived as

$$\rho_{opt} = 0.3445\frac{f_{cu}}{f_y} - 0.3585\frac{f_{cu}}{f_y}\frac{1}{1-r} + \frac{1+r}{2q} \tag{3.27}$$

The reinforcement ratio for the compressive steel ρ' is calculated to satisfy equation (3.21), setting ρ equal to ρ_{opt}. In this derivation it is assumed that the ratios r and r' are constant and equal to each other.

The optimum effective depth is calculated from equation (3.26) back substituting equation (3.27) for ρ_{opt} giving

$$d_{opt} = \sqrt{\frac{M}{f_{cu}b\left[0.156 + \left(0.87\rho_{opt}f_y / f_{cu} - 0.2\right)(1-r)\right]}} \tag{3.28}$$

Figure 3.4 is a graphical representation of the optimum reinforcement ratio given by equation (3.27), and shows the family of q-lines for typical values of r and r' of 0.15.

Plotted values of ρ_{opt} are constrained between ρ_{max} and ρ_{bound}. The compressive steel reinforcement ratio is obtained from equation (3.21) taking account of the minimum allowable value of 0.2% as specified by BS 8110. As for singly reinforced beams, a series of similar graphs can be plotted for different ratios of r and r'. However, the research found that the minimum cost is not significantly sensitive to changes in these ratios and hence a family of graphs is not essential.

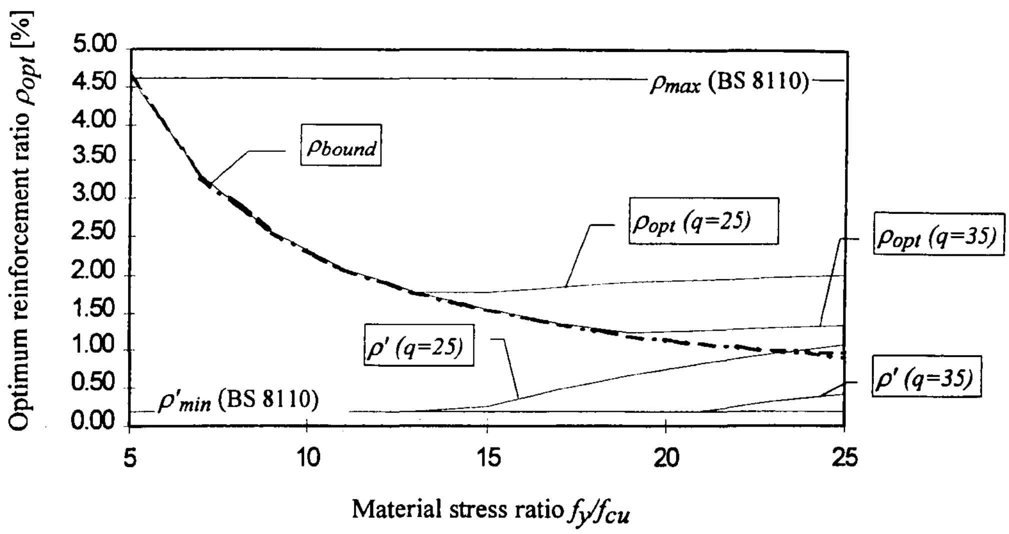


Figure 3.4 *Optimum reinforcement ratio versus ratio f_y/f_{cu} for doubly reinforced beams*

Figure 3.4 shows that for an increase in the material cost ratio q, the optimum solution requires a corresponding reduction in the reinforcement ratio ρ_{opt}. The conclusion is that under identical loading conditions, this reduction in ρ_{opt} is compensated by an increase in the effective depth of the section d. For $q > 45$ the optimum solution will be a singly reinforced beam. The q-lines are valid until they intersect the boundary reinforcement ratio curve. Below this line the optimum solution is given by a singly reinforced beam and hence Figure 3.2 should be used.

3.3.3 *Cost sensitivity analysis*

To identify the distinctive zones for which a particular solution gives a minimum cost, the research compared the optimum solutions for singly and doubly reinforced beams for different values of the material stress ratio f_y/f_{cu}. To ensure a valid singly reinforced optimum solution, the amount of reinforcement given by equation (3.16) has to be taken to be less than the boundary value given by equation (3.19), or more precisely

$$\frac{f_y}{f_{cu}} \leq 0.422\frac{q}{1+r} \tag{3.29}$$

Similarly, for the optimum solution to be a doubly reinforced beam the reinforcement ratio for the tension steel given by equation (3.27) has to be greater than the boundary value given by equation (3.19). Therefore, we have

$$\frac{f_y}{f_{cu}} \geq 2\left[0.3585/(1-r)-0.1135\right]\frac{q}{1+r} \tag{3.30}$$

Using equations (3.29) and (3.30), three distinct zones of optimum reinforcement ratio were identified by the research over the defined range of the material stress ratio f_y/f_{cu}. Figure 3.5 shows these zones for q=25 and r=0.15, with f_y/f_{cu} ratio between 5 and 25 covering the possible range of values given in BS8110.

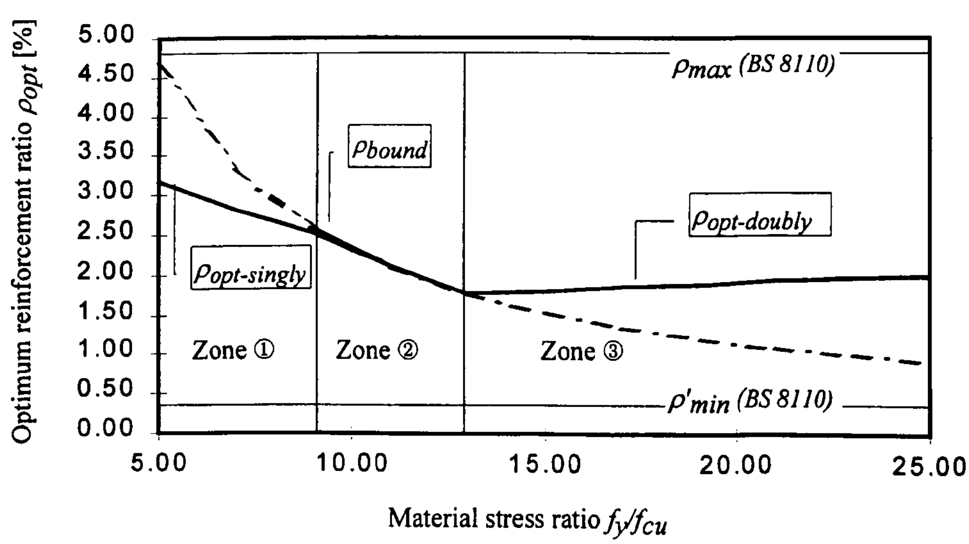


Figure 3.5 *Optimum reinforcement ratio for q=25 and r=0.15*

Zone 1 corresponds to a singly reinforced section with the ratio of f_y/f_{cu} between its lower bound value of 5 and the point of intersection with the boundary curve at 9.2. Zone 2 corresponds to a singly reinforced section with its optimum reinforcement ratio being set at ρ_{bound} for the range of f_y/f_{cu} between 9.2 and 13.4. Zone 3 corresponds to a doubly reinforced section with the ratio of f_y/f_{cu} between the point of intersection with the boundary curve at 13.4 and its upper bound value of 25. For any other values of q, it is possible to determine the valid material stress ratio range for different optimum solutions. For example, the results in Table 3.1 have been derived using values of r and r' equal to 0.15.

Material Cost Ratio (q)	Single Reinforcement Optimum Range f_y/f_{cu}	Boundary Reinforcement Optimum Range f_y/f_{cu}	Double Reinforcement Optimum Range f_y/f_{cu}
25	5.0-9.2	9.2-13.4	13.4-25.0
35	5.0-12.8	12.8-18.8	18.8-25.0
45	5.0-16.5	16.5-24.1	24.1-25.0
55	5.0-20.2	20.2-25.0	
65	5.0-23.8	23.8-25.0	
75	5.0-25.0	Outside the practical range (>25)	
85	5.0-25.0		
95	5.0-25.0		

Table 3.1 *Valid ranges of f_y/f_{cu} for different optimum reinforcement ratios*

A series of tables of this type can be produced for different values of r and r', which by definition must be less than 0.215 if the compression reinforcement is to have reached yield. Hence, for a given design problem, it is possible to select the optimum reinforcement ratio formula directly without recourse to repetitive calculations. The proposed approach therefore offers a convenient and easy method of selecting the appropriate optimum solution and corresponding formulae. In practice, the material stress ratio f_y/f_{cu} has discrete values which are predetermined by the possible combinations of f_{cu} and f_y that are permitted by BS8110. To assist the designer in the selection of an appropriate optimum solution, a graph showing the optimal zones for singly (SRO), boundary (BRO) and doubly reinforced (DRO) sections have been developed in this research. An example of such a graph is given in Figure 3.6, for typical values of r and r' ranging from 0.05 to 0.20.

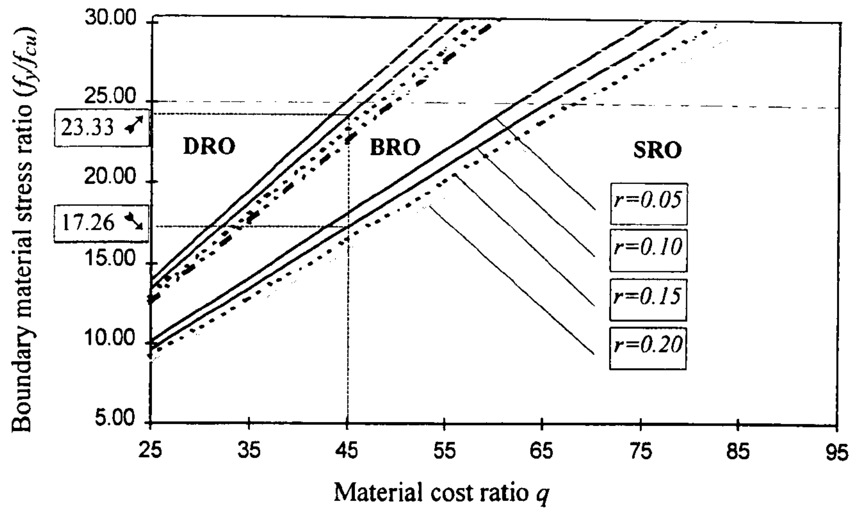


Figure 3.6 *Optimum solutions - r-curves*

Having selected values of q and r, the boundary allowable material stress ratios are read off the vertical axis. For example, with q=45 and r=0.10, the upper bound f_y/f_{cu} value for a singly reinforced section is 17.26 and the lower bound value for a doubly reinforced section is 23.33. If the values of f_y and f_{cu} are chosen such that their ratio is less than 17.26, the optimum solution will give a singly reinforced beam. If the values of f_y and f_{cu} are selected so that their ratio is greater than 23.33 then the optimum solution will give a doubly reinforced beam. Ratios between 17.26 and 23.33 result in a singly reinforced beam with boundary reinforcement as the optimum solution.

To compare the individual material costs with their total cost at the optimum solution, the research has introduced cost factors C_{tc}/C_t and C_{ts}/C_t. The cost of the concrete per unit length C_{tc} is given by

$$C_{tc} = C_c bd(1+r) \tag{3.31}$$

For a singly reinforced section the ratio C_{tc}/C_t is derived to be

$$\frac{C_{tc}}{C_t} = \frac{1}{1+\rho_{s\,opt}\, q/(1+r)} \tag{3.32}$$

where C_t is given by equation (3.7) and $\rho_{s\ opt}$ is the optimum reinforcement ratio for a singly reinforced beam.

The ratio C_{ts}/C_t is derived to be

$$\frac{C_{ts}}{C_t} = \frac{1}{1 + (1+r)/\rho_{s\,opt}q} \tag{3.33}$$

For a doubly reinforced section, the ratio C_{tc}/C_t is derived to be

$$\frac{C_{tc}}{C_t} = \frac{1}{1 + q\rho_{d\,opt}(2\rho_{d\,opt} - 0.231f_{cu}/f_y)/(1+r)} \tag{3.34}$$

where $\rho_{d\,opt}$ is the optimum reinforcement ratio for a doubly reinforced beam.

Correspondingly, ratio C_{ts}/C_t is derived to be

$$\frac{C_{ts}}{C_t} = \frac{2\rho_{d\,opt} - 0.231f_{cu}/f_y}{2\rho_{d\,opt} - 0.231f_{cu}/f_y + (1+r)/q} \tag{3.35}$$

For different values of stress ratio f_y/f_{cu}, r, and q, the material costs ratios can be compared. The example given by Figure 3.7 shows the material cost factor comparison for f_y=460 N/mm^2, f_{cu}=30 N/mm^2 and r=0.10.

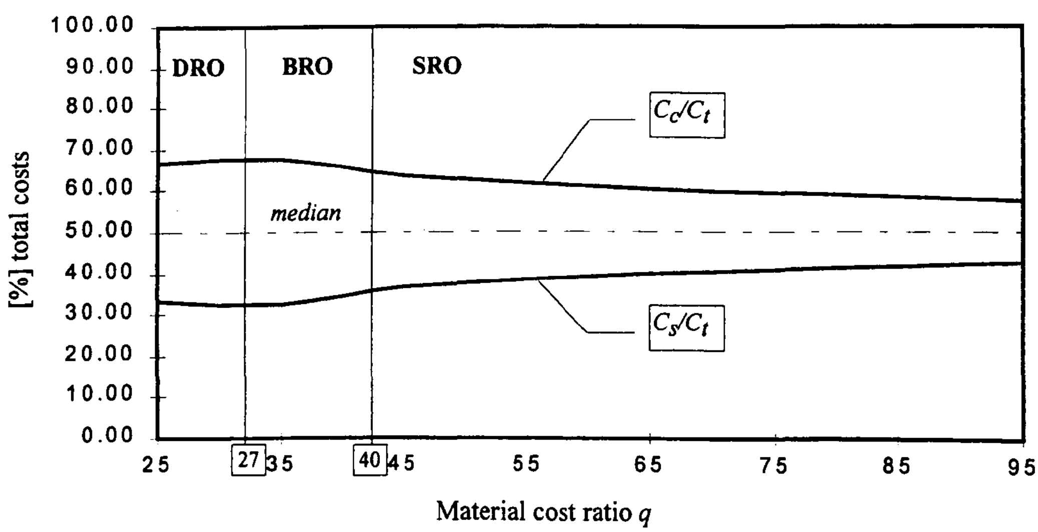


Figure 3.7 *Percentage material costs for* f_y=460 N/mm^2, f_{cu}=30 N/mm^2 and r=0.10

Three distinct zones are defined, depending on the beam having a singly (SRO), doubly (DRO) or boundary (BRO) reinforcement ratio as the optimum solution. The lower bound value of q for a singly reinforced section is 40. At the interface between boundary and double reinforcement the optimum reinforcement ratio for a doubly reinforced section is equal to ρ_{bound} plus 0.002 (minimum compression steel ratio specified by BS8110). For this condition, it can be calculated that the upper bound value of q is 27.

The research results point to the conclusion that the cost of concrete compared to the total costs shows a steady decrease as the value of q increases, behaving asymptotically to the median in the zone of the singly reinforced optimum solution. To further investigate this behaviour the ratio C_{tc}/C_t for a singly reinforced section was re-defined by substituting equation (3.16) into equation (3.32) to give

$$\frac{C_{tc}}{C_t} = 1 - \frac{q/(1+r)}{2q/(1+r) + 1.96 f_y / f_{cu}} \tag{3.36}$$

Considering the limited practical range of f_y/f_{cu} between 5 and 25 it can be shown that

$$\lim_{q \to \infty} \left(\frac{C_{tc}}{C_t} \right) = 0.5 \tag{3.37}$$

Hence, for the practical range of q values in the singly reinforced zone it is concluded that the material costs of the concrete will never fall below 50% of the total costs regardless of the values of f_y/f_{cu}.

3.3.4 *Design Examples*

Three typical design examples are given, illustrating situations where the optimum solution is either a singly, boundary or doubly reinforced section. For given values of q, r and f_y/f_{cu}, the optimum solution is obtained and presented graphically. The optimum solution developed within this research is compared with the standard design approach and the results are presented in a tabular form.

Design Example 1 - Singly Reinforced Beam A beam of width b=260 *mm* is subjected to the maximum bending moment of 185 *kNm*. The ratio r is taken as 0.15, material cost ratio q as 75, and the costs of concrete as *50 £/m*^{3}. Characteristic strength of steel and concrete are 460 and 30 *N/mm*^{2} respectively, giving a material stress ratio f_y/f_{cu} of 15.33. The lower (d_l) and upper bound (d_u) effective depths are taken to be 300 *mm* and 800 *mm*, respectively.

Using Figure 3.6, the optimum solution is shown to be a singly reinforced section. Hence, from equation (3.16) ρ_{opt} is 0.0105 giving the corresponding optimum effective depth of the section d_{opt} obtained from equation (3.17) as 448 *mm*.

The required area of the reinforcement $A_{s\ req}$ is calculated to be 1223 *mm*^{2}. The corresponding total material cost of beam per unit length C is then obtained from equation (3.7) to be $0.2256C_c$ *£/m* at its minimum.

A graphical representation of the results is given in Figure 3.8, showing the optimum to lie on the bending stress constraint boundary with the cost objective function being tangential to the curve.

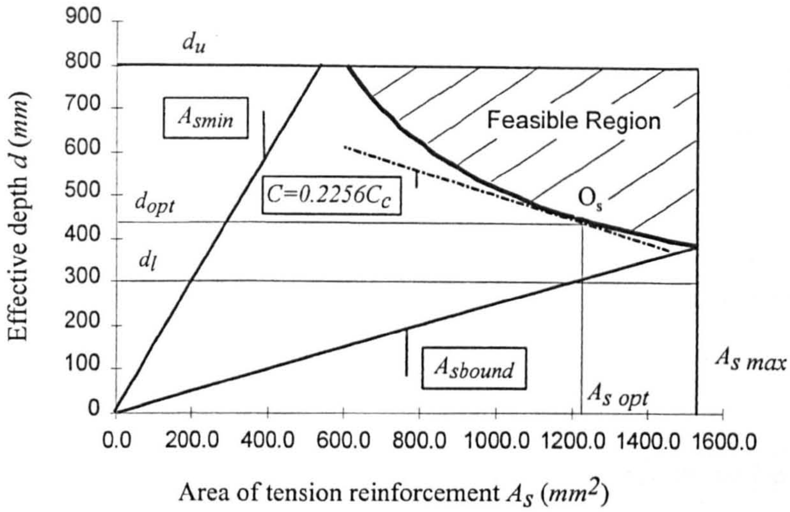


Figure 3.8 *Singly reinforced optimum solution*

The feasible region is bounded by the bending stress constraint, the upper bound effective depth and the maximum area of reinforcement $A_{s\,max}$ which corresponds with the intersection of the boundary reinforcement line with bending stress constraint. Table 3.2 shows the results using the standard design approach. It is evident from this table that the derived formulae for the singly reinforced optimum solution gives an accurate estimate of the minimum material cost of the beam.

Effective Depth d (mm)	Area of Tension Reinforcement A_s (mm^2)	Tension Reinforcement Ratio ρ_s	Total Material Costs(*C_c) (£/m)	LMM Solution (*C_c) (£/m)
390	1525.5	0.0150	0.2310	
400	1459.1	0.0140	0.2290	
440	1254.5	0.0110	0.2256	
448	1221.8	0.0105	0.2256	0.2256
460	1176.5	0.0098	0.2258	
480	1109.2	0.0089	0.2267	
500	1050.3	0.0081	0.2283	
540	951.7	0.0068	0.2328	
580	871.7	0.0058	0.2388	
640	775.9	0.0047	0.2496	
680	723.7	0.0041	0.2576	
760	640.3	0.0032	0.2753	
800	608.2	0.0029	0.2848	

Table 3.2 *Comparison between the optimum and standard design approach - Example 1*

Design Example 2 - Boundary Reinforced Beam The same design parameter values are used as in the previous example with the following exceptions. The material cost ratio q is 45, characteristic strength of concrete is 25 N/mm^2 and the lower and upper bound effective depths are 340 mm and 680 mm respectively.

With f_y/f_{cu}=18.4, Figure 3.6 indicates that the optimum solution is a boundary reinforced section. From equation (3.19) ρ_{opt} is 0.01255 giving a corresponding d_{opt} obtained from equation (3.17) of 428 mm.

The required area of the reinforcement is therefore calculated to be 1397 mm^2. The corresponding total material cost of beam per unit length C is then obtained from equation (3.7) to be $0.1904C_c$ *£/m* at its minimum.

The optimum result is presented graphically on the 2D-design surface (A_S,d) in Figure 3.9.

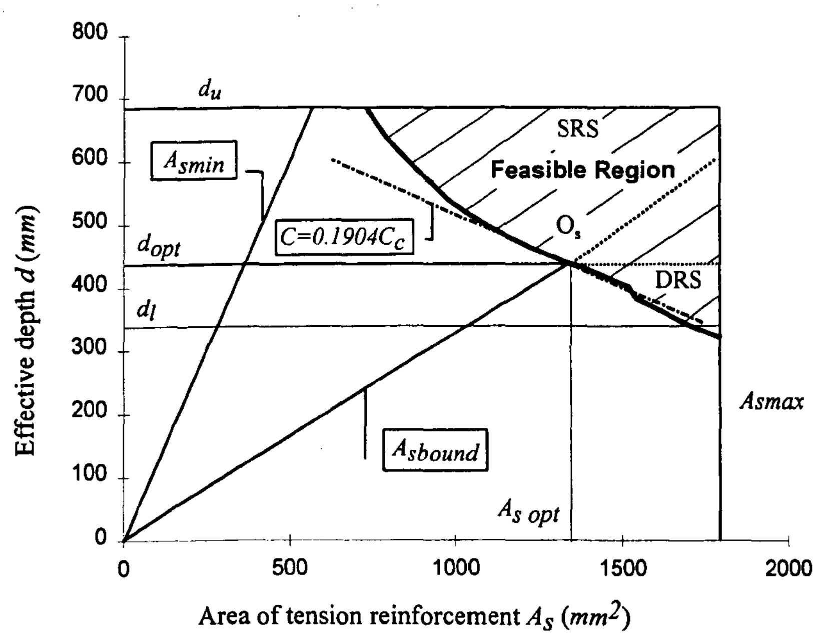


Figure 3.9 *Boundary reinforced optimum solution*

Figure 3.9 shows that the design space is discontinuous with the feasible region consisting of a singly (SRS) and a doubly (DRS) reinforced solution space. The optimum solution lies on the bending stress constraint boundary at the point of intersection with the boundary reinforcement. As in the previous example the cost objective function is tangential to the bending stress constraint curve. Table 3.3 shows the results using the standard design approach, with the optimum solution being comparable to that given by the Lagrangian Multiplier Method.

Effective Depth d (mm)	Area of Compression Reinforcement A_s' (mm^2)	Area of Tension Reinforcement A_s (mm^2)	Total Material Costs(*C_c) (£/m)	LMM Solution (*C_c) (£/m)
340	586.1	1697.6	0.2044	
360	437.6	1614.5	0.2000	
380	298.4	1540.8	0.1964	
400	208.0	1515.7	0.1972	
428	0.00	1388.0	0.1904	0.1904
440	0.00	1322.6	0.1911	
460	0.00	1229.9	0.1929	
480	0.00	1152.3	0.1954	
500	0.00	1085.9	0.1984	
540	0.00	977.0	0.2054	
580	0.00	890.6	0.2135	
620	0.00	819.7	0.2223	
660	0.00	760.3	0.2316	
680	0.00	734.0	0.2364	

Table 3.3 *Comparison between the optimum and standard design approach - Example 2*

Design Example 3 - Doubly Reinforced Beam The design parameter values are as those specified in Example 1 with the exception that the material ratio q is 25 and the lower bound effective depth is 300 *mm*.

With f_y/f_{cu}=15.33, Figure 3.6 indicates that the optimum solution is a doubly reinforced section. Applying equation (3.27) ρ_{opt} is 0.01796 giving a corresponding optimum effective depth of the section d_{opt} from equation (3.28) as 354 *mm*.

The required area of tension reinforcement is calculated to be 1653 mm^2. The corresponding total material cost of beam per unit length C is then obtained from equation (3.22) to be 0.1541C_c *£/m* at its minimum.

The optimum result is presented graphically on the design surface (A_S,d) in Figure 3.10.

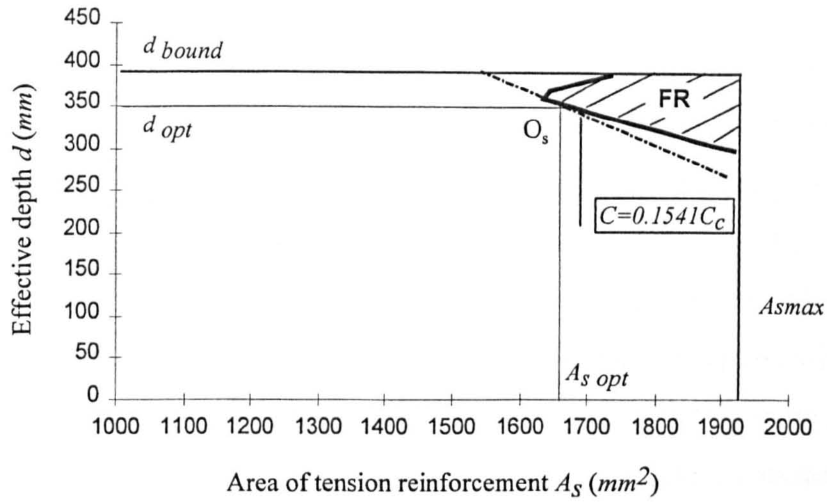


Figure 3.10 *Doubly reinforced optimum solution*

The optimum solution lies on the doubly reinforced stress constraint boundary with the objective function being tangential to the curve. The feasible region is bounded by the effective depth corresponding to a boundary reinforced section, its corresponding area of steel and the bending stress constraints for doubly reinforced section. Table 3.4 shows that the Lagrangian Multiplier Method and the standard design approach give comparable solutions.

Effective Depth d (mm)	Area of Compression Reinforcement A_S' (mm^2)	Area of Tension Reinforcement A_S (mm^2)	Total Material Costs(*C_c) (£/m)	LMM Solution (*C_c) (£/m)
300	739.7	1916.7	0.1561	
310	645.5	1861.7	0.1554	
320	554.9	1810.3	0.1548	
330	467.6	1762.3	0.1544	
340	383.4	1717.2	0.1542	
354	270.0	1658.8	0.1541	0.1541
370	192.4	1644.0	0.1565	
380	197.6	1688.4	0.1608	
390	202.8	1732.8	0.1650	

Table 3.4 *Comparison between the optimum and standard design approach - Example 3*

3.4 Conclusions

This chapter demonstrates that the Lagrangian Multiplier Method can be successfully employed to determine the minimum cost design of both singly and doubly reinforced concrete beams, offering an approach that can be used without prior knowledge of mathematical optimisation. A comparison of the solutions achieved with the Lagrangian Multiplier Method to those obtained using the standard design approach, clearly showed that the former gives the minimum material cost. The research method has identified three distinct optimal solutions that are dependent on whether the beam is singly, boundary or doubly reinforced. The boundaries between these zones are defined over the practical range of the material stress ratio f_y/f_{cu}, and are shown to be dependent on the adopted values of ratios q and r. The flexural stress constraints are critical with the minimum cost contour being a tangent to its boundary. For an increase in the material cost ratio q, it is concluded that the minimum material costs are achieved through a reduction of the percentage reinforcement in the beam. Under identical loading conditions, this reduction is compensated by an increase in the effective depth of the section.

To help the designer to select the optimum reinforcement ratio, parametric design curves and tables have been developed in the research to simplify the design process. However, in using either these design aids or the optimum design formulae, consideration should be given to the assumptions made. It is important to note that the cost sensitivity analysis has been performed on the material costs only and do not include the additional costs of formworking and labour, which in practice often make a significant contribution to the total costs. In contrast to the precast concrete industry, where labour and formworking costs are significantly lower than those of concreting *in situ*, the inclusion of these additional costs is of essential importance for an economical approach to design and manufacture.

A constant reinforcement ratio along the length of a beam has been assumed taking no account of potential cost savings through curtailment. Furthermore, under identical loading and geometrical conditions, the assumption of a constant applied bending moment is correct only for statically determinate structures. In the case of statically indeterminate structures such as continuous beams, member forces will vary as a function of the beam's geometric properties and should therefore be treated as variables. The ratio of cover to reinforcement to effective depth (r) is also a variable, although it has been found within the research to have a non-significant effect on the optimum solution.

Despite the simplification of the cost model and the assumptions made, the approach proposed in this research has the significant advantages of being simple and effective, without the need for iterative trials. Satisfactory and reliable results have been obtained when compared with the standard design approach. Moreover, the results have provided a valuable insight into the minimum material cost design of reinforced concrete beams for more complex skeletal structures that are considered in Chapter 5. The results show that the Lagrangian Multiplier Method has great potential as an effective approach for the optimum design of simple structural problems. It offers a reliable and relatively straightforward optimum design approach for flexural elements, and this work has been successfully employed in later research (see Section 5.7) for estimating upper bound reinforcement ratios.

4.

Volume Optimisation of Skeletal Systems

This chapter investigates the use of volume optimisation as the stepping stone towards a more complex cost optimisation formulation. Volume objective functions and design constraints for beams, columns and frames are derived. An improved approximation approach for sequential linear programming (SLP) has been derived. Sensitivity analysis and detailed testing are performed and the results reported.

4.1 Introduction

Reinforced concrete beams are one of the major components in skeletal structural systems, being rigidly jointed with the columns to constitute the form of the skeletal structure. For skeletal structural systems the number of design variables and constraints increases significantly with the total number of these components. The beam member forces and distribution of the reinforcement is more complex than the model proposed in Section 3.3, each being dependent on the frame topology and geometrical configuration. A survey of research work relating to the application of Lagrangian Multiplier Method (*see* Section 2.2.4), found that as the number of variables or constraints increases, achieving optimal solutions often becomes cumbersome or impractical. Given that the optimum design of these skeletal systems deals with complex and often discontinuous relationships between the large number of design variables and design equations, it was evident that the application of the LMM method was limited, and hence other approaches needed to be investigated. This chapter investigates an approximation technique based on sequential linear programming (SLP). The literature survey indicated a significant number of successful applications of the SLP method, although predominantly for steel skeletal structures. However, this survey gave a valuable insight into the implementation of the SLP method for reinforced concrete skeletal structures. Being cognisant of the complexity of formulating the minimum cost design problem for these skeletal systems, this research initially considered volume optimisation as an intermediate stage towards cost optimisation. The results show that the volume optimisation formulation offers a solution that is robust and mathematically stable whilst reducing the size of the programming problem compared to that of minimum cost design. Furthermore, it was found that the cost of the structure can also be evaluated from the minimum volume solution, being sensitive to the upper bound reinforcement ratio limits imposed on the structural elements (Fryer and Ceranic 1997). Further investigations also indicated that a good estimate of the minimum cost of volume optimised structures can be obtained (*see* Section 5.7), with the values of the upper bound beam reinforcement ratios calculated from equations derived using the LMM.

4.2 Sequential linear programming (SLP)

In this research, the method of Approximation Programming based on SLP, originally proposed by Griffith and Stewart (1961), is investigated to linearise the non-linear problem at the initial design point (x^o). This method belongs to a family of SLP techniques with a proven track record in structural optimisation. Schittkowski *et al.* (1994) in their detailed numerical comparison of a wide range of non-linear programming algorithms state that this method is quite robust and efficient and can be applied to solve real life design problems successfully, allowing for a large number of variables.

4.2.1 *Linearisation Approach*

The principle advantage of the Method of Approximation Programming is that the linearised problem can be solved by any standard linear programming software. In order to avoid high linearisation errors, particularly at the beginning of the algorithm, moving limits are introduced as proposed by Griffith and Stewart (1961). By introducing moving limits, the values of an independent variable may change by only a small amount at each step, keeping the linearisation errors within controllable and allowable bounds. Furthermore, Griffith and Stewart (1961) have formalised this linear approximation procedure into a well-defined mathematical algorithm

Minimise $\quad Z = f(x_1, x_2, \ldots\ldots\ldots\ldots, x_n) \qquad i = 1, 2 \ldots, n \qquad (4.1)$

subject to
$$h_j(x_i) = b_j \qquad j = 1, 2 \ldots, p$$
$$g_k(x_i) \le b_k \qquad k = 1, 2 \ldots, q$$
$$g_l(x_i) \ge b_l \qquad l = 1, 2 \ldots, r$$
$$k_{1i} \le x_i \le k_{2i} \tag{4.2}$$

where n is the number of design variables, p is the number of equality constraints, q and r are the number of inequality constraints respectively, b_j, b_k and b_l are constants, and k_{1i}, k_{2i} are the moving limits imposed on the design variables.

This method applies equally well to equality or inequality constraints. The mathematical problem defined in this research is to minimise the objective function in the region of the initial design point (x_i^o). The linearised objective function can be expressed as

$$Z = f\left(x_1^0, \ldots, x_n^0\right) + \sum_{i=1}^{n}\left(x_i - x_i^0\right)\left[\frac{\partial f\left(x_1^0, \ldots, x_n^0\right)}{\partial x_i}\right] \tag{4.3}$$

subject to linearised constraints of the form

$$h_j\left(x_1^0, \ldots, x_n^0\right) + \sum_{i=1}^{n}\left(x_i - x_i^0\right)\left[\frac{\partial h_j\left(x_1^0, \ldots, x_n^0\right)}{\partial x_i}\right] = b_j$$

$$g_k\left(x_1^0,\ldots,x_n^0\right)+\sum_{i=1}^{n}\left(x_i-x_i^0\right)\left[\frac{\partial g_k\left(x_1^0,\ldots,x_n^0\right)}{\partial x_i}\right]\leq b_k$$

$$g_l\left(x_1^0,\ldots,x_n^0\right)+\sum_{i=1}^{n}\left(x_i-x_i^0\right)\left[\frac{\partial g_l\left(x_1^0,\ldots,x_n^0\right)}{\partial x_i}\right]\geq b_l$$

$$k_{1i} \leq x_i \leq k_{2i} \tag{4.4}$$

To achieve a formulation that is suitable for solution using linear programming, equations (4.3) and (4.4) are modified as detailed in Appendix A.

4.3 Formulation of Structural Problem

The matrix *joint displacement* method is employed in this research to determine the member forces for the reasons outlined in Section 1.2.3. The volume objective function and the stress constraints are derived and presented in a form that is suitable for implementation using SLP. The stress constraints are derived according to BS8110 for both beam and column members. Other practical constraints in this formulation are the stiffness, deflection, and upper and lower bound dimensional constraints.

4.3.1 *Volume Objective Function*

In the design of reinforced concrete structures it is common to group elements having the same cross-sectional dimensions depending on their location within the skeletal system. Whilst this often increases the structural volume, the costs associated with the formworking are considerably reduced, resulting in a more economical structure. A non-linear volume objective function developed in this research takes advantage of this principle by grouping those structural members with the same cross-sectional dimensions. The volume objective function is therefore expressed as

$$V=\sum_{j=1}^{NBG}\sum_{k=1}^{NBBG}b_j h_j L_{jk}+\sum_{m=1}^{NCG}\sum_{n=1}^{NCCG}bc_m hc_m H_{mn} \tag{4.5}$$

where NBG and NCG are the total number of beam and column groups respectively, $NBBG$ is the number of beams in the j-th beam group, $NCCG$ is the number of columns in the m-th column group, b_j is the beam breadth, h_j is the overall beam depth , bc_m and hc_m are the breadth and depth of the column section respectively, L_{jk} is the length of the beam and H_{mn} is the height of the column.

Figure 4.1 shows the diagrammatic representation of the adopted notation for the three storey-two bay frame, with three beam and two column groups.

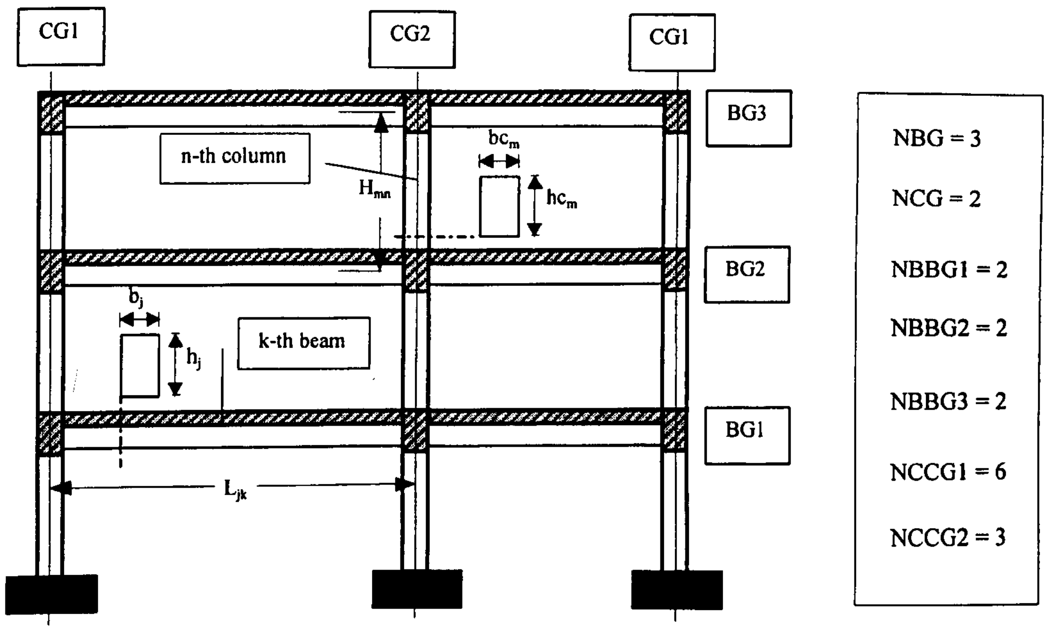


Figure 4.1 *Diagrammatic representation of the objective function notation*

4.3.2 The Stiffness Equality Constraints

During an analysis, it is necessary to select the overall stiffness matrix $[K]$ so that the structure is capable of resisting the applied load $\{W\}$. This is normally achieved by imposing a set of *stiffness* constraints of the type

$$\{W\} = [K]\,\{X\}$$

or

$$\{W\} = [D]^{\mathrm{T}}\,[k]\,[D]\{X\} \tag{4.6}$$

where $\{W\}$ is the vector of external joint loads, $[D]$ is the displacement transformation matrix, $[k]$ is the unassembled element stiffness matrix and $\{X\}$ is the vector of joint displacements.

The number of stiffness constraints required for the formulation of the optimisation problem can be significant, being directly related to the number of degrees of freedom for a particular structural problem. As reported by Fryer (1985), these constraints influence the efficiency and stability of the corresponding linearised problem to be solved by the Simplex Method, as often the artificial variables refuse to leave the simplex table. In such cases, infeasible solutions are encountered and the stability of the whole optimisation algorithm is degraded. Although removal of the stiffness equalities significantly reduces the number of artificial variables, this requires that joint displacements are not considered as design variables. In the design of reinforced concrete skeletal structures, the deflection limitations are normally satisfied using a simple approach based on the limiting span/effective depth ratios as detailed in BS8110 and explained in section 4.3.7. By omitting deflection constraints it is no longer necessary to introduce the stiffness equalities as design constraints. However, Fryer (1985) states that this approach is only possible if the member forces are kept constant between global iterations and with the following assumptions

(i) Changes in the member forces are small after each global iteration.

(ii) As the design vector approaches the optimum, the member forces converge and remain stable not significantly changing from one iteration to the other.

Assumption (i) is essential if the programming problem is to be well behaved and converge to an optimum. Large changes to the internal member forces could encourage divergence and instability due to the erratic changes in the bending moments and shear forces. As the design vector approaches the optimum, the element dimensions converge and variations in the self-weight of the structure are insignificant. Since the applied external loads are constant the internal forces must converge as the optimum is approached. Assumption (ii) is therefore a statement of this fact.

This research adopts the principle of only retaining the critical constraints by removing the stiffness equalities from the problem formulation. This approach not only reduces the size of the programming problem but also considerably simplifies the stress constraints, resulting in a more robust and mathematically stable problem formulation.

4.3.3 *Bending Stress Constraints in Beams to BS8110 - Rectangular Section*

In Section 3.3, bending equilibrium equations were developed for both singly and doubly reinforced rectangular sections as follows:

Singly reinforced section

$$\frac{M}{bd^2} = 0.87 f_y \rho \left[1 - 0.979 \frac{f_y}{f_{cu}} \rho \right] \tag{4.7}$$

Doubly reinforced section

$$\frac{M}{bd^2} = 0.156 f_{cu} + f_{cu} \left[0.87 \frac{f_y}{f_{cu}} \rho - 0.2 \right] \left[1 - \frac{d'}{d} \right] \tag{4.8}$$

where M is the ultimate design moment, f_y is the characteristic strength of steel, f_{cu} is the characteristic concrete strength, b is the breadth of the beam, d is the effective depth of the beam, d' is the depth from the top of the compression face to the centroid of the compression reinforcement and ρ is the tension reinforcement ratio.

BS8110 specifies lower and upper bound values for the tension reinforcement ratio as part of the serviceability limit state requirements. This ensures adequate strength and avoids the possibility of instantaneous collapse. In this research, the area of tension reinforcement is removed as a design variable to simplify the constraints and thus equations (4.7) and (4.8) are modified by specifying a limiting value on the tension reinforcement ratio. Equations (4.7) and (4.8) are thus transformed from equality to inequality constraints of the form

$$\frac{M}{bd^2} \le f_{ub} \tag{4.9}$$

where f_{ub} is an upper bound bending stress obtained using the right hand expressions of equations (4.7) and (4.8). This approach requires the designer to decide at the outset on the limiting value of the tension reinforcement ratio, i.e. whether the beam will be restricted to single reinforcement or otherwise. In determining the value of f_{ub} it is important to note that equation (4.7) is only applicable between ρ_{min} and ρ_{bound} (*see* equation 3.19), whilst equation (4.8) only applies between ρ_{bound} and ρ_{max}.

4.3.4 *Bending Stress Constraints in Beams to BS8110 - (T- or L-) Section*

Two possible section designs are considered in this research assuming that each individual beam under the slab has either a *'T'* or an inverted *'L'* section, consisting of a vertical web (or rib) surmounted by a flange as shown in Figure 4.2. When the beams are resisting sagging moments, part of the slab acts as a compression flange and the members may be designed as *T*- or *L*- beams. With hogging moments the slab will be in tension and assumed to be cracked, therefore the beam must be designed as a rectangular section of width b_w and overall depth h. In all cases, the effective depth d is given as the distance from the top of the compression zone to the centroid of the tension reinforcement. The depth of the flange h_f relates to the surmounting superstructure and hence is treated as constant in this problem formulation.

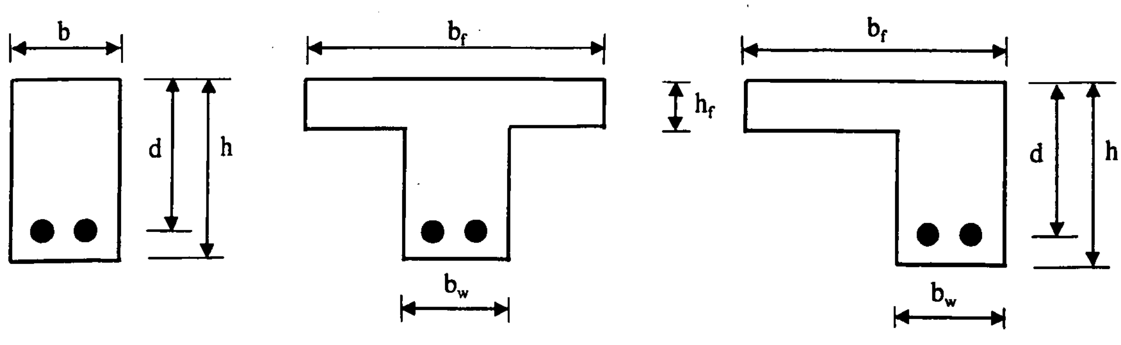


Figure 4.2 *Rectangular, T - and L - beam sections*

When the slab acts as the flange its effective width b_f is defined by empirical rules that are specified in BS8110 as follows:

'T' beams - the lesser of the actual flange width, or the width of the web (b_w) plus $l_z/5$

'*L*' beams - the lesser of the actual flange width, or the width of the web (b_w) plus $l_z/10$

where l_z is the distance taken between points of zero moments, and for a continuous beam may be taken as 0.7 times the effective span.

For *T*- or *L*- sections the derivation of the upper bound stress f_{ub} requires consideration of three cases.

Case 1 - Neutral axis within the flange (sagging moment) The design corresponds to the case of a singly reinforced section with breadth equal to the effective width of the flange, i.e. $b = b_f$. The expression for the upper bound bending stress f_{ub} is given by equation (4.7) with the tension reinforcement ratio being a function of the effective width of the flange.

Case 2 - Neutral axis within the web (sagging moment) Using the simplified rectangular stress block, the upper bound bending stress has been derived as

$$f_{ub} = 0.87 f_y \rho [1\text{-}0.5(h_f/d)] - 0.1 f_{cu} [0.45\text{-}(h_f/d)] \qquad (4.10)$$

This formula has been derived assuming that the redistribution of the moments does not exceed 10%, and that the depth of the flange is less than or equal to 0.45*d*.

Case 3 - Neutral axis within the web (hogging moment) The section is designed as a rectangular beam section with the breadth equal to the width of the web, i.e. $b=b_w$. Tension reinforcement is placed in the flange, and compression reinforcement, if required, is placed in the web. The upper bound bending stress is given by equation (4.7) with the tension reinforcement ratio being a function of the actual width of the web b_w.

4.3.5 *Shear Stress Constraints for Beams to BS8110*

To prevent shear failure in the section, the average shear stress *v* must not exceed the minimum value given by BS8110. The shear stress due to the shear forces only (excluding torsional shear stresses) is expressed as

$$v = \frac{V}{bd} \le v_{max} \qquad (4.11)$$

where V is the ultimate design shear force, b is the breadth of the web for a *T*- or *L*-section and v_{max} is the upper bound shear stress given by

$$v_{\max} = \min\left(0.8\sqrt{f_{cu}},\ 5\frac{N}{mm^2}\right) \tag{4.12}$$

The upper bound shear stress value has been derived according to BS8110, and a coefficient of reduction of 0.5 has been applied to prevent shear *over-stress*.

4.3.6 *Columns Stress Constraints*

Column design is distinctively different to that of beams as the internal forces consist of a combination of axial forces and bending moments. They can be considered as either *braced* or *unbraced*, and *short* or *slender*. This research only considers the design of *short braced* columns with combined bending and axial load as these represent the majority of columns in practice. Columns are considered *braced* in a particular plane when other structural elements such as shear walls have been designed to resist all the horizontal forces acting on the structure in that plane. Furthermore, a column is considered as *short* when both ratios l_{ex}/h and l_{ey}/h are less than 15 for braced and 10 for unbraced columns, where l_{ex} and l_{ey} are the effective heights of a column bending about the *x-x* and *y-y* axis respectively. Columns should otherwise be considered as *slender*. In deriving the column stress constraints, this research considered the equilibrium of a section when subjected to a combination of flexure and axial force as shown in Figure 4.3.

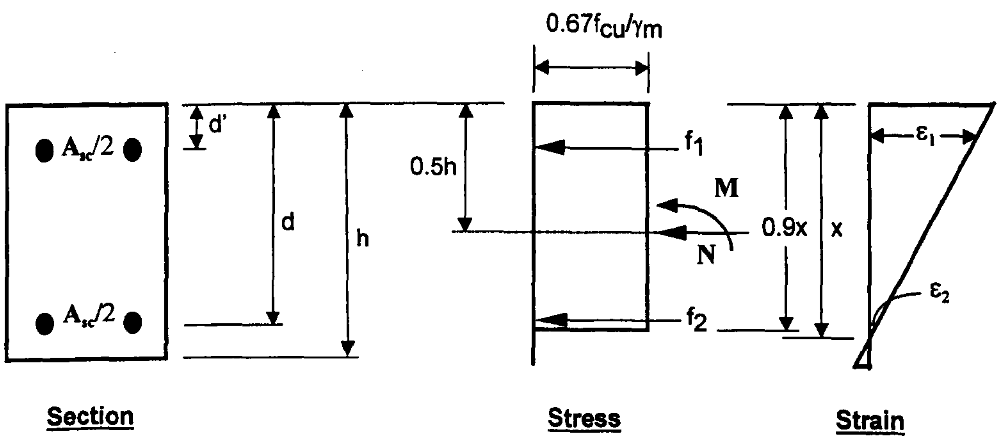


Figure 4.3 *Distribution of stress and strain in a rectangular column section*

Considering a simplified rectangular stress block distribution with the neutral axis within the section, the area of both sets of reinforcement are equal and are in compression, with the compressive stresses and strains being taken to be positive. The following procedure for deriving the column stress constraints has been developed.
Resolving the horizontal forces acting upon the section it can be shown that

$$\frac{N}{bhf_{cu}} = 0.402\frac{x}{h} + n\alpha \tag{4.13}$$

Similarly, taking moments about the centre line of the section we have

$$\frac{M}{bh^2 f_{cu}} = 0.402\frac{x}{h}\left(0.5 - 0.45\frac{x}{h}\right) + m\alpha \tag{4.14}$$

where

$$n = (f_1+f_2)/2f_y$$
$$m = (2d/h - 1)(f_1-f_2)/4f_y$$
$$\alpha = A_{sc}f_y/bhf_{cu} = \rho_{sc}f_y/f_{cu} \tag{4.15}$$

Here, N is the ultimate design axial force, M is the ultimate design bending moment, x is the depth of neutral axis, and f_1 and f_2 are the stresses in the top and bottom reinforcement set respectively.

Equations (4.13) and (4.14) are the *design equations* for columns with combined bending and axial load. These equations are complex and can only be solved by an iterative procedure, requiring numerous combinations of possible stresses in the column to be considered. In BS8110 the solution is therefore presented graphically, by a series of column graphs depending on the values of N/bhf_{cu}, M/bh^2f_{cu} and d/h, assuming that high yield steel f_y=460 N/mm^2 is always used. In this research however, an analytical formulation of the constraints is required for implementation by the SLP method. A mathematical model for short braced column design constraints is proposed in the following form

Constraint 1 $$\frac{N}{b_c h_c} + m_1\frac{M}{b_c h_c^2} \le c_1 f_{cu}$$

Constraint 2 $$-\frac{N}{b_c h_c} + m_2 \frac{M}{b_c h_c^2} \leq c_2 f_{cu} \tag{4.16}$$

where m_1, c_1, m_2 and c_2 are constants determined from the column design equations and are a function of the limiting percentage reinforcement ratio as follows:

$$\begin{aligned} m_1 &= \mathrm{abs}\{[d(y) - a'(y)]/d(x)\} \\ m_2 &= \mathrm{abs}\{[d(y)-e(y)]/ [d(x)-e(x)]\} \\ c_1 &= \mathrm{abs}[a'(y)] \\ c_2 &= \mathrm{abs}\{-e(x)[d(y) - e(y)]/ [d(x)-e(x)]\} \end{aligned} \tag{4.17}$$

where

$$\begin{aligned} a'(y) &= f_{cu}\,[0.45 + 0.87\alpha] \\ d(y) &= 0.256(d/h)f_{cu} \\ d(x) &= f_{cu}\,[0.2558d/h(0.5 - 0.2864d/h) + (0.87d/h - 0.4348)\alpha] \\ e(y) &= 0.1f_{cu} \\ e(x) &= f_{cu}\,[0.03883 + 0.435(2d/h - 1)\alpha] \\ \alpha &= \rho(f_y/f_{cu}) = [A_s/bd](f_y/f_{cu}) \end{aligned} \tag{4.18}$$

These constants represent the gradients and intercepts of the constraints, as shown in Figure 4.4.

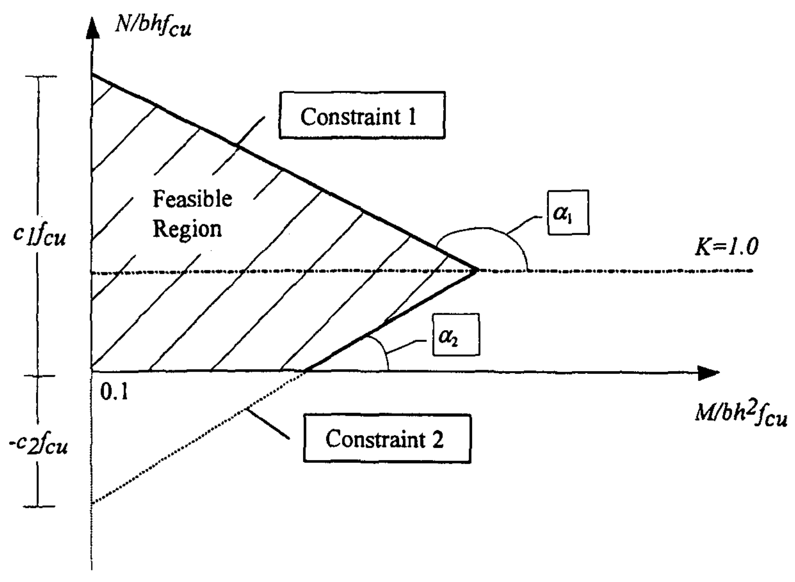


Figure 4.4 *Mathematical model for column constraints*

It may be observed that the column design constraint equations (4.16), (4.17) and (4.18) require the designer to assume a limiting value for the reinforcement ratio at the beginning of the optimisation process.

4.3.7 *Deflection Constraints*

BS8110 states that neither the efficiency nor the appearance of a structure should be harmed by the deflections which occur during its life. However, as stated by Fryer (1985) the deflection constraints only become effective if flexural or shear stresses derived from the ultimate limit states do not influence the final design of a structural element. As stated in Section 4.3.2, deflection requirements in BS110 are satisfied using a simplified approach where span/effective depth ratios are specified. The lower bound value on the depth of beams is set to ensure that the span/effective depth ratios are satisfied. These limitations ensure that the deflections are not excessive, and hence the need for explicit deflection constraints is avoided. Since the optimisation procedure deals with overall depths as design variables rather than effective depths, the following relationship is used throughout this research

Overall Depth $\geq$ Span/Basic Ratio + Depth to the Centroid of the Reinforcement from the tension face

where the basic ratio is obtained from BS8110.
Long term deflections in the structure are not considered in this research.

4.3.8 *Dimensional Constraints*

The imposed constraints are the lower and upper bound limitations placed upon the beam and column group cross-sectional dimensions. The lower bound dimensional constraints are formulated such that they limit excessive deflections or satisfy the maximum reinforcement ratios allowed by BS8110. The upper bound dimensional constraints take into account practical design considerations and buildability requirements that are specific to each structural design. In this research, the upper bound beam depths have been initially estimated to ensure that their resulting reinforcement ratios are greater than the minimum specified in BS8110. However, this is only a guideline for setting the upper bound dimensions, the final limiting values requiring the engineer to take account of other design considerations.

4.4 Implementation of SLP

The implementation of the method as outlined in Section 4.2 has been thoroughly investigated in this research. Derivations of the design constraints and the volume objective function are given, together with an outline of the method's implementation. The final results have been extensively tested, compared and a sensitive analysis was performed.

The objective function and design constraints are simultaneously approximated by their tangent planes at a given initial design point. The essence of the methodology is the assumption that these linear approximations are acceptable only over a narrow range. Therefore, the values of the independent variables are allowed to change by only a small amount at each step, controlled by the use of moving limits (*see* Appendix A). Linearised forms of the volume objective function and the design constraints are derived and their final forms presented.

4.4.1 *Problem Statement*

In this research, the volume optimisation problem is defined in the following non-linear form

Minimise $$V = f(x_1, x_2, \ldots, x_n) \qquad i = 1,2 \ldots n \tag{4.19}$$

subject to
$$g_k(x_i) \leq b_k \qquad k = 1,2 \ldots p$$
$$g_l(x_i) \geq b_l \qquad l = 1,2 \ldots r$$
$$k_{1i} \leq x_i \leq k_{2i} \tag{4.20}$$

where $V = f(x_1, x_2, \ldots, x_n)$ is the volume objective function to be optimised, $g_k(x_i)$ and $g_l(x_i)$ are the set of inequality design constraints.

4.4.2 *Linearisation of the objective function*

Applying the approximation programming technique, the objective function V is linearised using the expression

$$V = f(x_o) + Df(x_o)\,(\{x_i\}-\{x_o\}) \tag{4.21}$$

where $f(x_o)$ is the original function evaluated at the initial design point, $Df(x_o)$ is the gradient vector at the initial design point, $\{x_i\}$ is a vector of the new unknown section variables and $\{x_o\}$ is a vector of known section variables at the initial design point.

Considering the i-th beam and k-th column, the linearised volume function given by equation (4.21) can be expressed as

$$V_{ik} = (b_i^o h_i^o L_i + bc_k^o hc_k^o H_k) + [\, h_i^o L_i \;\; b_i^o L_i \;\; hc_k^o H_k \;\; bc_k^o H_k \,] \begin{bmatrix} b_i^{'} - b_i^0 \\ h_i^{'} - h_i^0 \\ bc_k^{'} - bc_k^0 \\ hc_k^{'} - hc_k^0 \end{bmatrix} \tag{4.22}$$

where the superscript (°) represents those variables evaluated at the initial design point.

By rearranging equation (4.22), the constant values $\sum_{j=1}^{NBG}\sum_{k=1}^{NBBG} b_j^0 h_j^0 L_{jk}$ and $\sum_{m=1}^{NCG}\sum_{n=1}^{NCCG} bc_m^0 hc_m^0 H_{mn}$ may be disregarded, since these expressions have no influence on the corresponding volume objective function.

Summing for the whole structure, the form of the linearised volume objective function in this research is given by

$$V = \sum_{j=1}^{NBG} b_j^{'} \sum_{ki=1}^{NBBG} h_j^0 L_{jk} + \sum_{j=1}^{NBG} h_j^{'} \sum_{ki=1}^{NBBG} b_j^0 L_{jk} + \sum_{m=1}^{NCG} bc_m^{'} \sum_{n=1}^{NCCG} hc_m^0 H_{mn} + \sum_{m=1}^{NCG} hc_m^{'} \sum_{n=1}^{NCCG} bc_m^0 H_{mn} \tag{4.23}$$

It can be shown that to minimise a function is the same as to maximise it's negative. Equation (4.23) therefore, is multiplied by minus one, and then maximised.

4.4.3 *Linearisation of the Stress Constraints*

The stress constraints involve the use of highly non-linear expressions due to the member forces being functions of the second moment of area and the reciprocals of characteristic dimensional variables. However, this formulation of the stress constraints has been considerably simplified by the removal of the associated deflection design variables, such that the following matrix form is obtained

$$\{G\} = \{M : C\} = \{b_1\, h_1 \;.\;.\;.\; b_i\, h_i \;\; bc_1\, hc_1 \;.\;.\;.\; bc_k \;\; hc_k\,\} \tag{4.24}$$

Here, the sub-matrices {M} and {C} refer to the beams and columns respectively, defined as

$\{M\} = \{b_1\, h_1 \;.\;.\;.\; b_i\, h_i\,\}$ - $2i$ beam design constraints

$\{C\} = \{bc_1\, hc_1 \;.\;.\;.\; bc_k \;\; hc_k\}$ - $2k$ column design constraints

where i is equal to the number of beam groups *NBG*, and k is equal to the number of column groups *NCG*.

With design variables specified, the stress constraints are expressed as:-

$$G\{M{:}C\} = \frac{[k\{M{:}C\}]\{X\}}{f(\{M{:}C\})} - f_{up} \leq 0 \tag{4.25}$$

where $G\{M{:}C\}$ is the element stress function expressed in the terms of the design variables, $[k\{M{:}C\}]\{X\}$ represents the member forces that are assumed constant between global iterations, $f(\{M{:}C\})$ is a function of the element dimensions required to calculate the stress and f_{up} is the associated upper bound stress.

The following procedure for linearisation of the stress constraints has been applied in this research.

Each constraint is expressed in the form

$$G_i(x_j) \leq f_{up}$$

with $\qquad x_j \geq 0 \qquad j = 1, 2, \ldots, n \qquad (4.26)$

Equation (4.26) is first written as

$$g_i(x_j) = G_i(x_j) - f_{up} \leq 0 \tag{4.27}$$

and then linearised to become

$$g_i(x_0) + Dg_i(x_0)\,[\{x_i\} - \{x_0\}] \leq 0 \tag{4.28}$$

where $g_i(x_0)$ is the value of the constraint evaluated at the initial design point, $Dg_i(x_0)$ is the gradient vector evaluated at the same point, $\{x_i\}$ is the new unknown set of design variables and $\{x_0\}$ is an initial (known) set of design variables.

Rearranging equation (4.28) in terms of the variables and constants results in

$$Dg_i(x_0)\{x_i\} = Dg_i(x_0)\{x_0\} - g_i(x_0) \tag{4.29}$$

Evaluating equation (4.27) at the initial design point x_0 we obtain

$$g_i(x_0) = G_i(x_0) - f_{up} \tag{4.30}$$

The value of any constraint evaluated at the initial design point is expressed as

$$G_i(x_0) = f_0 \tag{4.31}$$

where f_0 is the actual stress in the element.

Substituting equations (4.30) and (4.31) into equation (4.29) yields

$$Dg_i(x_0)\{x_i\} = Dg_i(x_0)\{x_0\} + (f_{up} - f_0) \tag{4.32}$$

where $Dg_i(x_0)\{x_0\}$ is a product of the gradient vector and the initial design point vector, and f_{up} is the upper bound stress. The expression $Dg_i(x_0)\{x_0\}+(f_{up} - f_0)$ represents the right-hand side of the Simplex table.

To linearise the stress constraints it is necessary to construct the gradient vector $Dg_i(x_0)$ and calculate the initial stress f_0. The stress constraints for beams and columns are functions of dimensional variables only, as the joint displacement design variables are removed. For rectangular beams in bending this can be expressed as

$$G_b(M) = \frac{[k_1\{M\}]\{X\}}{f_1\{M\}} - f_{ub} \leq 0 \tag{4.33}$$

whilst in shear

$$G_s(M)=\frac{[k_2\{M\}]\{X\}}{f_2\{M\}}-f_{us}\leq 0 \tag{4.34}$$

where $G_b\{M\}$ and $G_s\{M\}$ represents the bending and shear stress constraints respectively. Matrices $[k_1\{M\}]\{X\}$ and $[k_2\{M\}]\{X\}$ represent the member forces that are presumed constant, whilst f_{ub} and f_{us} are the associated upper bound stresses in bending and shear respectively.

Derivates of $G\{M\}$ are then obtained with respect to the design variables breadth b and depth h. For example, the bending derivates are obtained as

$$\frac{\partial G_b}{\partial b}=[k_1\{M\}]\{X\}\frac{\partial}{\partial b}\left(\frac{1}{f_1\{M\}}\right)$$

$$\frac{\partial G_b}{\partial h}=[k_1\{M\}]\{X\}\frac{\partial}{\partial h}\left(\frac{1}{f_1\{M\}}\right) \tag{4.35}$$

For columns, the stress constraints consist of a combination of the axial and bending stresses, written as

$$G_c(C)=\frac{[k_3+k_4\{C\}]\{X\}}{f_c\{C\}}-f_{up}\leq 0 \tag{4.36}$$

Here $G_c\{C\}$ represents the stress constraints for the columns. Matrix $[k_3]$ contains the stiffness coefficients of the axial stresses and $[k_4\{C\}]$ contains the coefficients of the bending stresses. Derivates of $G_c\{C\}$ with respect to the dimensional variables are given by

$$\frac{\partial G_c}{\partial b_c}=[k_3+k_4\{C\}]\{X\}\frac{\partial}{\partial b_c}\left(\frac{1}{f_c\{C\}}\right)$$

$$\frac{\partial G_c}{\partial h_c}=[k_3+k_4\{C\}]\{X\}\frac{\partial}{\partial h_c}\left(\frac{1}{f_c\{C\}}\right) \tag{4.37}$$

Once linearisation has been undertaken, the resulting linearised problem can be solved by any standard linear programming software. In this research, the modified Two-Phase Simplex Method has been adopted as a powerful and robust solver for linear programming problems, that is adaptable to computer solution. Theory and implementation of this method is presented in Appendix B.

4.5 Development of the Computer Program

In the initial stages of the research a plane frame structural analysis computer program using the matrix *joint displacement* method for assembling the overall stiffness matrix was developed. Special care was paid to the storage of the stiffness equations. Using a structured addressing code, a significant saving in memory storage has been achieved by storing only the lower triangular elements of the stiffness matrix, as an irregular band beginning with the first non-zero element in each row. *Gaussian* elimination and *triangular* decomposition has been used for solving the simultaneous system of linear equations without involving any additional variable arrays. This helps to minimise memory requirements within the program. The method uses a *row-wise* compact elimination procedure to obtain the upper *Gaussian* triangle in transposed form. The decomposition is carried out with the variable bandwidth storing the final solution in the right hand side vector.

Further development of the computer program concentrated on solving the volume optimisation problem. Using the theory presented in this chapter, an optimisation solver was developed and incorporated within the existing computer program structure. The adopted approach for incorporating the optimisation process was based on the objective of creating an optimum design method comparable to the conventional heuristic approach, but using mathematical techniques in order to obtain an optimum solution. The design procedure flowchart of this approach is shown in Figure 4.5.

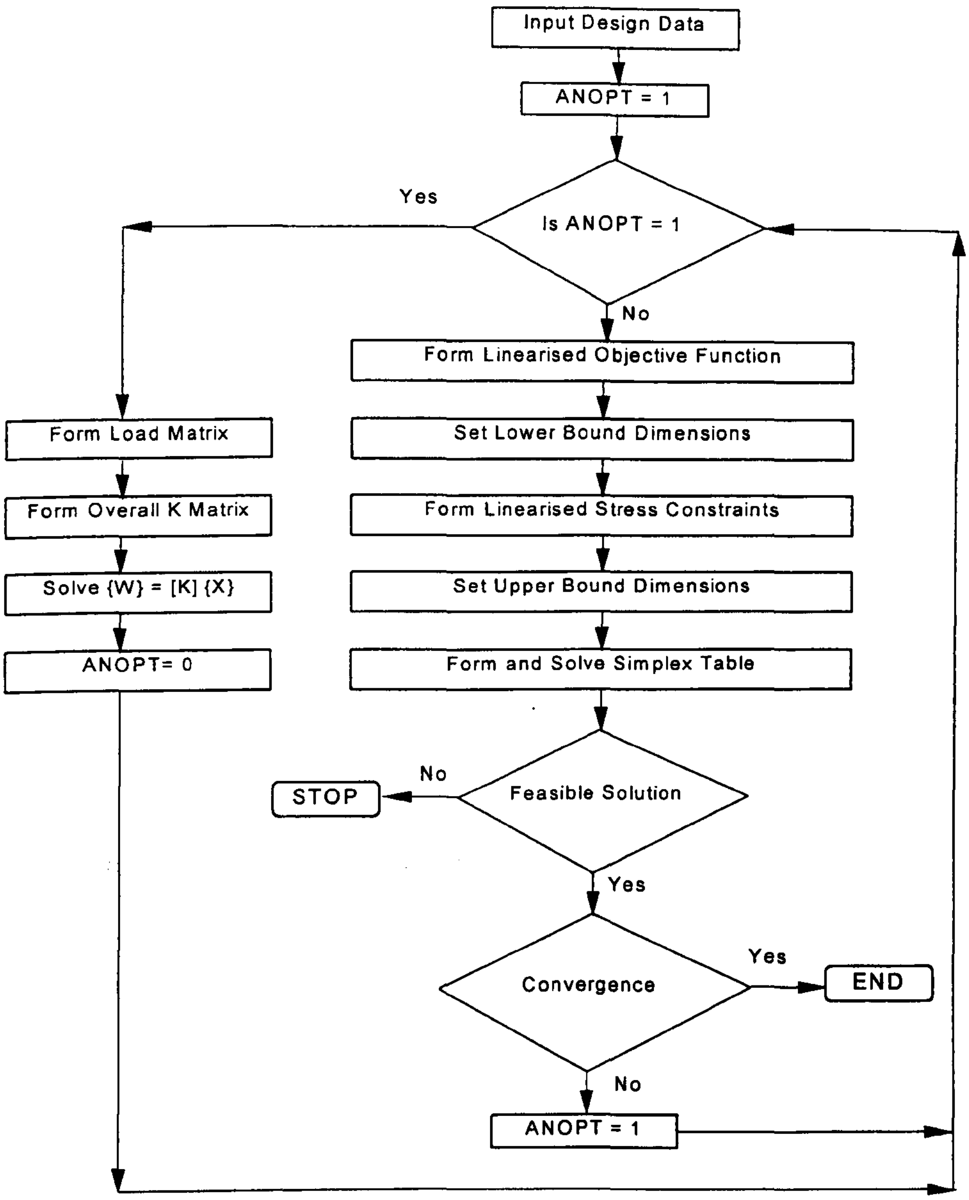


Figure 4.5 *Flow Diagram of the Computer Structural Optimisation Approach*

Figure 4.5 shows the two main functions of the resulting computer program; the full structural analysis and the solution of the programming problem. The cyclic sequence of operations is controlled by operator ANOPT, which effectively is a two-way switch allowing either an analysis or an optimum design to take place. The structural analysis (ANOPT=1) is performed after each successful solution of the simplex table providing that the convergence criteria has not yet been satisfied. Although this approach increases the computational time required, the ability to monitor true stresses and deflections in the structure ensures that the linearisation errors are kept within allowable tolerances. Hence, the resulting design is more accurate and the benefit of this outweighs the disadvantage of the increase in computational time.

4.6 Testing and Sensitivity Analysis

Detailed testing and sensitivity analysis of the algorithm's control parameters has been performed on a number of representative continuous beams and small to medium size reinforced concrete frames. In general, the minimum volume solution was reached when one or more of the stress constraints became critical, with the breadth of the both beams and columns being driven to their lower bound values. The results of a sensitivity analysis have shown a respectable degree of robustness in the performance of the implemented algorithm, with both the mathematical stability and computer efficiency being considerably improved due to the removal of the *stiffness* constraints. It was observed that the behaviour of the bending stress constraints for beams and both stress constraints for columns were dependent on the choice of the upper bound reinforcement ratio. This relationship had a direct influence on the minimum volume solution. Hence, to assist the designer in this choice further research concentrated on investigating algorithms that took account of the reinforcement ratio within the objective function so as to minimise the cost of a particular structural element. The results of this investigation are presented in Section 5.7 and compared to both cost optimisation and to the exhaustive search of standard design solutions using genetic algorithms.

4.6.1 Results - Upper Bound Reinforcement Ratio

Figure 4.6 shows a three-span continuous beam, with the length of each span and the corresponding loads (excluding self-weight) indicated in the figure. The partial safety factors for the imposed and dead loads are 1.6 and 1.4 respectively with a minimum partial safety factor of 1.0. The cover to reinforcement is 50 *mm* and the characteristic concrete and steel strengths are f_{cu}= 30 $N/mm2$ and f_y= 460 $N/mm2$, respectively. The example consists of one beam group with a lower bound breadth of 250 *mm*, a lower bound depth of 350 *mm*, an upper bound breadth of 500 *mm* and an upper bound depth of 800 *mm*. The results presented are representative of all the continuous beam problems investigated in this research.

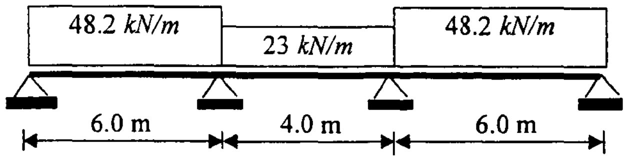


Figure 4.6 *Three-Span Continuous Beam*

The continuous beam was optimised over a range of upper bound bending stresses corresponding to reinforcement ratios between the minimum imposed by BS8110 (0.13%), and the boundary value at the singly/doubly reinforced interface. Figure 4.7 presents the relationship between the upper bound reinforcement ratio values and corresponding optimum depth and minimum volume. The breadth of the section was driven to its lower bound value of 250 *mm* in all of the investigated cases.

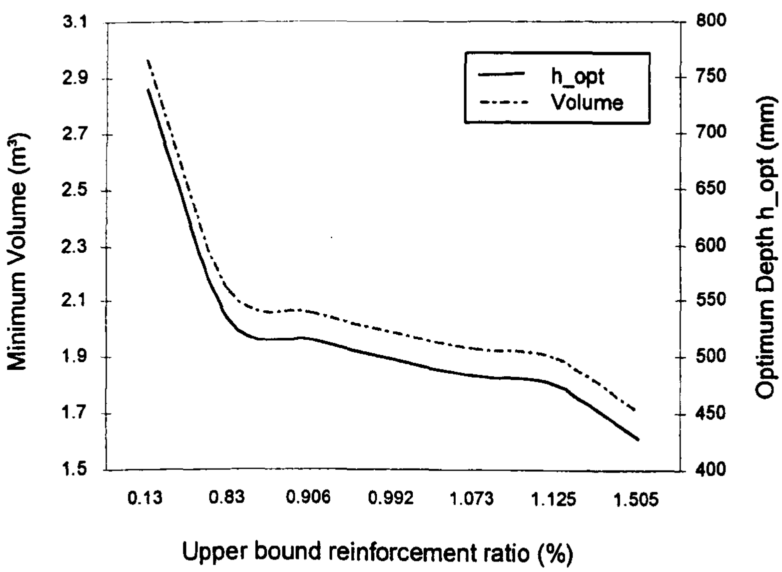


Figure 4.7 *Minimum volume and optimum depth versus upper bound reinforcement ratio*

Figure 4.7 shows that for an increase in the upper bound reinforcement ratio, the optimum solution exhibits a reduction in both the optimum depth and the minimum volume. Hence, the final optimum solution is directly dependent on the value of the upper bound reinforcement ratio chosen. Despite the decrease in the volume, as the upper bound reinforcement ratio increases there will be a corresponding increase in the reinforcement. When structural costs are considered, the chosen value of the upper bound reinforcement ratio becomes particularly important and this aspect is further investigated in Section 5.7. For each upper bound reinforcement ratio the bending stress was critical yielding a fully stressed design at the point of maximum design moment. For other beam problems where the shear constraint was found to be critical, it was observed that the choice of the upper bound reinforcement ratio has no influence on the final solution. This was due to the fact that the shear constraint is a function of the characteristic strength of the concrete only.

Figure 4.8 shows a heavily loaded braced frame structure with the geometrical properties and corresponding loads (excluding self-weight) as indicated.

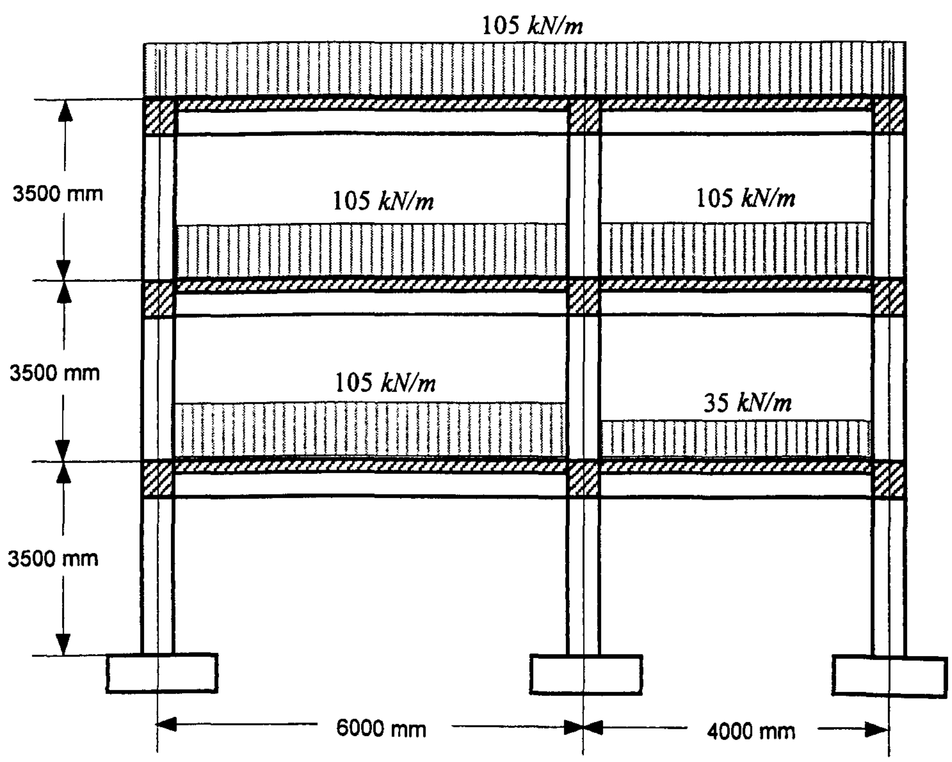


Figure 4.8 *Three Storey - Two Bay Frame*

The partial safety factors for the live and dead loads are 1.6 and 1.4 respectively with a minimum partial safety factor of 1.0. The modulus of elasticity is 28 *kN/mm²* with characteristic material strengths of f_{cu}=30 *N/mm²* for the concrete and f_y=460 *N/mm²* for the steel. The cover to the reinforcement is 50 *mm* for the both beams and columns. The frame consists of one beam and one column group with the breadth of the columns being the same as their connecting beams. The results presented in this chapter are representative of all the frame structures investigated for volume optimisation.

Table 4.1 shows the results for high (2%), medium (1.4%) and low (0.6%) values of the upper bound reinforcement ratio ρ_{upp} for beams, whilst keeping this ratio constant (2%) for columns.

Beam Design				Column Design		
ρ_{upp} (%)	h (mm)	Bending Constr. (N/mm2)	Shear Constr. (N/mm2)	ρ_{upp} (%)	h_C (mm)	Constraint 1 (N/mm2)
4	798	2.454	***2.188***	2.0	432	***21.533***
1.4	798	2.454	***2.188***	2.0	432	***21.533***
0.6	845	***2.401***	2.088	2.0	432	***21.533***

Table 4.1 *Upper bound reinforcement ratio varied on beams and constant on columns*

For beam design, it was concluded that when shear stresses are critical and upper bound reinforcement ratio for columns is kept constant, the upper bound beam reinforcement ratio had no influence on the final solution. As previously stated, the reason for this is that the upper bound shear stress in beams is only a function of the characteristic strength of concrete. However, when the bending constraints became critical (ρ_{b_upp}=0.6%), a different optimum solution was obtained. It was also observed that for lower values of the upper bound reinforcement ratios the optimum depth tends to increase, whilst for higher values it tends to decrease. The breadths of the sections were driven to their lower bound values of 300 *mm* for both the beams and the columns.

Further investigation was undertaken to analyse the influence of changing the value of the upper bound column reinforcement ratio whilst keeping it constant for beams (ρ_{b_upp}=1.4%). The results are presented in Table 4.2 below.

Beam Design				Column Design		
ρ_{upp} (%)	h (mm)	Bending Constr. (N/mm^2)	Shear Constr. (N/mm^2)	ρ_{upp} (%)	h_C (mm)	Constraint 1 (N/mm^2)
1.4	806	2.806	***2.188***	4	308	***29.508***
1.4	798	2.454	***2.188***	2.0	432	***21.533***
1.4	781	2.482	***2.188***	0.8	563	***16.704***

Table 4.2 *Upper bound reinforcement ratio varied on columns and constant on beams*

The results show that different optimum solutions were obtained. It was observed that the optimum column depth was inversely proportional to the value of ρ_{c_upp}. Beams dimensions adjusted accordingly until either shear or bending stresses became critical.

4.6.2 Results - Initial Design Point

For all cases of the continuous beams investigated in this research, the minimum volume optimisation algorithm has exhibited very stable convergence behaviour regardless of the choice of the initial design point. In all cases, the minimum volume solution was obtained with little difference in the computational time required. However, for the frame structures that have a larger number of design variables and constraints, the algorithm behaviour was more sensitive. A selection of the results from these investigations is presented in Table 4.3 for the frame structure shown in Figure 4.8.

Beam Design				Column Design		
Move Limits	h (mm)	Bending Constr. (N/mm^2)	Shear Constr. (N/mm^2)	Move Limits	h_C (mm)	Constraint 1 (N/mm^2)
0.1♣	798	2.454	***2.188***	0.8	432	***21.533***
0.1♠	No Feasible Solution			No Feasible Solution		
0.5♠	798	2.454	***2.188***	0.8	432	***21.533***
0.5♠	798	2.454	***2.188***	0.5	432	***21.533***

Table 4.3 *Influence of the initial design point on the algorithm convergence*

In first instance, the case of an initial design point close to the optimum (♣) of b=350*mm* and h=700*mm* was selected for the beam group, and b=350*mm* and h=450*mm* for the column group. To minimise linearisation errors for the beams whilst initially allowing a large search area for the columns to assist a more rapid convergence, tight moving limits of 0.1 for the beams and loose moving limits of 0.8 for the columns were chosen. The values of the upper bound reinforcement ratio were set to 1.4% and 2.0% for the beams and the columns respectively. The optimum was achieved in five global iterations, with the shear stress in the critical beam and Constraint 1 for the critical column reaching their upper bound values.

To study the convergence behaviour of an initial design point far from optimum (♠), a starting point of b=750*mm* and h=1000*mm* was selected for the beam group (*see* Table 4.3). The initial column depth was increased to 750*mm*. At this stage no further action in encouraging the convergence to the optimum was taken, with the moving limits being set at the previous values. In this instance, convergence was not obtained, as the tight moving limits would not allow the algorithm to move towards the feasible design region. To encourage convergence the moving limits for the beams were changed to 0.5 and the optimum solution was obtained in total of twelve global iterations. However, a solution requiring only eight global iterations was achieved by reducing the moving limits for the columns to 0.5. The moving limit for the beams was left unchanged. Hence, it was concluded from this research that the choice of initial design point in combination with the moving limit values have a significant influence on the rate of convergence and stability of the algorithm.

In general, when analysing the convergence characteristics of the algorithm with regard to the initial design point selection it was observed that a high degree of feasibility was exhibited. A low failure rate to converge was recorded, with problems only occurring when the initial point was far from the optimum and combined with tight moving limits. If the choice of initial design point was reasonably close to the optimum then no combination of moving limits influenced achieving the optimum solution. The research also concluded that an appropriate choice of initial design point and moving limits considerably improved the efficiency of the algorithm, measured in terms of the number of global iterations required to achieve an optimum solution.

4.6.3 Results - Moving Limit Values

This research uses dynamic moving limits that automatically adjust their values, decreasing as they approach the optimum and hence keeping the linearisation errors within allowable tolerances. As stated earlier, the continuous beams investigated have demonstrated that the convergence behaviour of the algorithm is stable for any value specified for the moving limits. However, when investigating frame structures the algorithm has shown to be more sensitive to the value of the moving limits. The SLP algorithm was tested for a range of moving limits applied to both the beam and column groups, for the structure shown in Figure 4.8. It has to be emphasised however, that the behaviour of the algorithm is influenced by the combination of initial design point and the value of the moving limits. Table 4.4 shows the results for the case when the initial design point (IDP) is close to the optimum for a range of moving limit values.

Beam Design		Column Design		Algorithm Performance
Move Limits	h_{opt} (mm)	Move Limits	$h_{c\ opt}$ (mm)	N° of Global Iterations Required
0.1	798	0.8	432	5
0.8	798	0.1	432	6
0.8	798	0.8	432	6
0.1	798	0.1	432	6
0.5	798	0.5	432	5

Table 4.4 *Influence of moving limits on algorithm convergence - IDP close to the optimum*

The breadths of the sections for both the beams and columns were driven to their lower bound values of 300 *mm*. Feasible solutions were obtained for all the cases investigated, with there being little difference in the number of global iterations required. However, when the initial design point was far from the optimum the algorithm exhibited greater sensitivity to the value of the moving limits, as shown in Table 4.5.

Beam Design		Column Design		Algorithm Performance
Move Limits	h_{opt} (mm)	Move Limits	$h_{c\ opt}$ (mm)	N° of Global Iterations Required
0.1	NON FEASIBLE	0.8	NON FEASIBLE	/
0.8	798	0.8	432	14
0.5	798	0.5	432	8

Table 4.5 *Influence of moving limits on algorithm convergence - IDP far from the optimum*

When loose initial moving limits were specified for both the beams (0.8) and columns (0.8), the optimum solution was achieved but at the expense of efficiency measured in terms of the number of global iterations required. Moderate moving limits (0.5) on both the beams and columns gave a decrease in the number of global iterations possibly as a result of reducing the linearisation errors. However, applying tight moving limits (0.1) on either the beams or columns when the initial design point was far from the optimum led to infeasible solutions, as the algorithm search area was over-restricted and hence incapable of providing an optimum solution.

4.7 Conclusions

An approximation programming method based on SLP has been developed in this research for the minimum concrete volume design of skeletal structures. Structural analysis was performed using the matrix joint displacement method and incorporated within the developed approach to multi-level optimisation. Beams are designed to resist bending moments and the effects of shear forces. Short braced columns are designed to resist axial forces and uniaxial bending moments. The stress constraints were derived using the design procedures outlined in BS8110. The upper bound stresses are controlled by limiting the upper bound percentage steel ratios. Dimensional and deflection constraints are defined to satisfy code of practice requirements and practical design considerations.

An optimum design method which does not include stiffness equalities was proposed based on the principle of retaining critical constraints only. In this way the original set of constraints was considerably simplified with the resulting programming problem being more mathematically stable and robust, and computer efficient. Feasible solutions were obtained in all the cases investigated where the initial design point was close to the optimum, regardless of the value of the moving limits. When the initial design point was far from the optimum the convergence behaviour of the algorithm showed an increased sensitivity to the values of the moving limits. In these instances, it was concluded that this choice had a direct influence on the efficiency of the algorithm measured in terms of the number of global iterations required to achieve a final solution. When the initial moving limits were chosen to be too tight (0.1 - 0.3) on either the beam or column dimensions, and the initial design point was far from the optimum, the algorithm failed to converge. However, this behaviour was considered to be an extreme situation. By loosening the move limits an optimum solution could be obtained. Detailed testing for additional member groups was not performed at this stage, as it was obvious that their influence would only affect the computational time required.

Whilst the concept of using upper bound reinforcement ratios to specify stress constraint criteria for the beams and columns significantly reduces the complexity of the programming problem, the model takes no account of the differential costs of the materials. Hence, further research was conducted to extend the minimum volume design methodology towards the development of a more sophisticated cost optimisation algorithm that includes multiple load cases, additional member groups and *T*- and *L*-beam solutions. This is presented in Chapter 5.

5.

Cost Optimisation of Skeletal Systems I

This chapter investigates a rationale for the minimum cost design of skeletal structures. Cost objective functions and design constraints for beams, columns and frames are derived, together with a modified approach to the application of the SLP method. Sensitivity analysis and detailed testing are performed and the results reported. A comparative study between cost and volume optimisation is presented, with the minimum cost results being verified using a genetic algorithm search incorporating conventional design office methods.

5.1 Introduction

This research has shown that the cost of a structure evaluated from volume optimisation is sensitive to the limits imposed on the upper bound reinforcement ratios for both beams and columns (Fryer and Ceranic 1997). For beams, a good estimate of minimum cost can be obtained by setting the upper bound reinforcement ratio to that given by the LMM solution as detailed in Section 3.3. However, given the complex behaviour of beam and column elements within skeletal systems, the research concluded that reinforcement areas should be incorporated as design variables within a cost optimisation problem formulation. This conclusion led to the consideration of the unit material costs for steel and concrete, and how they could be incorporated within a cost objective function. This inevitably led to investigating the development of a minimum cost design approach to more truly reflect both the material and construction costs. A novel approach to multi-level cost optimisation has been developed, incorporating many of the algorithms and techniques already developed in the previous chapters. On the elemental level the optimisation is carried out based on the principle of retaining critical constraints only, whilst on the structural level, member grouping is utilised by the modified formulation of the objective function. In this approach, those structural members specified to have identical cross-sectional dimensions are grouped, and hence the efficiency of the optimisation algorithm is significantly improved. For columns, the optimisation problem formulation takes into account the practical design considerations of topology and loading arrangements, by allocating groups to both cross-sectional dimensions and reinforcement ratios.

5.2 Formulation of Structural Problem

The minimum cost design approach proposed in this research allows the topology, material properties and loading conditions to be pre-assigned. Member sizes and reinforcement ratios are treated as continuous design variables. The cost objective function and the design equality constraints for both beams and columns are derived and presented in a form that is suitable for implementation by SLP. The method of structural analysis is the same as that used for volume optimisation but with the addition of multiple load cases.

5.2.1 *Cost Objective Function*

The objective function has been derived to include the material costs of concrete and steel. For beams, the cost of the main reinforcement takes account of the simplified rules for curtailment defining the layout of reinforcement. The cost function for columns takes into account practical design considerations such as topology and loading arrangements, by separately grouping both the cross-sectional dimensions and reinforcement ratios. To avoid impractical design configurations, the same reinforcement ratio group cannot be allocated to different cross-sectional dimension groups. To further model typical design practice, only critical columns within each group are required to be optimised, the remaining columns being duplicated. The proposed design procedure not only allows for the optimisation of columns specified by the designer, but also the optimisation of all columns if so required. The cost objective function derived in this research is a natural extension of the volume objective function given by equation (4.5) combined with the elements of the cost objective function derived in Section 3.3.1, yielding the following expression

$$Z = \sum_{j=1}^{NBG} \sum_{k=1}^{NBBG} C_c b_j d_j L_{jk} \left[\left(1 + r_{jk} \right) + C_{jk} q \rho_{\max j} \right] + \sum_{m=1}^{NCG} \sum_{n=1}^{NRG} \sum_{p=1}^{NCRG} C_c bc_m hc_m H_{mnp} \left(1 + q \rho_{mn} \right) \quad (5.1)$$

where NCG and NRG are the number of column dimension and reinforcement ratio groups respectively, $NCRG$ is the number of columns in the n-th reinforcement ratio group, r_{jk} is the cover to reinforcement over effective depth ratio, q is the ratio of cost of steel to cost of concrete, ρ_{maxj} is the maximum reinforcement ratio of the critical member in the j-th beam group, ρ_{mn} is the reinforcement ratio of the member in the m-th column dimension and the n-th column reinforcement ratio group.

Figure 5.1 shows a diagrammatic representation of the adopted notation for the three storey-two bay frame, with three beam and two column groups.

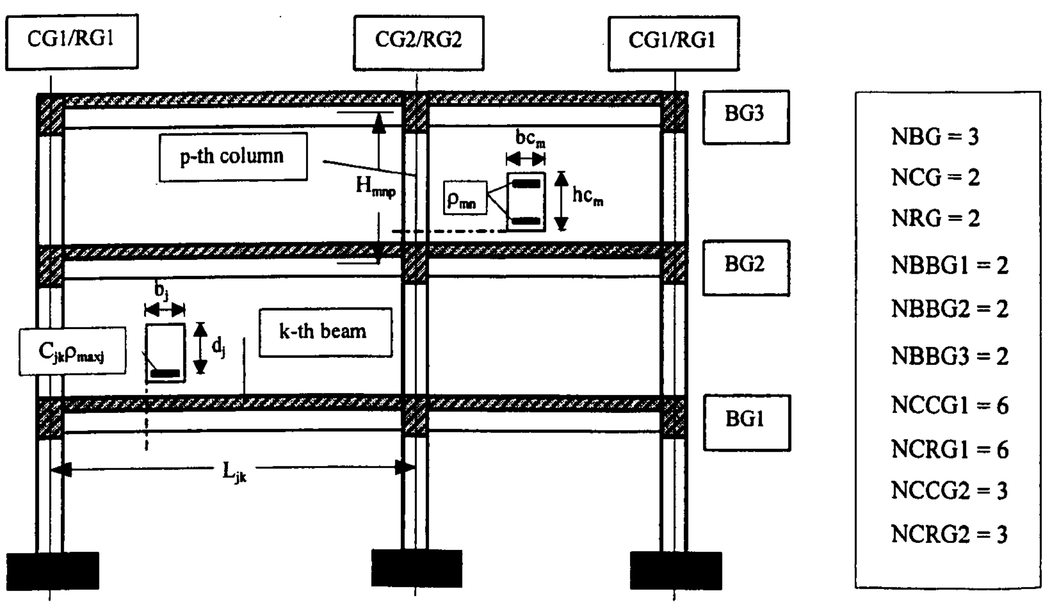


Figure 5.1 *Diagrammatic representation of the objective function notation*

The coefficient C_{jk} developed in this research relates to the total volume of main reinforcement required in the non-critical j-th beam group as a sum of that required at the left and right hand supports, and the mid-span. This is given as a function of the area of reinforcement in the corresponding location of the critical beam in the j-th beam group. It is expressed as

$$C_{jk} = \sum_{i=1}^{3} s_{ki}\, c_{ki} \tag{5.2}$$

where s_{ki} are coefficients presented as percentages of the maximum reinforcement ratio in the beam related to the simplified rules of curtailment, as shown in Figure 5.2. These coefficients are estimated depending on the values of the end moments in the beam, the sign of the mid-span bending moment, and the supporting conditions.

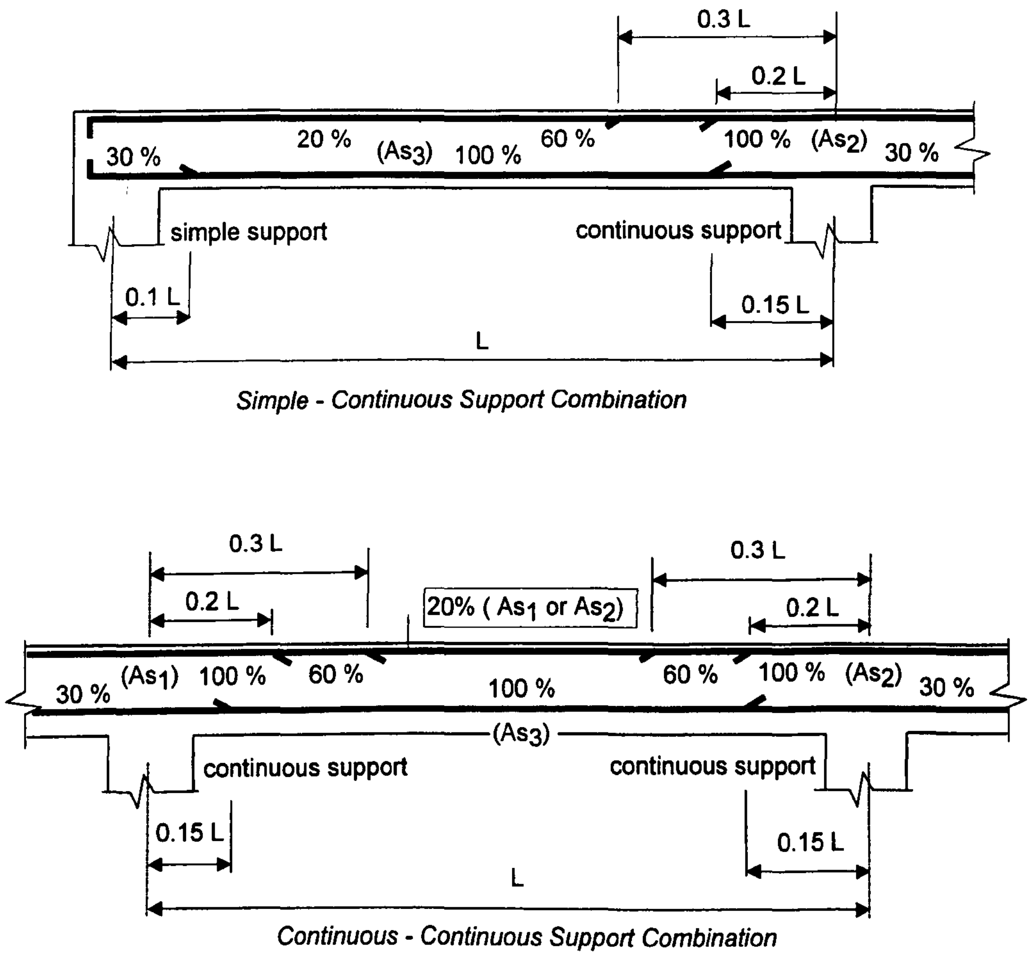


Figure 5.2 *Simplified rules of curtailment of bars in beams to BS 8110*

For example, considering the simple-continuous support combination, and setting M_1 and M_2 to be the hogging moments at the left-hand and right-hand supports respectively, and M_3 to be the mid-span sagging moment, when $M_1 = 0$, we get

$$s_{k_1} = 0\%$$

$$s_{k_2} = [(100\% \times 0.2L) + (60\% \times 0.1L) + (20\% \times 0.7L)]/L = 40\%$$

$$s_{k_3} = [(100\% \times 0.75L) + (30\% \times 0.1L) + (30\% \times 0.15L)]/L = 82.5\%$$

For the case of the continuous-continuous support combination, when $M_1 < M_2$

$$s_{k_1} = [(100\% \times 0.2L) + (60\% \times 0.1L)]/L = 26\%$$

$$s_{k_2} = [(100\% \times 0.2L) + (60\% \times 0.1L) + (20\% \times 0.4L)]/L = 34\%$$

$$s_{k_3} = [(100\% \times 0.7L) + (30\% \times 0.15L) + (30\% \times 0.15L)]/L = 79\%$$

Table 5.1 shows the complete list of s_{ki} values for various support conditions and moment configurations.

Moment Condition	s_{k1} (%)	s_{k2} (%)	s_{k3} (%)
$M_1 = 0$	0	40	82.5
$M_2 = 0$	40	0	82.5
$M_1 < M_2$	26	34	79
$M_1 > M_2$	34	26	79
$M_1 = M_2$	30	30	79

Table 5.1 *Percentage reinforcement coefficients*

For those occasions when the mid-span moment (M_3) is hogging, the percentage reinforcement ratio s_{k3} is set to zero.

The coefficients c_{ki} relate to the areas of reinforcement A_{ki} in the sections of the non-critical beams expressed as a function of the maximum area of reinforcement A_{max} in the corresponding section of the critical beam in the j-th beam group, as follows

$$c_{ki} = \left(\frac{A_{ki}}{A_{\max}}\right)_j = \left(\frac{M_{ki}}{M_{\max}}\right)_j \left(\frac{z_{\max}}{z_{ki}}\right)_j \tag{5.3}$$

where M_{ki} and z_{ki} are the bending moment and corresponding lever arm for the i-th location in the k-th beam respectively, whilst M_{max} and z_{max} are the maximum bending moment and corresponding lever arm for the critical beam in j-th beam group.

Here, the critical beam in each group is considered to be the one with maximum bending moment. In this way, optimisation is performed at the beam group level rather than at the individual beam level, resulting in significant improvements in the efficiency and robustness of the implemented computer algorithm.

For rectangular beam sections, the ratio z_{max} / z_{ki} is a function of the corresponding bending moment ratios for the critical and k-th beam in the j-th group, as derived below

$$\left(\frac{z_{max}}{z_{ki}}\right)_j = \frac{0.5+\sqrt{0.25-\left(M_{max}/M_{ki}\right)_j\left(K_{ki}/0.9\right)}}{0.5+\sqrt{0.25-K_{ki}/0.9}} \tag{5.4}$$

where $K_{ki} = M_{ki} / bd^2 f_{cu}$. Although the bending moment ratio $(M_{max} / M_k)_j$ is presumed constant between global iterations (*see* Section 4.3.2), there will be a small change in the lever arm ratio given by equation (5.4) due to K_{ki} being a function of the beam dimensions. Results from standard design solutions for reinforcement ratios ranging from the minimum to the boundary value have shown that these small changes do not significantly affect the assumption of c_{ki} being constant between global iterations.

For T- or L- beam sections, three possible cases need to be considered depending on the position of the neutral axis within the section, as detailed in Section 4.3.4. However, *Case 2* (when the neutral axis is within the web), occurs so rarely that it was omitted in this research. For *Cases 1* and *3* the beams are considered as rectangular sections with breadths equivalent to the effective width of the flange b_f and the width of the web b_w respectively. When applying equation (5.3), for moment ratios of the same sign *i.e. hogging to hogging* or *sagging to sagging*, equation (5.4) is valid. When however, the moments differ in sign i.e. *hogging to sagging*, the following expression for the ratio z_s / z_h was derived

$$\frac{z_s}{z_h} = \frac{0.5+\sqrt{0.25-\left(M_s/M_h\right)\left(b_w/b_f\right)\left(K_h/0.9\right)}}{0.5+\sqrt{0.25-K_h/0.9}} \tag{5.5}$$

where $K_h = M_h / b_w d^2 f_{cu}$ and the subscripts s and h stand for sagging and hogging respectively. The lever-arm ratio is a function of both the sagging and hogging bending moment ratio M_s/M_h and the breadth ratio b_w/b_f. The breadth of the flange b_f is a function of the breadth of the web and effective span and hence equation (5.5) is a more complex expression than equation (5.4) for rectangular sections.

Hogging to hogging ratio

1. For each beam group locate the point of the maximum hogging moment.
2. For all other hogging moments in the beam group, the area of the reinforcement can be related to this maximum value using equations (5.3) and (5.4).

Hogging to sagging ratio

1. Locate the point of maximum hogging bending moment in the beam group.
2. For all other sagging moments in the beam group, the area of reinforcement can be related to the maximum value in the manner given by equation (5.3). Although the same equation is principally valid for *T*- and *L*- beam sections, it is important to note that each beam within the same beam group will have identical b_w, d and h, but not necessarily the same effective breadth b_f since this is dependant on the span of the beam (L).

5.2.2 *Multiple Load Cases*

For the volume optimisation approach, the problem formulation considered only a single load case. BS8110 however, requires designers to consider combinations of *dead* and *imposed* load so that the critical ultimate limit state forces are identified. For reinforced concrete braced frames, the combinations of *dead* and *imposed* loads specified by BS 8110 are:

(i) All spans with maximum loading ($1.6Q_k + 1.4G_k$)

(ii) Alternate spans with maximum ($1.6Q_k + 1.4G_k$) and minimum ($1.0G_k$) load

where G_k and Q_k are the characteristic *dead* and *imposed* loads respectively. The latter will produce two loading patterns per each floor level (storey). From the analysis of each load case, a design envelope is constructed for each structural element that shows at any point on a member the worst effect resulting from these loading arrangements. This design envelope is then used to determine the critical member forces required for the design process.

5.2.3 *Stiffness Constraints*

In this research, the stiffness equality constraints have been removed from the cost optimisation problem formulation. This is in accordance with the principle of retaining the critical constraints, as detailed in Section 4.3.2. The approach adopted extends the previous research that showed that the removal of the stiffness constraints not only reduces the size of the programming problem but also considerably simplifies the corresponding stress constraints, resulting in a more robust and mathematically stable problem formulation.

5.2.4 *Bending Equilibrium Equality Constraints to BS8110 – Rectangular Section*

In Section 4.3.3, equations (4.7) and (4.8) state the bending equilibrium relationship for singly and doubly reinforced beams respectively. As discussed in Section 3.3, these equations are only applicable over a specific range of tension reinforcement ratios with the discontinuity interface being located at ρ_{bound} (*see* equation 3.19). The SLP method requires that functions are continuous over the entire feasible design space, and hence the bending equilibrium constraint can only model singly or doubly reinforced beams. Research carried out in Section 3.3.3, together with studies of practical design solutions, indicated that for q values reflecting UK material costs, the minimum cost design is normally a singly reinforced beam. Hence, the formulation of the cost optimisation problem considers only singly reinforced sections. Re-writing equation (4.7) in a form more suitable for cost optimisation gives the following bending equilibrium equality constraint for the critical beam of the j-th beam group

$$M_{\max_j} = 0.87 f_y \rho_{\max_j} b_j d_j^2 \left[1 - 0.979 \frac{f_y}{f_{cu}} \rho_{\max_j} \right] \tag{5.6}$$

where $M_{\max_j}$ and $\rho_{max\,j}$ are the maximum bending moment and reinforcement ratio in the critical member of the j-th beam group, and f_{cu} and f_y are the characteristic strengths of the concrete and reinforcement, respectively.

5.2.5 *Bending Equilibrium Equality Constraints to BS8110 - (T- or L-) Section*

For *Case 1*, when the neutral axis is within the web (sagging moment), the breadth of the beam is taken to be the effective width of the flange, i.e. $b=b_f$. Equation (5.6) is modified to give the following bending equilibrium equality constraint for the critical beam of the j-th beam group

$$M_{\max_j} = 0.87 f_y \rho_{\max_j} b_{f_j} d_j^2 \left[1 - 0.979 \frac{f_y}{f_{cu}} \rho_{\max_j} \right] \tag{5.7}$$

For *Case 3*, when the neutral axis is within the web (hogging moment), the breadth of the beam is taken to be the width of the web, i.e. $b=b_w$. Equation (5.6) is modified to give the following bending equilibrium equality constraint for the critical beam of the j-th beam group

$$M_{\max_j} = 0.87 f_y \rho_{\max_j} b_{w_j} d_j^2 \left[1 - 0.979 \frac{f_y}{f_{cu}} \rho_{\max_j} \right] \tag{5.8}$$

5.2.6 *Shear Stress Constraints for Beams to BS8110*

To prevent shear failure in the section, the cost optimisation problem formulation adopts the same shear stress constraint as the one given by equation (4.11) in Section 4.3.5.

5.2.7 *Column Equilibrium Equality Constraints to BS8110*

As stated in Section 4.3.6, the column design equations proposed in BS8110 are not suitable for direct solution. Furthermore, the mathematical model proposed for volume optimisation becomes impractical to implement in the minimum cost design due to the cumbersome derivations. A different approach is proposed in this research based on a direct estimate of the column reinforcement ratio ρ_{mn}.

Referring to Figure 5.3, for the compression failure zone ($K \geq 1$), when M/bd^2f_{cu} equals zero the area reinforcement ratio ρ_{mn} is obtained from the following expression

$$\alpha_{mn} = \left[\frac{N_{mn}}{bc_m hc_m f_{cu}} - 0.45\right]\frac{1}{0.87} \tag{5.9}$$

where $\alpha_{mn} = \rho_{mn} f_y / f_{cu}$.

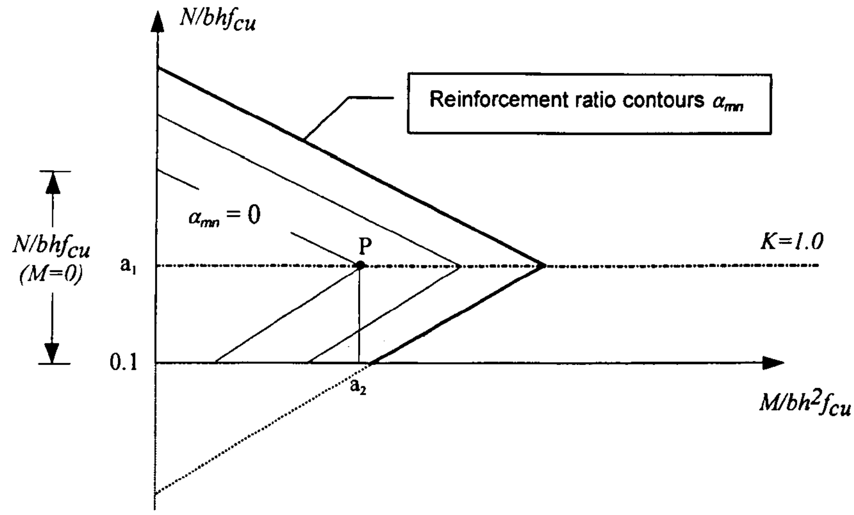


Figure 5.3 *Standard Column Design Graph*

Approximating the reinforcement ratio contours as parallel straight lines (*see* Figure 5.3), equation (5.9) is modified to provide an expression that takes account of the dimensionless factor M/bd^2f_{cu}, as follows

$$\alpha_{mn} = \left[\frac{N_{mn}}{bc_m hc_m f_{cu}} - 0.45\right]\frac{1}{0.87} + r_\alpha \frac{M_{mn}}{bc_m hc_m^2 f_{cu}} \tag{5.10}$$

where r_α represents the gradient of the area reinforcement ratio contours.

Three different techniques for estimating the value of r_α have been developed and tested in this research. The first uses the principle of *least squares*, whereby global values of

r_α are determined for *d/h* values of 0.8, 0.85, 0.9 and 0.95 respectively, and are given in Table 5.2.

d/h	0.8	0.85	0.9	0.95
R_α	3.905	3.420	3.038	2.727

Table 5.2 *Global values of r_α using the principle of least squares*

These values of *d/h* align with the column design charts provided in BS8110. Therefore r_α is taken to have discrete constant values for set ratios of *d/h*. The method is detailed in *Appendix* C. Whilst r_α is a good approximation for each chart, errors increase in predicting α_{mn} as its true value approaches zero.

To improve the approximation of r_α and take account of these localised errors, the second technique uses a more complex expression to estimate its value. Assuming that r_α is a function of the dimensionless factors (N/bhf_{cu}) and (M/bh^2f_{cu}), it was found that the following expression gave an improved estimate of its value and that the overall error was smaller.

$$r_\alpha = r_g + \left[r_p - r_g\right] a_1^3 \frac{1}{\left(N / bhf_{cu}\right)^3} \frac{a_2}{\left(M / bh^2 f_{cu}\right)} \tag{5.11}$$

where r_g is the global value of r determined using the principle of least squares, and r_p is the value of r at a point P located at the intersection of the K=1 line and the zero reinforcement contour, as shown in Figure 5.3. Furthermore, a_1 and a_2 are the values of N/bhf_{cu} and M/bh^2f_{cu} at the point P respectively. The method is detailed in *Appendix* C. As with the previous method, r_g and r_p are determined for the same predefined *d/h* values, as given in Table 5.3.

d/h	0.8	0.85	0.9	0.95
a_1	0.20465	0.217445	0.23020	0.24300
a_2	0.05560	0.05560	0.05560	0.05560
r_g	3.79100	3.31376	2.92549	2.59358
r_p	5.072	4.808	4.544	4.279

Table 5.3 *Values of r_g and r_p for the revised method*

Although these two methods provide good estimates of r_α over a wide range for each design chart, on a few occasions the level of error is unacceptable. To overcome these limitations, a third technique was developed whereby the value of r_α is modified at every global iteration, being recalculated to minimise the difference between the actual and approximate reinforcement ratios. These differences become insignificant as the solution converges towards the global optimum. The actual area reinforcement ratio ρ_{mn} is determined by balancing the internal forces calculated from the stress distribution with the known axial force and bending moment. This is an iterative process and is best implemented within a computer program. Having determined ρ_{mn}, r_α can be calculated at the start of each global iteration from the following expression

$$r_\alpha = \frac{\alpha_{mn} - \left[\left(N_{mn} / bc_m hc_m f_{cu}\right) - 0.45\right] / 0.87}{\left(M_{mn} / bc_m hc_m^2 f_{cu}\right)} \tag{5.12}$$

The results obtained using this technique overcame the problems associated with the two techniques based upon *least squares* minimisation. The technique was robust with changes to r_α becoming progressively smaller as convergence to the global optimum is achieved. To reduce the possibility of large changes in r_α occurring as a consequence of linearisation errors, tighter move limits have to be imposed to control the size of the search space.

For the tension failure zone ($K < 1$), an expression for ρ_{mn} was derived by first considering the stresses in the bottom and top reinforcement. The stresses in the bottom steel f_2 and in the top steel f_1 reach the design tensile stress of $-0.87f_y$ and design compressive stress of $0.87f_y$ respectively. For this case, the values of n and m in equation (4.15) become

$$n = 0\ ; \quad m = 0.435(2d/h\text{-}1) \tag{5.13}$$

Substituting these values into equation (4.13) and (4.14) gives

$$N = a_1 bhf_{cu}$$

$$M = a_1 a_2 bh^2 f_{cu} + m\alpha bh^2 f_{cu} \tag{5.14}$$

where $a_1 = 0.402\ x/h$ and $a_2 = 0.5\text{-}0.45x/h$, x is the depth to the neutral axis from the top of the compression zone in the column section.

Solving equations (5.14) in terms of α gives

$$\alpha = \frac{1}{0.435(2d/h-1)}\frac{M}{bh^2 f_{cu}} - \frac{0.5-0.45x/h}{0.435(2d/h-1)}\frac{N}{bhf_{cu}} \tag{5.15}$$

Interpolating for x/h between K=1.0 and N/bhf_{cu}=0.1, gives

$$x/h = c_1\frac{N}{bhf_{cu}}; \qquad c_1 = \frac{0.634d/h-0.249}{0.256d/h-0.1} \tag{5.16}$$

Substituting equation (5.16) into equation (5.15) and introducing $1/c_2$=0.435(2d/h-1), yields the following expression for ρ_{mn}

$$\rho_{mn} = \frac{f_{cu}}{f_y}\left[c_2\frac{M_{mn}}{bc_m hc_m^2 f_{cu}} - 0.5c_2\frac{N_{mn}}{bc_m hc_m f_{cu}} + 0.45\,c_1c_2\left(\frac{N_{mn}}{bc_m hc_m f_{cu}}\right)^2\right] \tag{5.17}$$

As before, the ratio d/h is only updated at the start of each global iteration and hence c_1 and c_2 are both treated as constants in equation (5.17).

5.2.8 *Deflection Constraints*

The deflection constraints are controlled by ensuring that the lower bound values on the depths of the beams are set to satisfy *span/effective depth* ratios as specified in BS8110. The rationale and principles behind this approach are detailed in Section 4.3.7.

5.2.9 *Cross-Sectional Variables Constraints*

The practical lower bound constraints imposed on the cross-sectional design variables are formulated as

$$b_j > max\ \{b_{jmin}, b0_j\ [1\text{-}m_b\ (j)]\};\ \ bc_k > max\ \{bc_{kmin}, bc0_k\ \ [1\text{-}m_c(k)]\}$$

$$h_j > max\ \{h_{jmin},\ h0_j\ [1\text{-}m_b\ (j)]\};\ \ hc_k > max\ \{hc_{kmin},\ hc0_k\ \ [1\text{-}m_c\ (k)]\}$$

$$\rho_j > \rho_{jmin}\ ;\ \ \rho c_k > \rho c_{kmin} \tag{5.18}$$

whilst the upper bound constraints are given by

$$b_j < min\ \{b_{jmax},\ b0_j\ [1+m_b\ (j)]\};\ \ \ bc_k < min\ \{bc_{kmax},\ bc0_k\ [1+m_c\ (k)]\}$$

$$h_j < min\ \{h_{jmax},\ h0_j\ [1+m_b\ (j)]\};\ \ \ hc_k < min\ \{hc_{kmax},\ hc0_k\ [1+m_c\ (k)]\}$$

$$\rho_j < \rho_{jmax}\ ;\ \ \rho c_k < \rho c_{kmax} \tag{5.19}$$

where $b0_j$, $h0_j$, $bc0_k$, $hc0_k$ are the initial values of the breadth and depth for the j-th beam and k-th column respectively, and b_{jmin} , h_{jmin}, bc_{kmin} , hc_{kmin} are the minimum values of the breadth and depth for the j-th beam and k-th column respectively. Similarly, b_{jmax}, h_{jmax}, bc_{kmax}, hc_{kmax} are the maximum values of the breadth and depth for the j-th beam and k-th column respectively, whilst ρ_{jmin}, ρ_{jmax}, ρc_{kmin}, ρc_{kmax} are the minimum and maximum values of the reinforcement ratio for j-th beam and k-th column respectively. The boundaries of the feasible design region defined by the move limits $m_b\,(j)$ and $m_c\ (k)$ for the j-th beam and k-th column respectively, should not exceed the upper or be less than the lower bounds specified by the designer or by the code of practice.

5.3 Implementation of SLP

Applying the approximation programming technique based on SLP, the objective function and the design constraints are linearised by using the first two terms of Taylor's series. From a specified value Z (x_0) of a function of several variables at $\{x\}=\{x_0\}$, the value of the term to give Z (x_1) at $\{x\}=\{x_1\}$ is found as

$$Z = Z(x_0) + \nabla Z(x_0)\ \left(\{x_1\} - \{x_0\}\right) \tag{5.20}$$

where $Z(x_0)$ is the original function evaluated at the initial design point, $\nabla Z\,(x_0)$ is the gradient vector at the initial design point, $\{x_0\}$ is a vector of initially known section variables and $\{x_1\}$ is the new vector of unknown section variables. The procedure for applying this method is explained in *Appendix* A.

5.3.1 *Linearisation of The Cost Objective Function*

The minimum cost objective function is given by equation (5.1). To simplify the expression of its linearised form, the following substitutions have been made

$$
\begin{aligned}
P_{jk} &= \left[(1 + r_{jk}) + C_{jk} q \rho_{\max j}\right] \\
Q_{mn} &= \left[1 + q \rho_{mn}\right]
\end{aligned}
\tag{5.21}
$$

The cost objective function can now be expressed as

$$
Z = \sum_{j=1}^{NBG} \sum_{k=1}^{NBBG} C_c b_j d_j L_{jk} P_{jk} + \sum_{m=1}^{NCG} \sum_{n=1}^{NRG} \sum_{p=1}^{NCRG} C_c bc_m hc_m H_{mnp} Q_{mn} \tag{5.22}
$$

Applying the approximation programming technique the objective function is linearised using the expression given by equation (5.20). Dividing equation (5.22) by the cost of concrete C_c, the linearised objective function is expressed in terms of the section variables of the k-th beam and p-th column, as follows

$$
Z = \underbrace{\left[b_j^o d_j^o L_{jk} P_{jk}^0 + bc_m^o hc_m^o H_{mnp} Q_{mn}^o\right]}_{\boxed{\mathbf{Z\,(X_0)}}} + \underbrace{\begin{bmatrix} d_j^o L_{jk} P_{jk}^o \\ b_j^o L_{jk} P_{jk}^o \\ b_j^o d_j^o L_{jk} C_{jk} q \\ hc_m^o H_{mnp} Q_{mn}^o \\ bc_m^o H_{mnp} Q_{mn}^o \\ bc_m^o hc_m^o H_{mnp} q \end{bmatrix}^T}_{\boxed{\mathbf{\nabla Z\,(X_0)}}} \times \underbrace{\begin{bmatrix} b_j' - b_j^o \\ d_j' - d_j^o \\ \rho_{\max j}' - \rho_{\max j}^0 \\ bc_m' - bc_m^o \\ hc_m' - hc_m^o \\ \rho_{mn}' - \rho_{mn}^o \end{bmatrix}}_{\boxed{\mathbf{\{X_1 - X_0\}}}} \tag{5.23}
$$

The superscripts (°) and (') denotes evaluation at both the initial design point and the new unknown design point, respectively.

Equation (5.23) is rearranged by summing for the whole of the structure and disregarding values which are constant, since those expressions have no influence on the optimisation of the corresponding cost objective function.

Hence, the final form of the linearised cost objective function Z_r' derived in this research is

$$Z = \sum_{j=1}^{NBG} b_j' \sum_{k=1}^{NBBG} d_j^o L_{jk} P_{jk}^o + \sum_{j=1}^{NBG} d_j' \sum_{k=1}^{NBBG} b_j^o L_{jk} P_{jk}^o + \sum_{j=1}^{NBG} \rho_{\max j}' \sum_{k=1}^{NBBG} b_j^o d_j^o L_{jk} C_{jk} q +$$

$$\sum_{m=1}^{NCG} bc_m' \sum_{n=1}^{NRG} \sum_{p=1}^{NCRG} hc_m^o H_{mnp} Q_{mn}^o + \sum_{m=1}^{NCG} hc_m' \sum_{n=1}^{NRG} \sum_{p=1}^{NCRG} bc_m^o H_{mnp} Q_{mn}^o + \sum_{m=1}^{NCG} \sum_{n=1}^{NRG} \rho_{mn}' \sum_{p=1}^{NCRG} bc_m^o hc_m^o H_{mnp} q \qquad (5.24)$$

where $P0_{jk}$ and $Q0_{mn}$ are obtained evaluating equation (5.21) at the initial design point. The notation for other variables is graphically represented in Figure 5.1.

5.3.2 *Linearisation of the Equilibrium Equality Constraints*

The equilibrium equality constraints for beams and columns are derived as functions of the dimensional variables and reinforcement ratios. For a rectangular section the equilibrium equality constraint for the critical beam in the j-th beam group is given by equation (5.6). Re-arranging this equation in a form suitable for linearisation gives

$$\frac{M_{\max j}}{b_j d_j^2} - 0.87 f_y \rho_{\max j} \left[1 - 0.979 \frac{f_y}{f_{cu}} \rho_{\max j} \right] = 0 \qquad (5.25)$$

The linearised equilibrium constraint takes the form

$$Dh_i(x_0)\{x_i\} = Dh_i(x_0)\{x_0\} - h_i(x_0) \qquad (5.26)$$

where $h_i(x_0)$ is the value of the constraint evaluated at the initial design point, $Dh_i(x_0)$ is the gradient vector evaluated at the same point, $\{x_i\}$ is the new unknown set of design variables and $\{x_0\}$ is an initial (known) set of design variables. The expression $Dh_i(x_0)\{x_0\} - h_i(x_0)$ represents the right hand side of the Simplex table.

Differentiating equation (5.25) with respect to the design variables, yields the elements of the gradient vector as follows

$$\frac{\partial h_i}{\partial b} = -\frac{M_{\max j}}{b_j^2 d_j^2}$$

$$\frac{\partial h_i}{\partial h} = -\frac{2M_{\max j}}{b_j d_j^3}$$

$$\frac{\partial h_i}{\partial \rho} = -0.87 f_y \left[1 - 1.958 \frac{f_y}{f_{cu}} \rho_{\max j}\right] \tag{5.27}$$

where $M_{max\,j}$ is the maximum bending moment in the j-th beam group, b_j and d_j are the breadth and effective depth of the critical beam in the j-th group, fy and f_{cu} are the characteristic strength of steel and concrete respectively and $\rho_{max\,j}$ is the reinforcement ratio in the corresponding section of the critical beam.

For T- and L- beams, the gradient vectors are derived by substituting b_j in equations (5.27) with b_f for the *Case 1* and b_w for the *Case 2*.
The shear stress constraints are linearised using the procedure set out in Section 4.4.3.

For columns, the equilibrium equality constraints consist of a combination of axial and bending stresses. For the compression failure zone (K ≥ 1), the equality constraint for the p-th column is obtained by re-arranging equation (5.10) to give

$$\left[\frac{N_{mn}}{bc_m hc_m} - 0.45 f_{cu}\right] \frac{1}{0.87 f_y} + r_{am} \frac{M_{mn}}{bc_m hc_m^2} - \rho_{mn} = 0 \tag{5.28}$$

Using equation (5.26) to form the linearised constraints, the elements of the gradient vector are obtained by differentiating equation (5.28) with respect to the design variables, to give

$$\frac{\partial h_i}{\partial bc_m} = -\frac{1}{bc_m^2}\left[\frac{N_{mn}}{0.87 f_y h_m} + r_{am}\frac{M_{mn}}{h_m^2}\right]$$

$$\frac{\partial h_i}{\partial hc_m} = -\frac{1}{bc_m}\left[\frac{N_{mn}}{0.87 f_y hc_m^2} + r_{am}\frac{2M_{mn}}{hc_m^3}\right]$$

$$\frac{\partial h_i}{\partial \rho_{mn}} = -1 \tag{5.29}$$

where N_{mn} and M_{mn} are the axial force and bending moment for the m-th column chosen to be optimised, bc_m and hc_m are the breadth and overall depth, r_{am} is the coefficient recalculated at each global iteration and ρ_{mn} is the reinforcement ratio of the m-th column calculated by iterative procedure based on the column design equations as outlined in BS8110.

For the compression failure zone (K < 1), the equality constraint for the p-th column is obtained by re-arranging equation (5.17) to give

$$\frac{f_{cu}}{f_y}\left[c_2\frac{M_{mn}}{bc_m hc_m^2 f_{cu}} - 0.5c_2\frac{N_{mn}}{bc_m hc_m f_{cu}} + 0.45\,c_1c_2\left(\frac{N_{mn}}{bc_m hc_m f_{cu}}\right)^2\right] - \rho_{mn} = 0 \tag{5.30}$$

with the corresponding gradient vector elements derived as

$$\frac{\partial h_i}{\partial bc} = -\frac{1}{bc_m^2}\left[c_2\frac{M_{mn}}{hc_m^2 f_y} - 0.5c_2\frac{N_{mn}}{hc_m f_y} + 0.45\,c_1c_2\frac{2N_{mn}^2}{bc_m hc_m^2 f_{cu} f_y}\right]$$

$$\frac{\partial h_i}{\partial hc} = -\frac{1}{bc_m}\left[c_2\frac{2M_{mn}}{hc_m^3 f_y} - 0.5c_2\frac{N_{mn}}{hc_m^2 f_y} + 0.45\,c_1c_2\frac{2N_{mn}^2}{bc_m hc_m^3 f_{cu} f_y}\right]$$

$$\frac{\partial h_i}{\partial \rho_{mn}} = -1 \tag{5.31}$$

Having linearised the objective function and equilibrium equality constraints, the design problem is solved using the modified *Two-Phase Simplex* method (*see Appendix* B).

5.4 Development of Computer Programme

The computer programme developed for minimum cost design is an extension of that developed for volume optimisation, as outlined in Figure 4.4. The revised computer programme incorporates many of the techniques already developed in the previous research as described in Section 4.5. The objective function and its linearised form are modified to reflect the cost optimisation approach. The equilibrium equality constraints and their linearised forms are assembled according to the procedures detailed in the previous section. Modifications to the structural analysis subroutines are implemented to allow for the solution of multiple load cases. The logical structure of the computer programme has been further improved by clearly separating subroutines according to the tasks that they perform within the complex optimisation process. Subroutines are hence classified into four groups according to their functions, namely; control routines, ancillary subroutines, element and speciality subroutines. The control routines are the main body of the computer programme and are responsible for structural analysis and the assembly of the corresponding non-linear optimisation problem. They construct the overall stiffness matrix, calculate the linearised objective function and linearised critical design constraints. The ancillary subroutines perform a variety of tasks required to support the operation of the main subroutines, and by doing so improve the functionality and efficiency of the computer programme. The element subroutines mainly evaluate member stiffness and stress matrices and are involved in assembling the Simplex table. The speciality subroutines control the implementation of the *Two-Phase Simplex* method and the solution of the linearised problem generated by the main routines.

5.5 Testing and Cost Sensitivity Analysis

The results obtained in this research showed that the developed algorithm is both reliable and accurate. For simple skeletal structures, the results were compared with solutions obtained from a direct search of standard office designs. For more complex structures, where this method was not practical to implement, a commercial genetic algorithm software package *Generator* (1995) was used to compare the results within the neighbourhood of the global optimum solution. Testing was performed on representative examples of continuous beams and small to medium sized reinforced concrete skeletal structures both for single and multiple loading conditions.

Furthermore, these structures were tested for one or more beam or column groups. Detailed results were produced for a three span continuous beam, a three-bay one-storey frame and a two-bay three-storey frame, as discussed in the following sections.

5.5.1 Design Example 1 - Three Span Continuous Beam

Figure 5.4 shows a three span continuous *T*-beam subjected to three loading combinations. The length of each span and the corresponding loads (excluding self-weight) are indicated in the figure. The lower and upper bounds of breadth and overall depth are given as 250 *mm* and 500 *mm*, and 350 *mm* and 800 *mm* respectively. The actual flange width is 4000 *mm*, thickness of flange is 200 *mm* and cover to reinforcement is 40 *mm*. The partial safety factors for the imposed and dead loads are 1.6 and 1.4 respectively with a minimum partial safety factor of 1.0. Cost of concrete is £50/m^3. Characteristic concrete and steel strengths are f_{cu} =30 N/mm^2 and f_y =460 N/mm^2, with the modulus of elasticity for concrete taken to be 28 kN/mm^2.

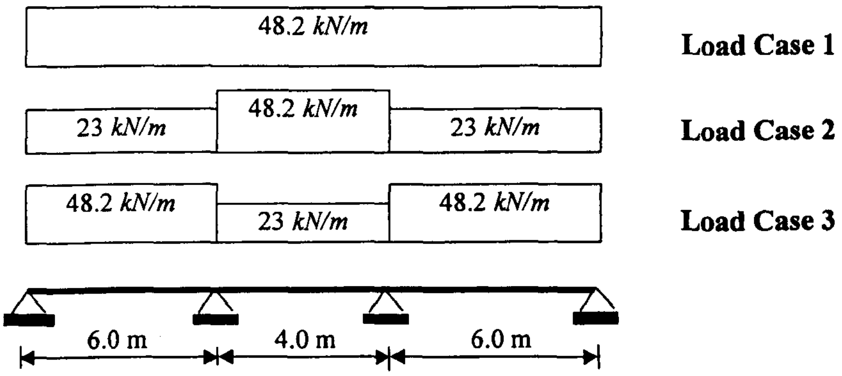


Figure 5.4 *Three-Span Continuous T- Beam*

The continuous beam was tested for both single (SLC) and multiple load cases (MLC) considering one beam group. The single load case is *Case 3* and the multiple load case considers all three possible loading combinations (*see* Figure 5.3).

The testing was performed by comparing the cost optimisation results for the *T*- section to those obtained using *Generator*. In addition, a minimum cost solution was obtained for the case when the beams were considered as rectangular sections with identical lower and upper bounds being imposed on the cross-section dimension variables. Results are presented in the Table 5.4 for different values of cost of steel to cost of concrete ratio q. The breadth of the web (b_w) was driven to its lower bound of 250 *mm* for all cases investigated.

q	Cost Optimisation Method			*Generator*			Cost Optimisation Method		
	T- section			*T*- section			Rectangular section		
	h_{opt} (mm)	A_s/b_wd (%)	Cost (£)	h_{opt} (mm)	A_s/b_wd (%)	Cost (£)	h_{opt} (mm)	A_s/b_wd (%)	Cost (£)
25	385	1.505	92.02	387	1.50	92.56	428	1.505	112.49
35	385	1.505	98.04	387	1.50	98.65	428	1.505	123.36
45	385	1.505	104.06	388	1.50	104.76	429	1.505	134.62
55	385	1.505	110.08	387	1.50	110.79	472	1.125	144.41
65	385	1.505	116.29	388	1.50	116.85	480	1.073	153.67
75	391	1.479	122.32	391	1.48	122.88	495	0.992	162.60
85	425	1.008	128.27	410	1.15	128.72	513	0.906	171.14
95	425	1.008	133.38	418	1.11	134.36	531	0.830	179.30

Table 5.4 *T- and Rectangular Section – One Beam Group - Single Load Case*

Table 5.4 shows that the results obtained from the developed cost optimisation algorithm are close and comparable to those obtained using *Generator* through an exhaustive search of standard design solutions. It was observed that for the *T*- beam sections an average cost reduction of 23 % could be achieved when compared to the minimum cost of a rectangular beam section for different values of q (*see* Figure 5.5). This is due to the optimisation process for *T*-sections taking account of the effective flange breadth (b_f) when calculating the required area of reinforcement to resist the sagging moments (*see* Section 4.3.4). Hence, it was concluded that a consideration of *T*- (or *L*-) sections is important when developing a minimum cost optimisation algorithm for realistic structural systems. This argument is based on the fact that for rigid, *in-situ* beam-slab connections, the corresponding beam uses the surmounting slab flange to more effectively resist sagging moments.

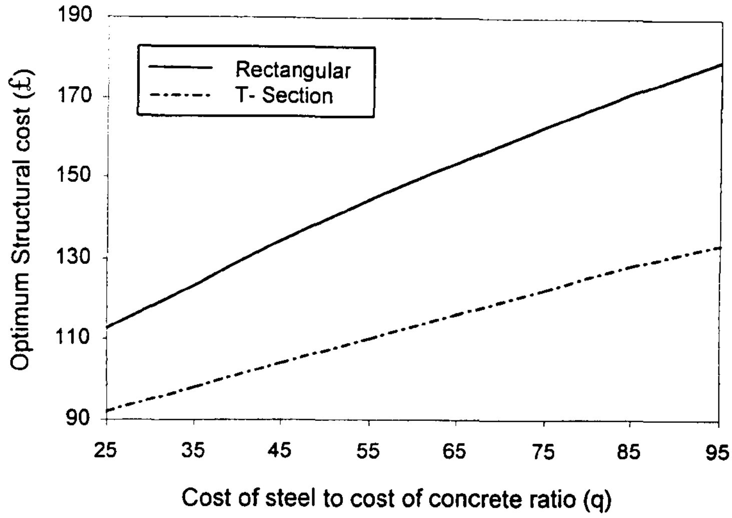


Figure 5.5 *Cost Comparison for T- and Rectangular Section – One Beam Group*

Further testing concentrated on the rectangular beam section, with emphasis on the behaviour of the final solution corresponding to the additional beam groups and multiple load cases. The results are presented in Tables 5.5 and 5.6, with the former considering one beam group and the latter considering two beam groups.

q	Cost Optimisation Method					Standard Design		Generator	
	Beam Group 1			SLC	MLC	SLC	MLC	SLC	MLC
	b_w (mm)	h_{opt} (mm)	A_s/bd (%)	Cost (£)	Cost (£)	Cost (£)	Cost (£)	Cost (£)	Cost (£)
25	250	428	1.505	112.49	114.31	112.82	115.11	112.49	114.32
35	250	428	1.505	123.36	125.82	123.57	125.97	123.36	125.83
45	250	429	1.505	134.62	137.22	135.02	137.64	134.82	137.18
55	250	472	1.125	144.41	147.81	144.91	148.14	145.62	147.76
65	250	480	1.073	153.67	157.70	154.03	157.85	155.46	157.64
75	250	495	0.992	162.60	167.01	162.76	167.61	163.58	166.94
85	250	513	0.906	171.14	174.64	171.24	175.04	173.53	175.76
95	250	531	0.830	179.30	182.77	179.81	182.98	180.69	184.18

Table 5.5 *One Beam Group - Single (SLC) and Multiple (MLC) Load Case*

Table 5.5 shows that both the direct search and genetic algorithm solutions give close and comparable results to those obtained using the developed cost optimisation approach. These results are verified by rigorously investigating a range of different problem settings, such as the q values. It was observed that for q values between 25 and

45 the reinforcement ratio reached the boundary value between a singly and doubly reinforced section (ρ_{bound}). This value was set as the upper bound of the reinforcement ratio as only singly reinforced sections are considered, as discussed in Section 5.2.4. Thus, the concrete section is driven to its minimum possible volume, since the costs of steel are insignificant in comparison to those of concrete. As q increases, and with it the cost of reinforcement relative to that of concrete, the depth of the beams increase and the percentage reinforcement ratio decreases accordingly to achieve a minimum cost solution. The breadth of the section was driven to its lower bound value of 250 *mm* regardless of the value of q.

When analysing the differences in the costs between single and multiple load cases for different values of q, it was observed that the latter gives slightly more expensive solutions for all q values (see Figure 5.6). This is due to the latter considering the critical bending moments that result from all three possible loading combinations, thus requiring an increase in the section dimensions (or reinforcement ratios) to satisfy the increased bending moments compared to those for the single load case. The differences were more significant when the second load combination (*Case 2*) was considered as a single load case, due to the lower bending moments at the supports and mid-spans.

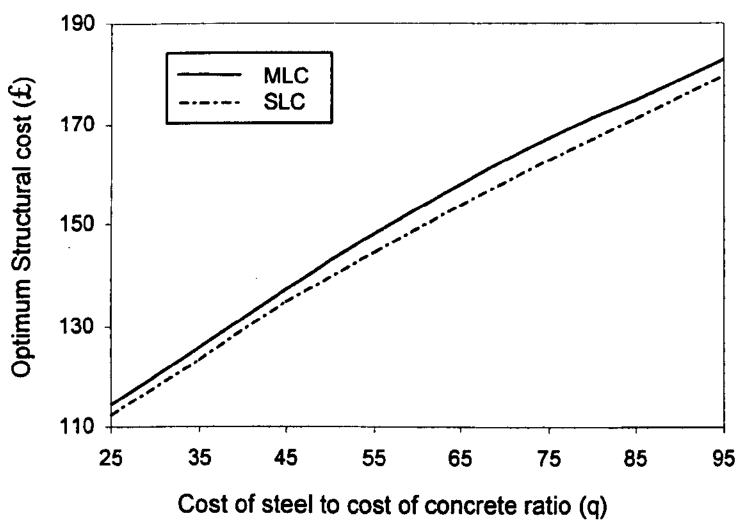


Figure 5.6 *Cost Comparison between SLC and MLC – One Beam Group*

Table 5.6 shows the results for two beam groups both for single and multiple loading conditions.

q	Cost Optimisation Method							Generator	
		Beam Group 1		Beam Group 2		SLC	MLC	SLC	MLC
	b_{opt} (mm)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	Cost (£)	Cost (£)	Cost (£)	Cost (£)
25	250	459	1.505	307	1.505	109.18	113.55	109.67	113.86
35	250	459	1.505	307	1.505	119.17	124.93	119.56	125.11
45	250	468	1.433	300	1.272	128.85	135.81	129.16	135.85
55	250	485	1.294	300	1.169	138.05	146.16	138.45	145.93
65	250	513	1.131	300	1.075	146.51	155.26	146.87	155.30
75	250	534	1.028	300	1.015	154.53	163.42	155.02	164.07
85	250	539	1.004	300	1.002	162.33	172.34	162.58	172.44
95	250	575	0.864	300	0.921	169.38	180.49	169.87	180.41

Table 5.6 *Two Beam Groups - Single (SLC) and Multiple (MLC) Load Case*

The end spans were assigned to the first beam group, having the same upper and lower dimension bounds as previously assigned. The internal span was assigned to the second beam group with the lower bounds for breadth and depth given as 250 *mm* and 300 *mm*, and the upper bounds given as 400 *mm* and 700 *mm*, respectively.

The same q value range was considered and the cost optimisation results were compared to those obtained using the direct search approach and genetic algorithms. For clarity, only the genetic algorithm solutions are presented in Table 5.6 as the direct search solutions gave similar results. For all values of q, the optimum breadth of each beam was driven to its lower bound value. As q increased, the depth of the second beam group was driven to its lower bound value.

When comparing the results of one beam group (Table 5.5) to those of two beam groups (Table 5.6), it was found that the latter gives a more efficient cost design. This was found to be due to the optimisation process having a greater choice of beam depth combinations to satisfy the bending moments more efficiently. When comparing SLC and MLC it was observed that the minimum cost solution for multiple load cases is consistently higher than that obtained for the single load case (see Figure 5.7). These increased costs are the result of considering the full design envelope as required by BS8110. It was concluded that the realistic minimum cost design for this frame can only be obtained by considering multiple load cases.

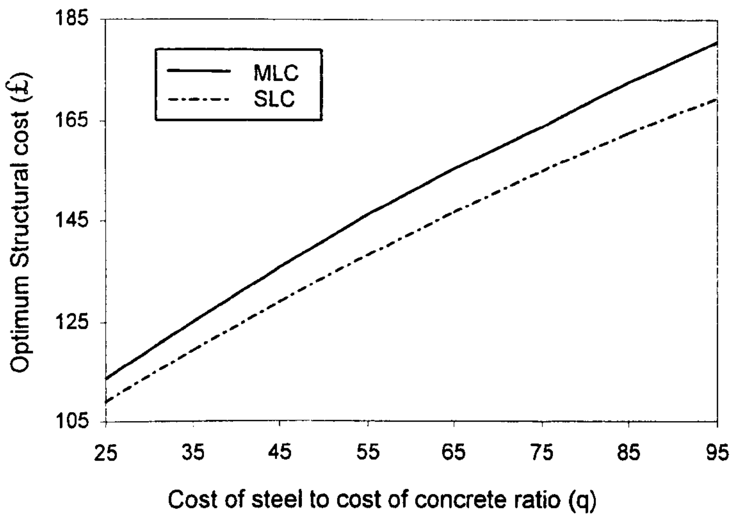


Figure 5.7 *Cost Comparison between SLC and MLC – Two Beam Groups*

5.5.2 Design Example 2 - Three Bay - One Storey Frame

Figure 5.8 shows a heavily loaded industrial frame subjected to three loading combinations. The frames, spaced at 4*m* centres, are braced against lateral forces and support a dead load g_k of 12.5 *kN/m²* (excluding self weight), live load q_k of 4.7 *kN/m²* and concentrated axial loads applied to each column as shown in Figure 5.8.

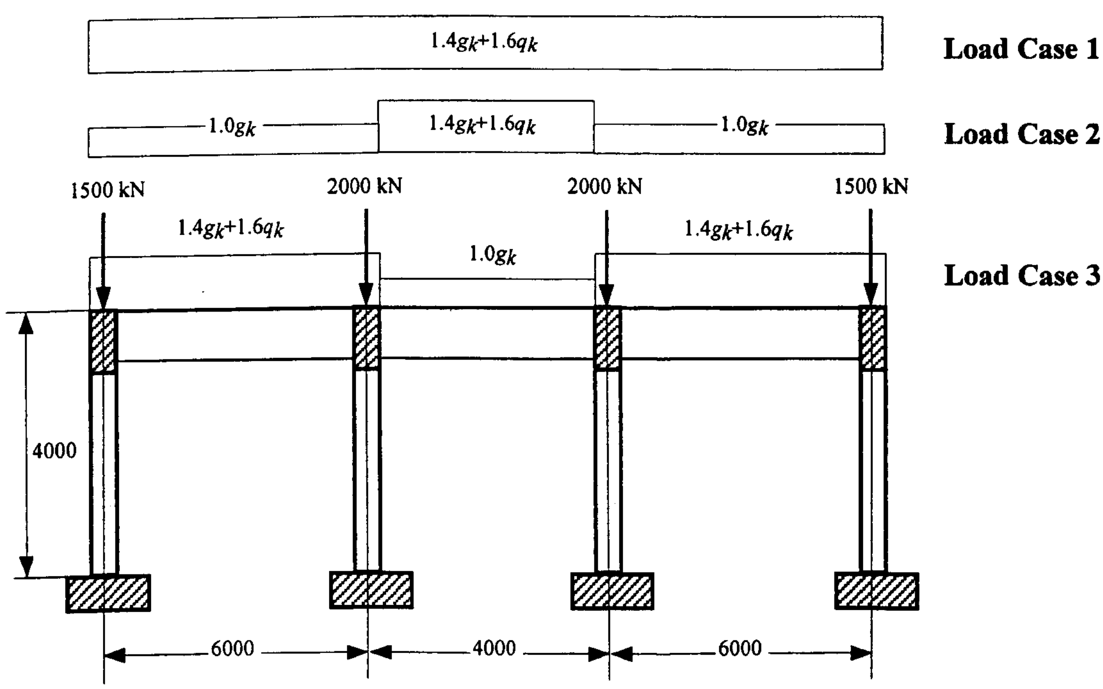


Figure 5.8 *Three Bay - One Storey Frame*

The partial safety factors for live and dead loads are 1.6 and 1.4 respectively with a minimum partial safety factor of 1.0. The modulus of elasticity is 28 kN/mm^2 with characteristic material strength f_{cu}=30 N/mm^2 for the concrete and f_y=460 N/mm^2 for the steel. The cover to the reinforcement is 50*mm* both for the beams and columns, with the cost of concrete being 50 *£/m³*.

The results for the one beam-one column group for single and multiple load cases are given in Table 5.7. The single load case is *Case* 3.

q	Cost Optimisation Method								Genetic Algorithms	
	Beam Group 1			Column Group 1			SLC	MLC	SLC	MLC
	b_{opt} (mm)	h_{opt} (mm)	A_s/bd (%)	$b_{c\,opt}$ (mm)	$H_{c\,opt}$ (mm)	A_s/bd (%)	Cost (£)	Cost (£)	Cost (£)	Cost (£)
15	300	490	1.46	250	325	5.90	258.57	261.57	258.38	260.36
25	300	484	1.46	250	484	2.42	300.56	314.14	301.30	312.36
35	300	484	1.46	250	484	2.42	335.43	343.64	335.92	346.64
45	300	503	1.50	250	738	0.40	362.42	372.44	362.44	373.44
55	300	503	1.50	250	740	0.40	383.24	395.23	382.70	397.23
65	300	503	1.50	250	740	0.40	403.99	417.91	403.56	418.52
75	300	497	1.50	250	740	0.40	421.04	440.57	424.41	442.55
85	300	580	0.98	250	740	0.40	440.09	459.98	440.86	458.82
95	300	601	0.98	250	740	0.40	457.44	480.93	458.56	484.23

Table 5.7 *One Beam Group - One Column Group - Single and Multiple Load Case*

As with the continuous beam, testing showed that the proposed cost optimisation algorithm gave close and comparable cost optimisation results to those obtained using genetic algorithms. The direct exhaustive search method had been abandoned by this stage as it was impractical to implement due to the increase in both the complexity of the problem formulation and number of design variables.

For all four combinations, and for all values of q, the breadth of the beams and columns were driven to their lower bound values. As q increases the depth of each structural element increases and the percentage reinforcement ratio decreases. For the one-beam-one column combination it was observed that for q values between 15 and 75 the reinforcement ratio in the beams reached the boundary value between a singly and doubly reinforced section. The columns on the other hand continued to increase their depth and reduce their reinforcement ratio. It was found that as q increased the columns became stiffer and more substantial, reducing the member forces in the beams which tended to be more expensive due to their higher reinforcement content. This process continued until the columns reached their minimum reinforcement ratios, after which the depth of the column remained constant and the beams adjusted their depths and reinforcement ratios accordingly to achieve a minimum cost. When single and multiple loading combinations were compared a similar conclusion to that for continuous beams was obtained.

Table 5.8 presents the results for the frame with two beam-two column groups subjected to both single and multiple load cases. The external beams were assigned to the first beam group, whilst the internal beam was assigned to the second beam group. The external columns were assigned to the fist column group whilst the internal columns were assigned to the second column group.

q	Cost Optimisation Method										Genetic Algorithm	
	Beam Gr 1		Column Gr 1		Beam Gr 2		Column Gr 2		SLC	MLC	SLC	MLC
	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	Cost (£)	Cost (£)	Cost (£)	Cost (£)
15	513	1.46	261	6.00	350	1.40	334	6.00	248.71	253.72	250.30	252.64
25	510	1.46	289	5.00	360	1.46	347	5.40	295.92	301.99	298.63	309.13
35	494	1.46	695	0.45	350	0.82	751	0.58	328.94	335.41	331.48	336.99
45	500	1.46	700	0.40	350	0.74	795	0.40	345.79	356.61	347.72	356.28
55	500	1.46	700	0.40	350	0.74	795	0.40	364.71	377.26	367.12	375.66
65	500	1.46	700	0.40	350	0.74	795	0.40	383.61	394.95	386.28	396.84
75	610	0.84	700	0.40	350	0.73	795	0.40	401.86	416.21	404.70	418.73
85	626	0.79	700	0.40	350	0.73	795	0.40	417.26	433.23	420.10	431.36
95	626	0.79	700	0.40	350	0.73	795	0.40	432.85	449.86	434.97	448.48

Table 5.8 *Two Beam Groups - Two Column Groups - Single and Multiple Load Case*

For the single load case it was observed that the depth of the internal beam was driven to its lower bound due to minimum loading being applied to the span. The internal columns were deeper than the external columns due to their higher bending moments and axial forces. When comparing SLC and MLC solutions, it was again concluded that the latter gave consistently higher material costs.

5.5.3 Design Example 3 - Two Bay - Three Storey Frame

Figure 5.9 shows a heavily loaded industrial two bay-three storey frame. The frames are spaced at 4*m* centres, braced against lateral forces and support a dead load g_k of 8.75 *kN/m²* (excluding self weight), and a live load q_k of 15 *kN/m²*. The modulus of elasticity is 28 *kN/mm²* with characteristic material strength of f_{cu}=30 *N/mm²* for the concrete and f_y=460 *N/mm²* for the steel. The cover to the reinforcement is 50 *mm* both for the beams and columns.

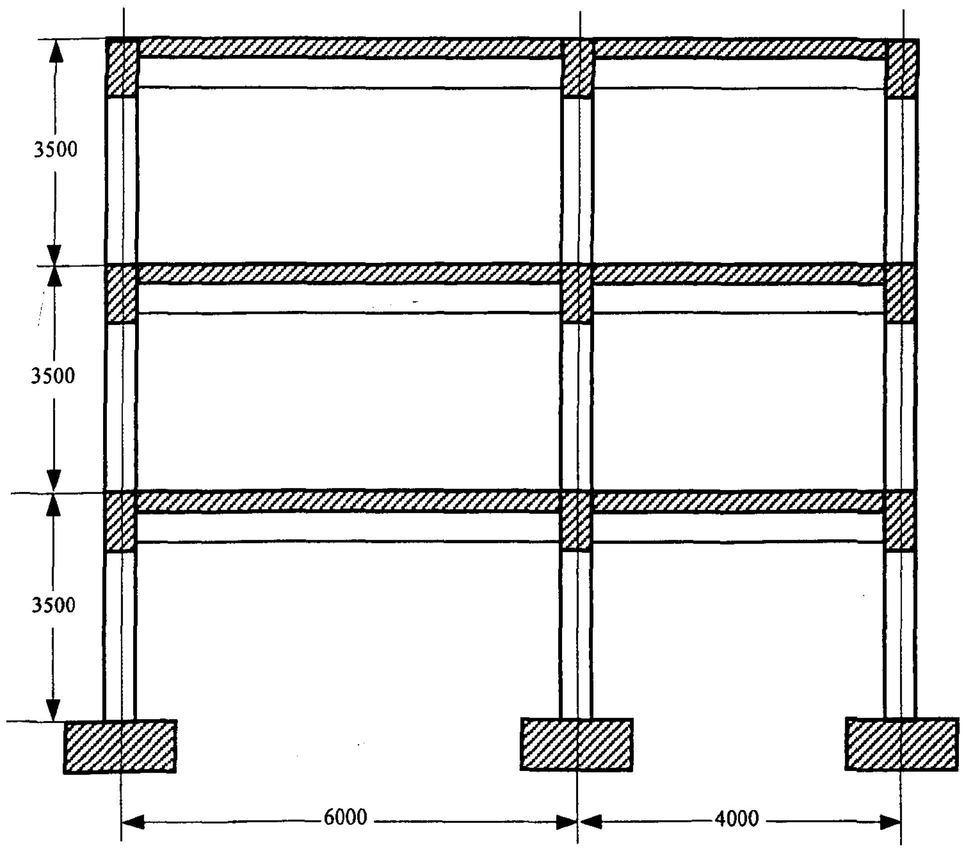


Figure 5.9 *Three Storey - Two Bay Frame*

The following combinations of uniform load are considered

(i) All spans with maximum loading ($1.6Q_k + 1.4G_k$)

(ii) Alternate spans with maximum ($1.6Q_k + 1.4G_k$) and minimum ($1.0G_k$) load

The second case produces two loading patterns per floor level (storey) giving altogether for this frame a total of 7 load combinations.

Table 5.9 shows the results for the one beam-one column group for both single and multiple loading conditions. To compare the cost optimisation results with those obtained using genetic algorithms, an approximate method of structural analysis based on the *subframes* approach was implemented within the *Generator* spreadsheet solution, as outlined in BS8110. This approach was adopted to overcome the limitations imposed by spreadsheets, in that the *joint displacement* method is difficult to implement.

q	Cost Optimisation Method								*Generator*	
	Beam Group 1			Column Group 1			SLC	MLC	SLC	MLC
	b_{opt} (mm)	h_{opt} (mm)	A_s/bd (%)	$B_{c\ opt}$ (mm)	$h_{c\ opt}$ (mm)	A_s/bd (%)	Cost (£)	Cost (£)	Cost (£)	Cost (£)
15	300	593	1.46	250	430	5.99	649.27	673.47	664.92	691.38
25	300	605	1.46	250	546	3.84	771.39	795.43	786.42	816.82
35	300	635	1.46	250	775	0.40	840.21	869.08	857.39	877.93
45	300	635	1.46	250	775	0.40	894.01	927.93	915.05	946.52
55	300	635	1.46	250	775	0.40	947.82	986.77	963.22	1001.6
65	300	635	1.46	250	775	0.40	1001.6	1045.6	1023.01	1061.9
75	300	637	1.46	250	777	0.40	1055.3	1104.5	1069.30	1126.3
85	300	637	1.46	250	777	0.40	1108.8	1163.3	1121.73	1178.5
95	300	637	1.46	250	777	0.40	1162.4	1217.5	1185.46	1235.7

Table 5.9 *One Beam Group - One Column Group - Single and Multiple Load Case*

Table 5.9 shows comparable results between the cost optimisation method and genetic algorithms but small percentage differences due to the approximate structural analysis used in the *Generator* solution. For all values of q, the breadth of the beams and columns were driven to their lower bound values, with the reinforcement ratio in the beams reaching the boundary value between a singly and doubly reinforced section. The columns continued to increase their depth until the corresponding reinforcement ratio reached its lower bound value. As with the previous frame, as q was increased the columns became stiffer and more substantial, reducing the member forces in the beams which tended to be more expensive due to their higher reinforcement content.

Table 5.10 shows the results for the same frame but with the two beam-two column groups combination. For this problem only multiple loading cases are considered, as this maps against the procedures specified in BS8110.

q	Cost Optimisation Method								
	Beam Group 1		Col. Group 1		Beam Group 2		Col. Group 2		MLC
	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	Cost (£)
15	635	1.46	318	5.99	555	1.46	463	5.99	649.50
25	623	1.46	747	0.57	467	1.46	800	1.86	782.92
35	623	1.46	785	0.42	459	1.46	800	0.43	841.49
45	623	1.46	790	0.40	457	1.46	800	0.40	889.53
55	623	1.46	790	0.40	457	1.46	800	0.40	937.17
65	623	1.46	790	0.40	457	1.46	800	0.40	986.87
75	786	0.78	790	0.40	457	1.46	800	0.40	1029.33
85	784	0.79	790	0.40	459	1.46	800	0.40	1070.43
95	785	0.78	790	0.40	457	0.76	800	0.40	1111.43

Table 5.10 *Two Beam Groups - Two Column Groups - Single and Multiple Load Case*

For each storey, beam group one and beam group two were allocated to the end and internal spans respectively. For the columns, group one applies to the external columns and group two to the internal columns. The lower and upper bound cross-sectional design variable constraints are given in Table 5.11.

Design Variable	Beam Group 1		Beam Group 2		Col. Group 1		Col. Group 2	
	lower	*Upper*	*lower*	*Upper*	*Lower*	*upper*	*lower*	*upper*
breadth (mm)	300	500	300	400	300	400	300	500
depth (mm)	400	900	300	700	300	800	300	800
A_s/bd (%)	0.13	1.46	0.13	1.46	0.4	6.0	0.4	6.0

Table 5.11 *Lower and upper cross-sectional design variable constraints*

Table 5.10 clearly shows that the optimum solution exhibits similarities with the one beam-one column group frame with respect to the behaviour of the beams and columns. In this case however, for $q \geq 75$ the beams in group one eventually increased their depths and decreased their reinforcement ratios accordingly to achieve a minimum cost. This was due to the columns reaching their upper bound depths with the external beams increasing their depth to satisfy the changes in the bending moments.

In all the cases tested, the breadths of the beams and columns were always driven to their lower bounds regardless of the vales of q. For low to medium values of q, it was observed that the reinforcement ratio in the beams reached the boundary value between a singly and doubly reinforced section, whilst the columns continued to increase their depth and reduce their reinforcement ratio. For increased values of q the columns became stiffer and more substantial, reducing the member forces in the beams. This process continued until the reinforcement ratio in each column reached its minimum, after which the depth of the column remained constant and the beams adjusted their depths and reinforcement ratios accordingly to achieve a minimum cost.

When the influence of the beam and column groups on the final design solution is considered, for low values of q the differences in the minimum cost between frames with one or more member groups were lesser than those for higher values of q. As q increased the cost difference showed a steady increase too. Material cost optimisation for frames with two or more member groups was more efficient than those with one group. In practice however, these cost differences would be offset against the potential increase in the formworking and labour costs.

5.6 Sensitivity Analysis

The behaviour of the algorithm has been tested for different parameter settings, such as the choice of initial design point and moving limits. Furthermore, the effects of both multiple load cases and member groups have been investigated. This sensitivity analysis has shown that the developed algorithm is both a mathematically stable and robust optimisation approach. The use of a genetic algorithm search has been invaluable in obtaining a greater understanding of those cases where the cost optimisation algorithm failed to reach an optimum solution, or became trapped at a local optimum.

5.6.1 Choice of Initial Design Point and Move Limits

Investigations on the choice of initial design point and move limits have indicated similar conclusions to those obtained from the sensitivity analysis in Section 4.6. Analysing the influence of the choice of initial design point on convergence to the optimum solution indicated that the algorithm performance is dependent also on the value of move limits (*see Appendix* D, Design Examples 1&2, Case 1&2). A low convergence failure rate was recorded, usually being associated with problems defined with an initial design point far from the optimum combined with tight moving limits. In these instances an infeasible design solution was often encountered. To overcome this problem, the research proposes the following procedure. If the optimum solution cannot be obtained, the search should be undertaken in two phases. In the first phase, loose moving limits (0.6 - 0.8) should be introduced to allow the algorithm to search a large area of the feasible region so that the neighbourhood of the optimum solution is detected. The size of the feasible region should then be reduced by imposing tighter move limits (0.1 - 0.3) starting from a new initial design point deduced from phase one. This second phase is essentially a *fine-tuning* process that helps to direct the search to the global optimum solution.

5.6.2 Multiple Load Cases

Results obtained from multiple load case analyses have shown the importance of considering realistic loading conditions for a structure. It was observed that the member forces and hence the final optimum solution depends on the critical force envelopes (*see also Appendix* D). Optimising a structure for only one load case therefore, does not produce a realistic design, although for that particular loading combination the obtained optimum solution is mathematically correct. In practice, structural design has to consider all the possible loading combinations and this should also apply to minimum cost design. Only then is it possible to argue that the resulting solution is representative of realistic structures with the design approach comparable to that of standard office design practice.

5.6.3 Multiple Beam (Column) Groups

Results obtained from the analysis of beams and frames with multiple beam (column) groups have indicated that the choice and approach to member grouping has a direct influence on the final member sizing and hence the final cost of a structure (*see also Appendix* D). The use of multiple beam (column) groups result in a more efficient cost design, allowing the optimisation process to have a greater choice of member dimension combinations to balance the external forces. This however, needs to be considered in the light of a potential increase in formworking costs.

5.7 Comparison with Volume Optimisation

Two alternative structural optimisation approaches have been considered; minimum volume of concrete and minimum material cost. Both methods are critically reviewed, tested and compared. Although the formulations differ in the choice of independent variables and in the formulation of the objective function and resulting constraints, both have shown to give similar results when the costs of the structure are compared for certain q values. The research has shown that the cost of a structure evaluated from the volume optimisation method is sensitive to the upper bound reinforcement ratio limits imposed on the structural elements. Therefore, different approaches in the choice of these reinforcement ratios have been investigated and are discussed. Finally, the results of the volume and cost optimisation algorithms have been extensively tested and compared with a sophisticated search of the feasible region of standard design solutions using genetic algorithms.

5.7.1 *Selection of Comparative Approaches*

To compare costs between the volume and cost optimisation methods, four different approaches were considered in the selection of these upper bound reinforcement ratios. In addition, structures were optimised for both single and multiple loading conditions with q ratios between 25 and 95 for a three span continuous beam example, and 15 and 95 for the frame example.

In the first approach (A_1), for each value of q, the upper bound reinforcement ratios used were those obtained from the cost optimisation method at the optimum. In the second approach (A_2), the minimum reinforcement ratios given in BS8110 were selected as the upper bounds for both beams and columns. The third approach (A_3) uses the maximum reinforcement ratio in columns allowed by BS8110. However, for beams the upper bound reinforcement ratio is set at the boundary between singly and doubly reinforced sections, as only singly reinforced beams were considered in this research. The fourth and final approach (A_4), uses upper bound reinforcement ratios obtained from a Lagrangian Multiplier solution previously developed by the research in Section 3.3.

5.7.2 *Design Examples*

Figure 5.10 shows the three-span continuous beam first encountered in section 5.5.1. As previously, the beam is subjected to three loading cases, and the lower and upper bound dimensional constraints together with the material properties have been kept the same.

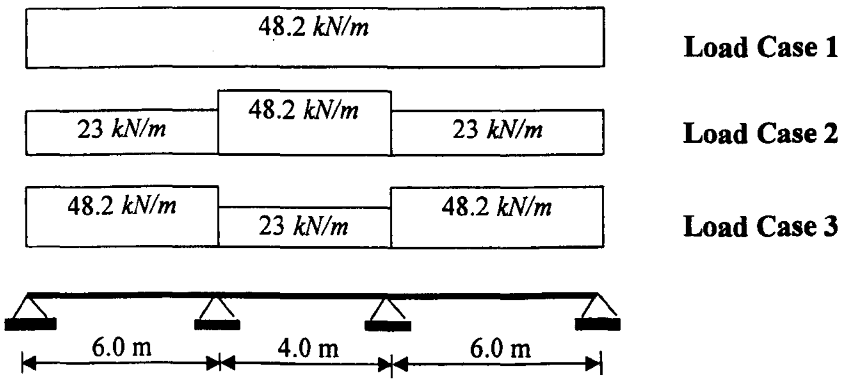


Figure 5.10 *Three-Span Continuous Beam*

Table 5.12 below shows the results of a cost comparison for the continuous beam optimised for minimum material cost and minimum volume (using the four different approaches).

q	Minimum Costs	Volume Optimisation				Genetic Alg.
		A_1	A_2	A_3	A_4	
Cs/Cc	(£)	Cost Factor = Volume Cost/Minimum Cost				(£)
25	112.49	1.004	2.320	1.004	1.004	112.56
35	123.36	1.004	2.143	1.004	1.004	123.39
45	134.62	1.003	1.989	1.003	1.006	134.82
55	144.41	1.005	1.876	1.015	1.009	145.62
65	153.67	1.012	1.785	1.027	1.012	155.46
75	162.60	1.013	1.707	1.039	1.013	163.58
85	171.14	1.011	1.638	1.051	1.014	173.53
95	179.30	1.013	1.584	1.066	1.013	180.69

Table 5.12 *Cost Comparison for Three Span Continuous Beam*

Figure 5.11 shows the geometry and loading conditions of the three bay-one storey frame considered in the section 5.5.2.

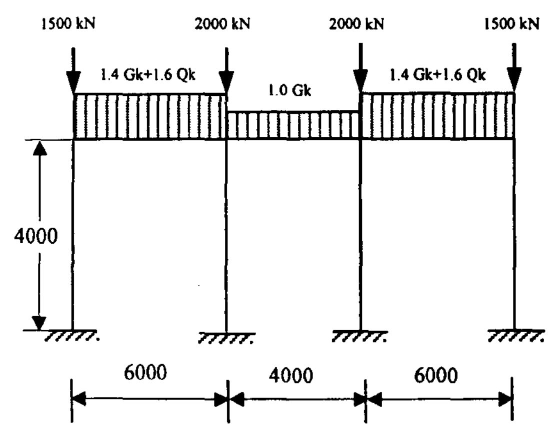


Figure 5.11 *Three Bay - One Storey Frame*

The partial safety factors for live and dead load are 1.6 and 1.4 respectively with a minimum partial safety factor of 1.0. The end beams are loaded with maximum load ($1.4G_k$+$1.6Q_k$), and the internal beam is loaded with minimum load of $1.0G_k$. The modulus of elasticity is 28 *kN/mm²* with characteristic material strengths of f_{cu}=30 *N/mm²* for the concrete and f_y=460 *N/mm²* for the steel. The cover to the reinforcement is 50 *mm* for the beams and columns. The cost of concrete is 50 *£/m³*.

Table 5.13 below shows the results of a cost comparison for the frame shown optimised for minimum material cost and minimum volume (using the four different approaches).

q	Minimum Costs	Volume Optimisation				Genetic Alg.
		A_1	A_2	A_3	A_4	
Cs/Cc	(£)	Cost Factor = Volume Cost/Minimum Cost				(£)
15	258.57	1.002	1.877	1.002	1.039	258.38
25	300.56	1.007	1.651	1.029	1.058	301.30
35	335.43	1.002	1.512	1.073	1.030	335.92
45	362.42	1.008	1.430	1.132	1.015	362.44
55	383.24	1.003	1.381	1.202	1.014	382.70
65	403.99	1.000	1.337	1.265	1.006	403.56
75	421.04	1.003	1.309	1.333	1.011	424.41
85	440.09	1.007	1.277	1.390	1.009	440.86
95	457.44	1.004	1.253	1.447	1.008	458.56

Table 5.13 *Cost Comparison for Three Bay - One Storey Frame*

For each example, a comparison between the minimum cost solution and approach (A_1), shows negligible cost differences for all values of *q*, indicating that it is possible for the volume optimisation method to provide a good estimate of minimum cost. Although this requires prior knowledge of the upper bound reinforcement ratios for each *q* value, it shows that a comparable solution can be obtained.

In the second approach (A_2), the cost ratios evaluated from the volume optimisation method are considerably greater than unity for all *q* values, especially in the case of the continuous beam, and the two methods are not well matched. In this approach the upper bound reinforcement ratio is set to be constant for all values of *q* and hence only one minimum volume solution is possible. Due to the severity of these reinforcement ratios, the optimum solution requires large beams and columns with minimum reinforcement. The minimum cost solution will therefore only give a similar optimum design once the cost of the reinforcement has become significant in comparison to that of concrete. Hence, as q increases the cost differences between the two methods reduces as shown in Tables 5.12 and 5.13.

In the third approach (A_3), for the three-span continuous beam the minimum volume solution is given when the reinforcement ratio in the beams reaches its boundary reinforcement ratio. For $q < 45$, this approach gives negligible differences when compared to the minimum cost solution. For the frame, the minimum volume solution is given when the reinforcement ratio in the critical column reaches its maximum allowed and the critical beam reaches its boundary reinforcement ratio. For $q < 25$, this fully stressed design gives negligible differences when compared to the minimum cost solution. As the value of q increases the costs from the two optimisation methods bifurcate due to the increasing cost of the reinforcement which cannot be modelled by the minimum volume solution. Approaches (A_2) and (A_3) are extreme cases, A_2 providing a good assessment of costs for high q values, whilst A_3 is compatible for low values of q. Both of these approaches suffer from having a fixed upper bound reinforcement ratio for all q values and therefore are unable to provide an estimate of minimum cost over the full range of q.

The fourth approach A_4, addresses this weakness in that for each value of q different upper bound reinforcement ratios are chosen. However, to avoid the random selection of these ratios and ensure that the process is systematic, the results from the research on the Langrangian Multiplier Method have been incorporated. The equations were derived for the optimum reinforcement ratio of beams and columns by minimising costs and assuming constant member forces. These equations which include q and the material strengths, are used to obtain an improved estimate of the upper bound reinforcement ratios. This approach gives close and reliable results as shown in Tables 5.12 and 5.13. Studies of other examples by this research have shown similar results for both single and multiple loading combinations. Initial results would suggest that for continuous beams a minimum volume design using the upper bound reinforcement ratios from the Langrangian solution, are very close to those obtained using the cost optimisation method.

5.8 Conclusions

A modified approximation programming algorithm based on the SLP method has been developed in this research for the minimum cost design of skeletal structures under multiple loading conditions. The proposed objective function incorporates the practical assessment of the material costs, taking into the account topology, loading arrangements and curtailment of reinforcement. The formulation of this objective function provides a novel approach to multi-level cost optimisation by the grouping of structural elements in the manner that mirrors design office practice. The results demonstrate that the SLP method can be successfully used to obtain the minimum material cost for reinforced concrete beams and frames. The algorithm has been developed to incorporate the design equations and procedures specified in BS8110, with the advantage of encapsulating both the analysis and design processes within a single operation. Dimensional, deflection and reinforcement ratio constraints have also been incorporated to comply with the codes of practice in a way that is intuitive to the designer. The implemented approximation programming technique proved to be mathematically robust and stable for a wide variety of structural problems. As with volume optimisation, the problem formulation was simplified by omitting the stiffness equalities and hence encouraging convergence to the optimum to be both rapid and stable.

The results have shown that optimising problems with single load cases do not give the realistic minimum material cost of a structure. The consideration of multiple load cases is both analysed and designed for the most unfavourable conditions. The algorithm developed in this research takes account of the multiple loading arrangements specified in BS8110 so that the resulting minimum cost solutions are representative of realistic structures and their loading conditions. When single and multiple beam and column groups are considered, the results showed that the latter are more cost efficient designs. In practice however, this needs to be considered in the light of potential increase in the formworking costs.

Research on volume optimisation has shown how it can be used for the minimum cost design of realistic 2D reinforced concrete structures. Although the *real* minimum cost design is considered to give the best results, volume optimisation is less complex in its formulation and is generally more robust and mathematically stable. However, it is important to emphasise that the volume optimisation method is not sensitive to the material costs but to the choice of the upper bound reinforcement ratios imposed. For the volume optimisation method to provide a good estimate of the minimum cost, it is necessary to provide a more systematic approach to the selection of these upper bound reinforcement ratios. Initial results from numerous studies would suggest that the use of the solution derived from a Langrangian Multiplier Method (*see* Section 3.3), combined with the volume optimisation method could provide a good estimate of the minimum material cost of reinforced concrete structures.

The results obtained in this research led to a study of how more general cost optimisation algorithms that take account of both material and additional construction costs could be developed. In this context, the potential of using genetic algorithms has been recognised, especially when realistic structures, loading conditions and limit states are considered.

6.

Cost Optimisation of Skeletal Systems II

This chapter describes an improved approach to the implementation of genetic algorithms (GA's) for the minimum cost design of reinforced concrete skeletal systems, set within an artificially intelligent computer design environment. The rationale of this method is explained, highlighting the limitations encountered in the application of traditional mathematical programming methods. Fitness functions for beams, slabs and frames are derived utilising a practical approach to the assessment of the total structural cost. This includes additional construction costs associated with the labour and formworking. Extensive testing results are reported, together with a sensitivity analysis for both GA control parameters and different unit component costs.

6.1 Introduction

The application of sequential linear programming as a traditional mathematical programming approach to the minimum material cost design of skeletal systems is discussed in Chapters 4 and 5. Although proven to be a suitable optimisation tool for non-linear programming problems with continuous feasible regions and design constraints, this method exhibits certain limitations when more realistic structural costs and design requirements are considered. These limitations are mostly related to the method's poor performance when dealing with discontinuous feasible regions and design equations. Furthermore, the formulation of the gradient vector for both the objective function and design constraints is often impractical to obtain. When unable to remove all the artificial variables, these algorithms encounter non-feasible solutions and hence terminate the search without any success. In that context, genetic algorithms (GA's) offer a promising approach to optimise structural problem formulations that have not been solved successfully using traditional methods. In particular, they are capable of ensuring a possible near-optimum solution when the algorithm stops short of reaching the global optimum. Furthermore, GA's are *blind* to the nature of the applied structural problem avoiding the need for linearisation with its associated gradient vector derivates. They can effectively deal with discontinuous design equations, and their ability to rapidly search the entire feasible region independent of the starting point makes them particularly well suited for the optimum design of skeletal systems.

The use of the commercial GA software (*Generator*) was considered and abandoned due to its inability to model complex structural optimisation problems. After extensive investigation of published work, it was concluded that this research required the development and implementation of its own GA code. This was designed to satisfy the specific requirements of the research resulting in a more bespoke suite of computer programmes being developed.

6.2 Formulation of Structural Optimisation Problems

Three distinct components of a skeletal structural system are considered within this research; reinforced concrete beams, slabs and frames. The fitness functions and corresponding design constraints are derived, highlighting those modifications and improvements to the problem formulation considered in Section 5.2.

6.2.1 Analysis and Design of Reinforced Concrete Beams

Simply supported beams are analysed using the basic equilibrium equations applicable to statically determinate structures. The analysis of continuous beams is based on the *slope-deflection* method, whereby the values of the support moments are calculated once the rotations at each support have been determined. Loading arrangements that give critical moments and shear forces within each span are considered, as discussed in Section 5.2.2.

Three possible section designs are considered assuming that the beam has either a rectangular , *'T'* or an inverted *'L'* section as shown in Fig. 6.1. For the latter two cases, when the beams are resisting sagging moments, the flange will be in compression and the members should be designed as *T*- or *L*- beams. For hogging moments, the flange will be in tension and therefore the beam should be designed as a rectangular section of width b_w and overall depth h. In all cases, the effective depth d is given as the distance from the top of the compression zone to the centroid of the tension reinforcement.

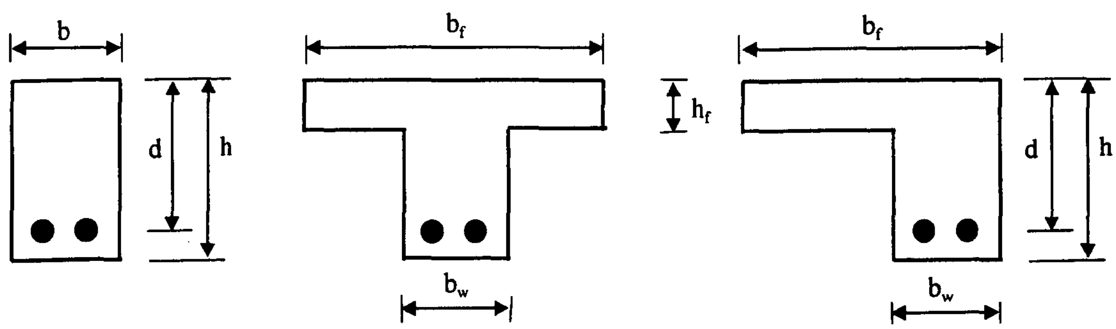


Figure 6.1 *Rectangular, T - and L - beam Section*

In Chapter 5, only the main tension reinforcement was considered when calculating the cost component for the steel. However, BS8110 states that additional steel should also be provided in the form of compression and shear reinforcement. The following types of reinforcement are therefore considered for simply supported and continuous beams:

Longitudinal reinforcement In ultimate limit state theory, it is assumed that on the tension side of the neutral axis the concrete is cracked and makes no contribution to the ultimate moment of resistance. In other words, the steel reinforcement in this zone carries the tensile force. In some cases, it is also necessary to provide longitudinal reinforcement in the compression zone. Beams are thus said to be either *singly* or *doubly* reinforced. Reinforcement is also required in the sides of beams with an overall depth greater than 750 *mm* to prevent excessive cracking.

For any beam, the area of main reinforcement required at the supports and mid-span is calculated using the maximum bending moment at the locations determined from the design envelope. The reinforcement is curtailed using the simplified rules given in BS 8110, following the same procedure described in Section 5.2.1.

Shear reinforcement Since the state of pure flexure rarely occurs in practice, it is required for the beams to also resist the effects of shear stresses arising from the transverse loads. To achieve this, shear reinforcement in the form of bent-up bars or vertical links is provided (*see* Figure 6.2). In this research, only vertical links are considered as these are more commonly used in design practice.

In doubly reinforced beams, vertical links also prevent compression bars from buckling.

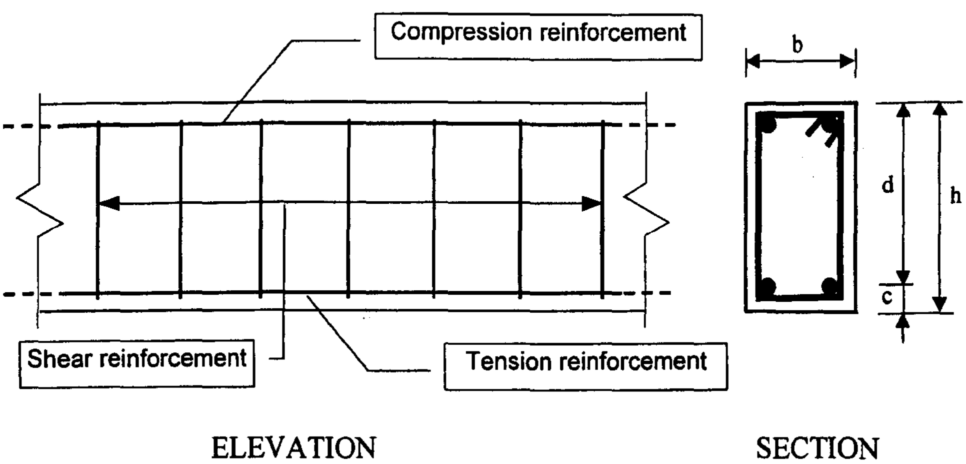


Figure 6.2 *Beam Reinforcement Layout*

The design and arrangement of shear reinforcement is influenced by both the magnitude of the maximum shear force at each support and the changes in the design envelope, as shown in Figure 6.3.

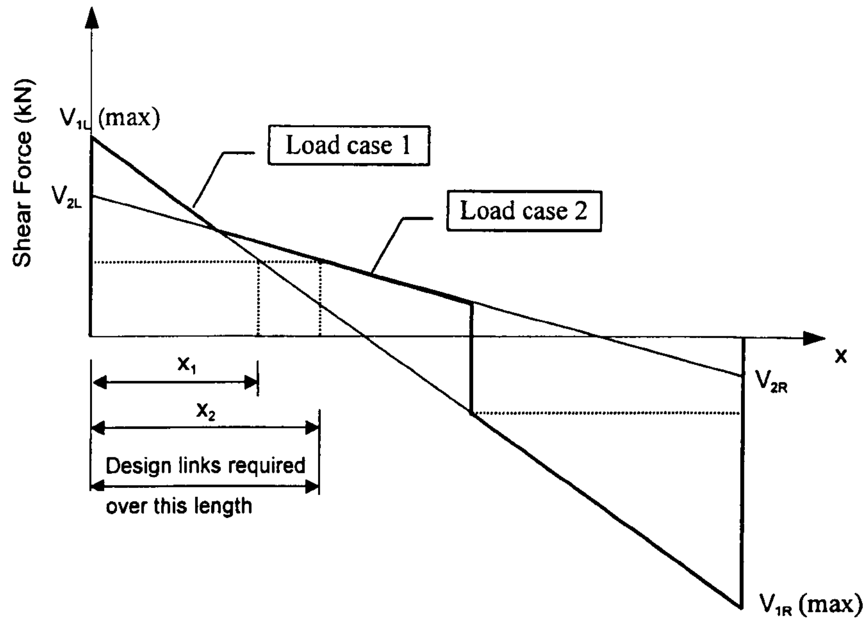


Figure 6.3 *Comparison of link arrangements for different shear stress cases*

From Figure 6.3, it is evident that at the left hand support design links are required to extend to a point determined by load case 2 (distance x_2), even though their size and spacing are determined from load case 1 using the maximum shear force $V_{1L(max)}$. Hence, when considering shear reinforcement for multiple load cases, it is important to determine those critical loading combinations for which the resulting shear forces give the greatest design link length on that part of a beam. Consideration has to be given to both the magnitude of the shear forces and the geometry of the design envelopes.

Maximum and minimum reinforcement In BS8110, it is stipulated that the area of reinforcement should never exceed 4.0% of the overall concrete section to avoid the potential for a sudden and catastrophic type of failure and practical difficulties in reinforcing and concreting. On the other hand, too little reinforcement indicates an *under-reinforced* section which is also undesirable. The area of reinforcement is therefore limited not to be less than 0.24 % for mild steel, and 0.13 % for high yield steel.

6.2.1.1 Singly Reinforced Rectangular Section – Design Procedure

For a section with moment redistribution less than 10%, single reinforcement only is provided if the factor K

$$K = \frac{M}{bd^2 f_{cu}} \tag{6.1}$$

is not greater than 0.156, where M is the ultimate design bending moment acting on beam, b and d are the breadth and effective depth of the section respectively, and f_{cu} is the characteristic strength of the concrete.

The area of the tensile steel A_s is given by

$$A_s = \frac{M}{0.87 f_y z} \tag{6.2}$$

where f_y is the characteristic strength of the steel and z is the lever-arm defined by

$$z = d\left[0.5 + \sqrt{(0.25 - K / 0.9)}\right] \tag{6.3}$$

The lever-arm should not exceed a value of $0.95d$.

6.2.1.2 Doubly Reinforced Rectangular Section – Design Procedure

The required area of the compression reinforcement A'_s for a doubly reinforced section is obtained from

$$A_s^{'} = \frac{\left(K - K^{'}\right) f_{cu} bd^2}{f_s^{'}(d - d')} \tag{6.4}$$

where $K' = 0.156$ and d' is the depth to the centroid of the compression reinforcement.

The stress in the compression reinforcement f_s' is equal to the design strength of $0.87f_y$, if the ratio

$$\frac{d'}{x} \leq \left[1 - \frac{0.87 f_y}{700}\right] \tag{6.5}$$

When this inequality is not satisfied, the stress in the compression reinforcement is in the elastic zone and is calculated from

$$f_s' = 700(1 - d'/x) \tag{6.6}$$

The total area of tension reinforcement A_s is therefore a sum of what is required for a corresponding singly reinforced section, and an additional area needed to balance the compression force in the top reinforcement, to maintain the equilibrium of the section. If the stress in both the tension and compression reinforcement are at design strength, this additional area must be equal to the area of the compression reinforcement, and hence

$$A_s = \frac{0.156 f_{cu} b d^2}{0.87 f_y z} + A_s' \tag{6.7}$$

where the level arm z is equal to $0.775d$.

6.2.1.3 *Flanged (T- or L-) Section*

In Section 5.2.5, three distinctive design cases were identified for calculating the area of tension reinforcement A_s. In the first case, when the section is resisting sagging moments and the neutral axis falls within the flange, the reinforcement ratio ρ is given by

$$\rho = \frac{A_s}{b_f d} = \frac{M}{0.87 f_y z} \frac{1}{b_f d} \tag{6.8}$$

where M is the ultimate design bending moment and b_f is the effective breadth of the flange (see Figure 6.1). When the neutral axis falls below flange whilst resisting sagging moments, the design equation for calculating the reinforcement ratio is given by

$$\rho = \frac{A_s}{b_w d} = \frac{M + 0.1 f_{cu} b_w d (0.45d - h_f)}{0.87 f_y (d - 0.5 h_f)} \frac{1}{b_w d} \tag{6.9}$$

where f_{cu} is the characteristic concrete material strength, b_w is the breadth of the web and h_f is the height of the flange. Finally, when the section is resisting hogging moments the flange is in tension and the design equation is then given by

$$\rho = \frac{A_s}{b_w d} = \frac{M}{0.87 f_y z} \frac{1}{b_w d} \tag{6.10}$$

6.2.1.4 Shear Reinforcement Design Equations

To prevent punching shear type failure in the section the average shear stress v must not exceed the minimum value given by

$$v = \frac{V}{b_w d} \leq \min\left(0.8\sqrt{f_{cu}}, 5\,N/mm^2\right) \tag{6.11}$$

where V is the shear force at the support due to ultimate loads.

Shear reinforcement in the form of nominal vertical links should be provided where the average shear stress v (N/mm^2) is less than (v_c +0.4), according to the expression

$$\frac{A_{sv}}{s_v} = \frac{0.4 b_w}{0.87 f_{yv}} \tag{6.12}$$

where A_{sv} is the cross-sectional area of two link legs, s_v is the spacing of the links and f_{yv} is the characteristic strength of the link reinforcement. Here, v_c is the ultimate shear stress given in BS8110 as

$$v_c = k_1 k_2 0.79 \left[100 A_s / b_w d\right]^{1/3} (400/d)^{1/4} / \gamma_m \tag{6.12a}$$

where k_1 and k_2 are the enhancement factors for depth/shear span ratio and the shear strength respectively, A_s is the area of tension steel, b_w is the thickness of the web, d is the effective depth of the section and γ_m is a combined materials safety factor for steel and concrete .

For those parts of the beam where the average stress v exceeds (v_c + 0.4) N/mm^2, the design links are provided according to

$$\frac{A_{sv}}{s_v} = \frac{b_w\left(v - v_c\right)}{0.87 f_{yv}} \tag{6.13}$$

Since the expression for v_c is a function of the area of the tension reinforcement A_s, the procedure for determining the critical shear forces has to be implemented after the critical bending moments and their corresponding areas of tension reinforcement have been established using the design envelope.

To determine the weight of the shear reinforcement W_{sj} required for the calculation of the reinforcement costs in the objective function (see equation 6.18), the following expressions are adopted from the IStructE Manual (1985)

Single Links (i.e. two legs) $\quad 8(b_w + h)\dfrac{A_{sv}}{s_v} \quad$ *(kg/m)*

Double Links (i.e. four legs) $\quad 8(1.5b_w + 2h)\dfrac{A_{sv}}{s_v} \quad$ *(kg/m)* $\quad$ (6.13a)

6.2.1.5 Objective Function

In this research, the fitness function for reinforced concrete beams includes the cost of concrete, cost of steel and the cost of formwork together with their associated labour costs. The total cost of the steel is the addition of the cost of the main reinforcement and the cost of the shear reinforcement. Taking account of all these costs the fitness function is proposed as

$$Z = Z_c + Z_s + Z_f \tag{6.14}$$

where Z_c, Z_s and Z_f are the total cost of concreting, reinforcing and formworking respectively. Furthermore, the breakdown in the costs of concreting is represented as

$$Z_c = Z_{cm} + Z_{cw} + Z_{cl} \tag{6.15}$$

where Z_{cm} is the material cost, Z_{cw} is the cost allowance for wastage and Z_{cl} is the labour cost. Relating these individual costs to the design variables, the total costs of concreting for flanged continuous beams is derived as

$$Z_c = \left[C_c\left(1 + w_{fc}\right) + C_{cl}\right]b_w\left(h - h_f\right)\sum_{j=1}^{NS} L_j \tag{6.16}$$

where NS is the number of spans, C_c is the cost of concrete per unit volume, w_{fc} is the wastage allowance factor, b_w is the breadth of the web, h is the overall depth of the section, L_j is the effective length of the span and C_{cl} is the cost of labour per unit volume of concrete.

For a rectangular section, equation (6.16) is modified, with the breadth of the beam equal to b_w and the height of the flange h_f equal to zero. Furthermore, for a simply supported beam the number of spans NS is equal to one.

The cost of steel is represented in a similar manner as

$$Z_s = Z_{sm} + Z_{sw} + Z_{sf} + Z_{sl} \tag{6.17}$$

where Z_{sm} is the material cost, Z_{sw} is the cost allowance for wastage, Z_{sf} is the steel fixing cost and Z_{sl} is the labour cost. Relating these individual costs to the design variables, the total costs of reinforcing are derived as

$$Z_s = \left[C_s\left(1 + w_{fs} + f_{fs}\right) + C_{sl}\right]\sum_{j=1}^{NS}\left(W_{l_j} + W_{s_j}\right) \tag{6.18}$$

where C_s is the cost of steel per unit weight, $W_{l\,j}$ and $W_{s\,j}$ are the weights of longitudinal and shear reinforcement respectively, w_{fs} is the wastage allowance factor, f_{fs} is the steel fixing allowance factor and C_{sl} is the cost of labour per unit weight of steel.

Finally, the cost of formwork is represented as

$$Z_f = Z_{tf} + Z_{tb} + Z_{wfp} + Z_{lm} + Z_{lfs} \tag{6.19}$$

where Z_{tf} and Z_{tb} are the material cost of timber framing and boarding respectively, Z_{wfp} is the cost allowance for wastage, fixing and props, Z_{lm} is the cost of labour to make the formwork and Z_{lfs} is the cost of labour to fix and strip the formwork. Relating these individual costs to the design variables, the total costs of formworking are derived as

$$Z_f = \left[\left(T_f\,C_{tf} + C_{tb}\right)\left(1 + w_{fp}\right) / T_u + C_{lm} / T_u + C_{lfs}\right]\left[b_w + 2\left(h - h_f\right)\right] \sum_{j=1}^{NS} L_j \tag{6.20}$$

where C_{tf} is the cost of timber framing per unit volume, C_{tb} is the cost of timber boarding per unit area, T_f is the volume of timber framing per unit area of timber boarding, T_u is the timber usage factor and C_{lm} and C_{lfs} are the labour costs to make and to fix and strip per unit area of timber respectively.

6.2.2 Analysis and Design of Reinforced Concrete Slabs

Reinforced concrete slabs can be defined primarily as flexural structural members that are wider than deep to a significant ratio. They are used in floors, roofs and walls of buildings and as the decks of bridges. Reinforced concrete slabs may span in one direction or two directions, and they may be supported on concrete beams, steel beams, walls or directly by the structural columns. The floor system of a structure can take many forms such as *in-situ* solid slabs, ribbed slabs or precast units. The structural behaviour of reinforced concrete slabs depends on the supporting conditions, loading patterns, presence of openings and geometrical shape. In this research, rectangular flat solid slabs are considered, supported on continuous beams and cast *in situ*, with no openings and carrying a regular pattern of uniform loading. When a flat solid slab is supported on all four sides it effectively spans in both directions and it is sometimes more economical to design it on that basis. In these circumstances the amount of bending in each direction will depend on the ratio of the two perpendicular spans and on the supporting conditions.

6.2.2.1 BS 8110 Moment Coefficient Method of Analysis

The orthogonal moments are calculated using the *Moment Coefficient Method*, as detailed in BS 8110. For a given slab geometry, supporting conditions and loading arrangements, the analysis procedure used in this research may be summarised into the following steps:

1. Calculate the ultimate limit state load n including the self-weight of the slab using the partial safety factors as recommended by BS 8110.
2. Calculate coefficients β_x, β_y and β_1 to β_4 according to BS8110 procedure.
3. Calculate moments per unit width by multiplying the corresponding coefficient β with the product of the total load n and the square of the span L_x^2 (see Figure 6.4), where L_x is always referred to as the shorter slab span.

$$m_i = \beta_i n L_x^2 \qquad i = 1, 2, 3, 4, x, y \tag{6.21}$$

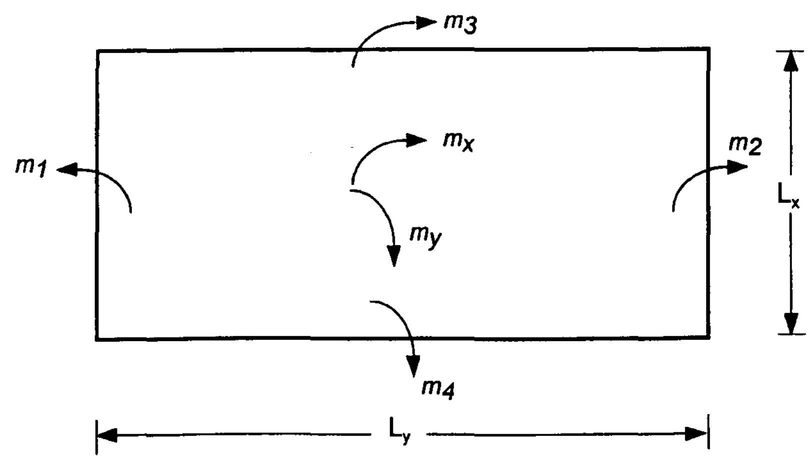


Figure 6.4 *Bending Moments and Slab Geometry*

For more complex slabs with non-rectangular plan geometry, irregular support conditions, openings or non-uniform loading arrangements, a more sophisticated analysis is required such as that offered by the *finite element* method. Two other principal analytical methods are applicable for certain slab configurations. For instance, the *yield-line* method is particularly suitable for slabs with a complex plan geometry or concentrated loading, whilst the *Hillborg strip* method (BS8110) is useful for slabs with openings. Both of these *plastic* or *collapse* methods of analysis are particularly valuable in assisting an understanding of failure mechanisms.

Continuous reinforced concrete slabs are in principle designed to withstand the most unfavourable loading arrangements, except in certain cases where simplified single load case analysis is used, the conditions of which are precisely defined by BS8110. The design of slabs is similar to that of beams, albeit somewhat simpler due to the following reasons

(i) the design breadth of the slab is fixed (b = 1000 mm).

(ii) the shear stresses are usually low except when there are heavy concentrated loads present.

(iii) compression reinforcement is seldom required.

Having determined the moments in each direction, the areas of reinforcement to resist these moments are determined independently, in a similar manner to reinforced concrete beam design. The slab is reinforced in both directions parallel to the spans, with bars for the shorter span being placed furthest from the neutral axis to maximise the section's effective depth.

6.2.2.2 Span - Effective Depth Ratios

Serviceability limit state requirements are imposed to prevent excessive slab deflections that may result in damage to ceilings, floor finishes or other architectural details. This is achieved by limiting span-depth ratios. Since slabs are usually slender members this limitation will often control the overall depth of the slab. The lower bound depth therefore, must be set greater than the minimum effective depth d_{min} given by

$$d_{\min} = \frac{L}{f_1 f_2} \tag{6.22}$$

where f_1 is the basic ratio and f_2 is the modification factor defined by BS8110.

The basic ratio is a function of the support conditions and the shape of the cross-section. The modification factor takes into account the level of service stress f_s in the tension steel spanning in the shorter direction.

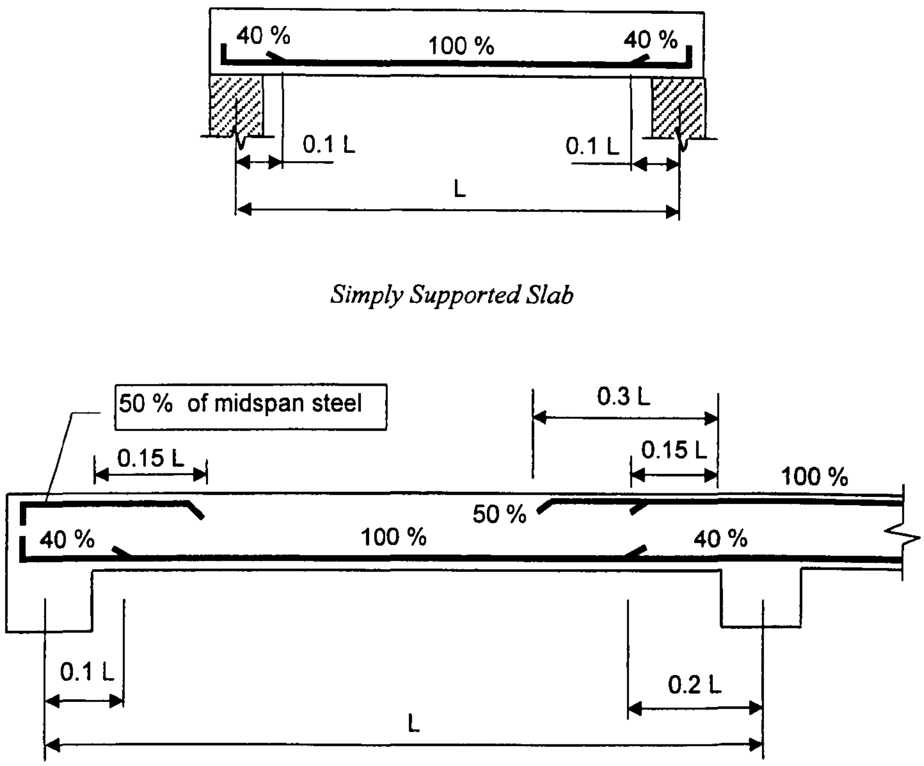
6.2.2.3 Shear Stresses

In general, the use of shear reinforcement is avoided by either increasing the amount of tension reinforcement or the slab depth, until the maximum shear stress in the slab v is less than the ultimate design shear stress v_c defined by BS8110. Shear resistance is generally not a problem in solid slabs, and only in special cases where shear stresses are high is shear reinforcement required. The shear resistance of solid slabs under concentrated loads is calculated in the same way as for *punching shear* around a column. This research however, considers only uniform loads as these are the most often assessed in professional practice. Hence, *punching shear* is not further elaborated.

6.2.2.4 Slab detailing and curtailment

The slab is divided into middle and edge strips in both directions and reinforcement required to resist the maximum mid-span moments is placed in the middle strips. In the edge strips only nominal reinforcement is necessary, such that the percentage tension reinforcement ratio equals 0.13 for high yield steel or 0.24 for mild steel.

No redistribution of the moments is allowed when this method is applied. BS8110 also specifies that torsion reinforcement should also be provided at any corner where the slab is simply supported on both edges. The weight of the longitudinal steel in both the top and bottom of the slab is calculated using the simplified rules of curtailment as shown in Figure 6.5.



Simply Supported Slab

Continuous Slab

Figure 6.5 *Simplified rules of curtailment of bars in slab spanning in one direction*

Once determined, the weight of the longitudinal reinforcement is used to calculate the total reinforcement costs, with the provision that if $L_y/L_x > 2$, the slabs should be designed as spanning in the shortest (L_x) direction.

6.2.2.5 Fitness Function

The elements of the slab cost objective function derived in this research reflect a practical and realistic approach to the assessment of the structural costs. They include the cost of concrete, reinforcement and formwork together with the costs associated with labour, making, fixing and stripping the formwork, steel fixing and material wastage. The total cost of the reinforcement is apportioned between that required to resist the orthogonal maximum moments, torsional corner reinforcement and shear reinforcement if required. Formwork costs apply to both the vertical faces and the underside of a slab, although those costs associated with the vertical formwork may often be included when casting the continuous beams that support such an integral structure. Taking account of all these costs the following fitness function is proposed

$$Z = Z_c + Z_s + Z_f \tag{6.23}$$

where Z_c, Z_s and Z_f are the total cost of concreting, reinforcing and formworking respectively. Furthermore, the costs of concreting can be broken down into their individual elements and derived as

$$Z_c = \left[C_c\left(1 + w_{fc}\right) + C_{cl}\right]h_s L_x L_y \tag{6.24}$$

where C_c is the cost of concrete per unit volume, w_{fc} is the wastage allowance factor, C_{cl} is the cost of labour per unit volume of concrete, h_s is the overall depth of the slab, L_x is the length of the shorter side and L_y is the length of the longer side.

Similarly, the cost of steel are derived as

$$Z_s = \left[C_s\left(1 + w_{fs} + f_{fs}\right) + C_{sl}\right]\left(W_{l_j} + W_{s_j}\right) \tag{6.25}$$

where C_s is the cost of steel per unit weight, $W_{l\,j}$ and $W_{s\,j}$ are the weights of the longitudinal and shear and/or torsional reinforcement respectively, w_{fs} is the wastage allowance factor, f_{fs} is the steel fixing allowance factor and C_{sl} is the cost of labour per unit weight of steel. Here, the weights $W_{l\,j}$ and $W_{s\,j}$ are calculated for the whole slab in both directions.

Finally, the cost of formwork are derived as

$$Z_f = \left[\left(T_f C_{tf} + C_{tb}\right)\left(1 + w_{fp}\right) / T_u + C_{lm} / T_u + C_{lfs}\right]\left[L_x L_y + 2h_s\left(L_x + L_y\right)\right] \tag{6.26}$$

where C_{tf} is the cost of timber framing per unit volume, C_{tb} is the cost of timber boarding per unit area, T_f is the volume of timber framing per unit area of timber boarding, T_u is the timber usage factor, and C_{lm} and C_{lfs} are the labour costs to make and to fix and strip per unit area of timber respectively.

6.2.3 Analysis and Design of Reinforced Concrete Frames

Reinforced concrete skeletal structures consist of a series of monolithic frames that are connected to walls and slabs to form an integral structure. When analysing such systems, they can be considered as complete space frames or can be divided into a series of plane frames. In the latter case, the slabs are analysed first as continuous members which span in one or two directions, and are supported by beams or structural walls. The rigid plane frames can be divided into two types; *braced* frames supporting vertical loads only, and *unbraced* frames supporting vertical and lateral loads. In this research, only *braced* plane frames are considered, assuming that the sway deflection is reduced substantially by the presence of shear walls or other forms of bracing. This situation is commonly encountered in design office practice.

6.2.3.1 Design of Reinforced Concrete Frames

Frames consist of rigidly jointed reinforced concrete beams and columns, and hence once analysed the structure is designed at elemental level. For a *braced* frame structure, beams are designed using the procedures outlined in Section 6.2. In this research, only braced short columns are considered as these are representative of the majority of columns used in design. BS8110 defines a *braced short* column as one for which the ratio of effective height to the least lateral dimension does not exceed 15.

In Section 4.3.6, the column design equations were derived and are summarised below for convenience.

$$\frac{N}{bhf_{cu}} = 0.402\frac{x}{h} + n\alpha \tag{6.27}$$

$$\frac{M}{bh^2 f_{cu}} = 0.402\frac{x}{h}\left(0.5 - 0.45\frac{x}{h}\right) + m\alpha \tag{6.28}$$

where

$$n = (f_1+f_2) / 2f_y$$

$$m = (2d/h - 1)(f_1 - f_2)/4f_y$$
$$\alpha = A_{sc}f_y/bhf_{cu} = \rho_{sc}f_y/f_{cu} \qquad (6.29)$$

This research has derived an analytical approach to the exact solution of these design equations, considering all the possible combinations of steel stresses in both sets of reinforcement. In this way, the iterative procedure that requires balancing the design equations with regard to x/h is avoided, as it can be time consuming and sometimes ill-conditioned. This approach offers an exact and direct solution to (x/h) without the need for iterative trials. This considerably reduces the computation time when optimising structures with a large number of columns.

For the case where the steel stresses in both sets of reinforcement are in the elastic range, the following approach for obtaining x/h has been derived.

1. Subtracting equation (6.28) from (6.27) and eliminating α, gives

$$\frac{M}{bh^2 f_{cu}} = \frac{N}{bhf_{cu}}\frac{m}{n} - 0.1809\left(\frac{x}{h}\right)^2 - 0.201\left(\frac{x}{h}\right)\left(2\frac{m}{n} - 1\right) \qquad (6.30)$$

The stresses in the reinforcement f_1 and f_2 can be presented in the following form

$$f_1 = 700\ (x/h - 1 + d/h)/(x/h)$$
$$f_2 = 700\ (x/h - d/h)/(x/h) \qquad (6.31)$$

2. Considering equations (6.29), the general expression for the ratio m/n is derived as

$$m/n = (d/h - 0.5)(f_1 - f_2)/(f_1 + f_2) \qquad (6.32a)$$

Replacing f_1 and f_2 with the expressions given by equation (6.31), we obtain

$$m/n = (d/h - 0.5)^2/\ (x/h - 0.5) \qquad (6.32b)$$

3. Substituting equation (6.32b) into the equation (6.30), a cubic expression with x/h as an unknown is obtained

$$(x/h)^3 - 1.6\dot{1}(x/h)^2 + K_1(x/h) + K_2 = 0 \qquad (6.33)$$

where factors K_1 and K_2 are constants derived as direct functions of N/bhf_{cu}, M/bh^2f_{cu} and d/h. This cubic equation is solved using the *Newton-Ralphson* method to obtain the root of the function. Using this value for x/h, α and hence the area of reinforcement is determined from equation (6.27).

Equation (6.33) only applies when the steel stresses in the top and bottom reinforcement are in the elastic range. Beyond the elastic range, the stress in the steel reaches the design yield stress ($\pm f_y/1.15$), as specified in BS8110. Four possible cases exist, namely

$$
\begin{aligned}
x/h &\geq (1-d/h)/(1-f_y/805) &&\Rightarrow && f_1 = +f_y/1.15 \\
x/h &\leq (1-d/h)/(1+f_y/805) &&\Rightarrow && f_1 = -f_y/1.15 \\
x/h &\geq (d/h)/(1-f_y/805) &&\Rightarrow && f_2 = +f_y/1.15 \\
x/h &\geq (d/h)/(1+f_y/805) &&\Rightarrow && f_2 = +f_y/1.15
\end{aligned}
\qquad (6.34)
$$

For the practical range of x/h values, the stresses f_1 and f_2 are rarely both elastic at the same time. Hence, the derived solution for the all elastic stress region, represented by equation (6.33) was further investigated. Five critical combinations of stresses f_1 and f_2 were established and solutions were derived using high yield steel for the column reinforcement. The derivation of the design equations for each combination is given in *Appendix* F.

Figure 6.6a shows the stress distribution for the five combinations indicating the boundary values for x/h for a typical d/h value of 0.9.

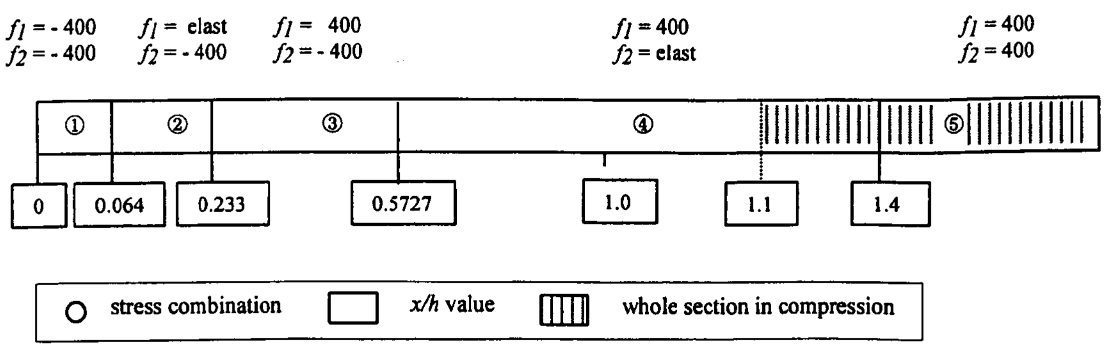


Figure 6.6a *Stress Distribution in column reinforcement for d/h =0.9*

Figure 6.6b shows the flow diagram for the column reinforcement ratio design algorithm that was extensively tested and results were compared with those obtained from a standard graphical solution. Table 6.1 shows the results for a column with breadth and depth of 300 *mm* and a cover to reinforcement of 28 *mm*. Values of f_{cu} and f_y are taken to be 30 and 460 N/mm^2, respectively. Axial force and bending moment are varied so that all possible combinations of steel stresses are analysed and compared.

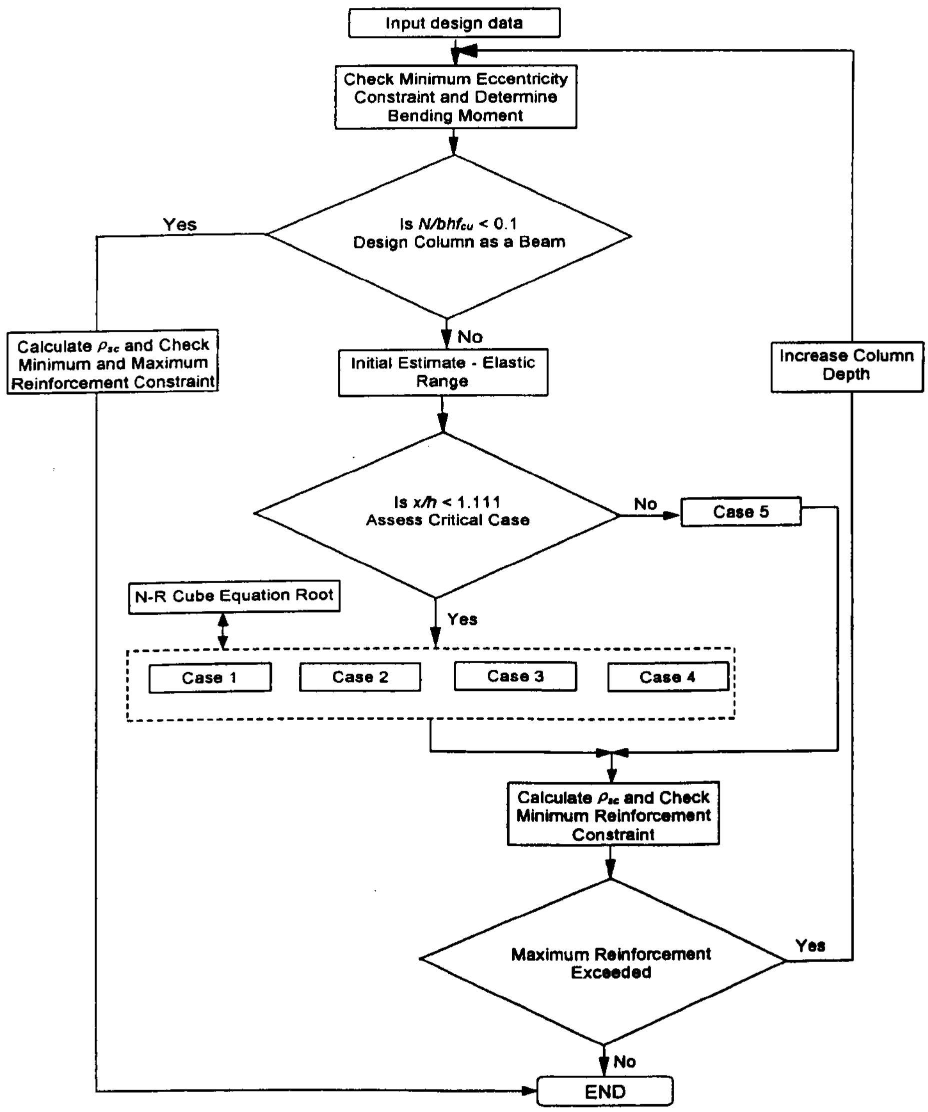


Figure 6.6b *Column Reinforcement Ratio Design Algorithm*

Combination	N (kN)	M (kNm)	N/bhf_{cu}	M/bh^2f_{cu}	x/h	f_1	f_2	n	m	α	$\alpha_{gr}^{(+)}$	α_diff (%)
1	-1360	7	-0.504	0.009	0.045	-400	-400	-0.87	0	0.60	0.6	0
2	300	10	0.111	0.012	0.264	351	-400	-0.05	0.33	0	0	0
3	300	280	0.111	0.3457	0.276	400	-400	0	0.35	0.86	0.86	0
4	1750	100	0.648	0.1235	0.945	400	34.8	0.47	0.16	0.57	0.57	0.4
5(*)	2620	0	0.969	0.0	>1.11	400	400	0.87	0	0.60	0.6	0.3

(*) - pure axial stress; (+) – standard graphical solution

Table 6.1 *Comparison between the developed* (α) *and standard graphical solution* (α_{gr})

The comparison demonstrates that the developed approach gives accurate results for reinforcement ratios across the whole range of possible *x/h* values.

The solution of the design equations is also subject to additional requirements specified in BS8110, that is *minimum eccentricity* and *minimum/maximum reinforcement ratios.*

Minimum eccentricity In practice, it is never possible to ensure that a column is perfectly straight or that the load is purely axial. Therefore, BS8110 states that at no section in a column should the design moment be taken as less than that produced considering the design ultimate axial load as acting at a minimum eccentricity e_{min} of 0.05 times the overall dimension in the plane of bending considered, but not more than 20 *mm*.

Minimum and maximum reinforcement ratios A rectangular column should not contain less that a total of 4 bars in the section, with compression reinforcement links diameter limited to 0.25 times the largest compression bar or 6 *mm*, whichever is greater. Minimum percentage of reinforcement for a rectangular section is 0.4% of the overall compressed section area, for both mild steel and high yield steel. The maximum percentage of longitudinal reinforcement should not exceed 6% of the overall section area for vertically cast columns.

6.2.4 *Multiple Load Case Analysis*

Account is taken of multiple load cases using the approach developed for minimum cost optimisation using the SLP method (*see* Section 5.2.2). As previously, braced frames are subjected to the following load cases

(i) All spans with maximum loading ($1.6Q_k + 1.4G_k$)

(ii) Alternate spans with maximum ($1.6Q_k + 1.4G_k$) and minimum ($1.0G_k$) load

The latter load case produces two loading patterns per each floor level (storey). The total number of load cases *TNLC* is therefore expressed as

$$TNLC = 2NS + 1 \tag{6.35}$$

where NS is a number of storeys.

For the special case of one-bay, *n*-storey frame structures, equation (6.35) needs to be modified to give

$$TNLC = NS + 1 \tag{6.36}$$

For these type of frames only one load pattern per storey can be produced.

6.2.5 *Objective Function*

The research derived fitness function for braced frame structures is represented as

$$Z = Z_c + Z_s + Z_f \tag{6.37}$$

where Z_c, Z_s and Z_f are the total cost of concreting, reinforcing and formworking respectively. Furthermore, the breakdown in the costs of concreting is represented as

$$Z_c = Z_{cm} + Z_{cw} + Z_{cl} \tag{6. 38}$$

where Z_{cm} is the material cost, Z_{cw} is the cost allowance for wastage and Z_{cl} is the labour cost. Relating these individual costs to the design variables, the total costs of concreting for reinforced concrete frames is derived to be

$$Z_c = \left[C_c\left(1+w_{fc}\right)+C_{cl}\right]\left(\sum_{j=1}^{NB} b_j h_j L_j + \sum_{k=1}^{NC} bc_k hc_k Hc_k\right) \tag{6. 39}$$

where NB and NC are the number of beams and columns respectively, C_c is the cost of concrete per unit volume, C_{cl} is the cost of labour per unit volume, w_{fc} is the wastage allowance factor, b_j , h_j and L_j are the breadth, overall depth and the effective length of the beam respectively and bc_k , hc_k and Hc_k are the breadth, overall depth and the effective height of the column respectively.

The cost of steel is represented in similar manner as

$$Z_s = Z_{sm} + Z_{sw} + Z_{sf} + Z_{sl} \tag{6. 40}$$

where Z_{sm} is the material cost, Z_{sw} is the cost allowance for wastage, Z_{sf} is the steel fixing cost and Z_{sl} is the labour cost. Relating these individual costs to the design variables, the total cost of reinforcing is derived as

$$Z_s = \left[C_s\left(1+w_{fs}+f_{fs}\right)+C_{sl}\right]\left[\sum_{j=1}^{NB}\left(W_{m_j}+W_{s_j}\right)+\sum_{k=1}^{NC}\left(Wc_{m_k}+Wc_{c_k}\right)\right] \tag{6. 41}$$

where C_s is the cost of steel per unit weight, w_{fs} is the wastage allowance factor, f_{fs} is the steel fixing allowance factor and C_{sl} is the cost of labour per unit weight of steel, W_{mj} and W_{sj} are the weights of main and shear reinforcement in the beam respectively and Wc_{mk} and Wc_{ck} are the weights of main reinforcement and nominal compressive links in the column respectively.

Finally, the cost of formwork is represented as

$$Z_f = Z_{tf} + Z_{tb} + Z_{wfp} + Z_{lm} + Z_{lfs} \tag{6. 42}$$

where Z_{tf} and Z_{tb} are the material cost of timber framing and boarding respectively, Z_{wfp} is the cost allowance for wastage, fixing and props, Z_{lm} is the cost of labour to make formwork and Z_{lfs} is the cost of labour to fix and strip formwork. Relating these individual costs to the design variables, the total cost of formworking is derived as

$$Z_f = \left[\left(T_f C_{tf} + C_{tb}\right)\left(1+w_{fp}\right)/T_u + C_{lm}/T_u + C_{lfs}\right]\left[\sum_{j=1}^{NB}\left(b_j+2h_j\right)L_j + \sum_{k=1}^{NC}\left(2bc_k+2hc_k\right)Hc_k\right] \tag{6. 43}$$

where T_f is the volume of timber framing per unit area of timber boarding, C_{tf} is the cost of timber framing per unit volume, C_{tb} is the cost of timber boarding per unit area, w_{fp} is the wastage allowance factor for fixing and props, T_u is the timber usage factor and C_{lm} and C_{lfs} are the labour costs to make and to fix and strip per unit area of timber respectively.

6.3 Implementation of Genetic Algorithms

For the implementation of GA used in this research it was important to identify the main design stages that are undertaken as part of a traditional design approach (see Figure 6.7).

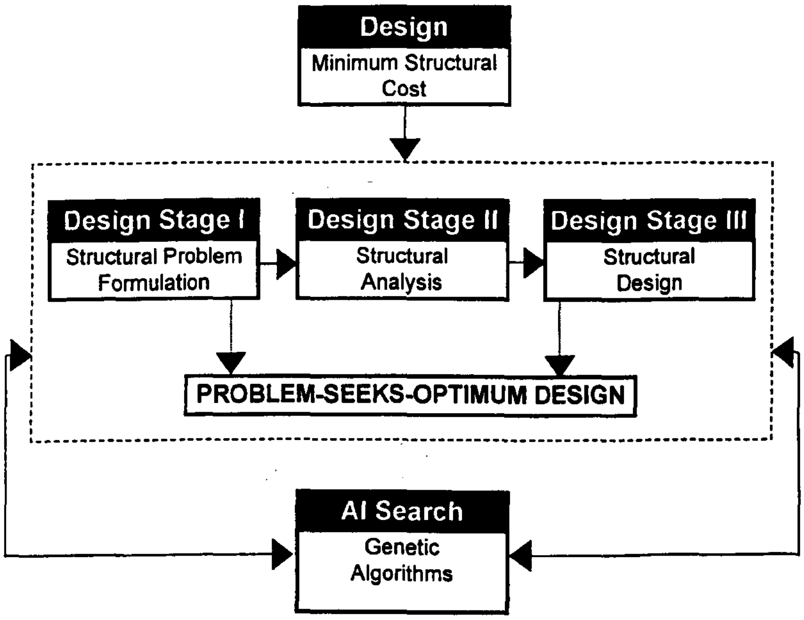


Figure 6.7 *Design Process Model*

As shown in Figure 6.7, the structural analysis and design are integrated with the effective GA search of standard design solutions, guided by the objective of finding minimum structural costs. Whilst the structural analysis and design phases are important activities within the developed optimum design computer programmes, their effectiveness within an artificially intelligent computer design environment requires that the structural problem, the objective function and the design constraints are clearly identified and mapped against 'real world' expectations. In this way, the resulting computer programme can simulate the decision-making process and model this against the design requirements specified within BS8110. The adopted *problem-seeks-optimum design* approach (*see* Figure 6.7) had as its aim to ensure that the implemented genetic algorithms would use practice-orientated, robust, reliable and efficient optimisation coding, taking into account the design procedures and analytical methods that are familiar to the designer.

6.3.1 *Fitness Function*

The component costs are specified within the computer programme developed in this research using the Cost Control Form, as shown in Figure 6.8.

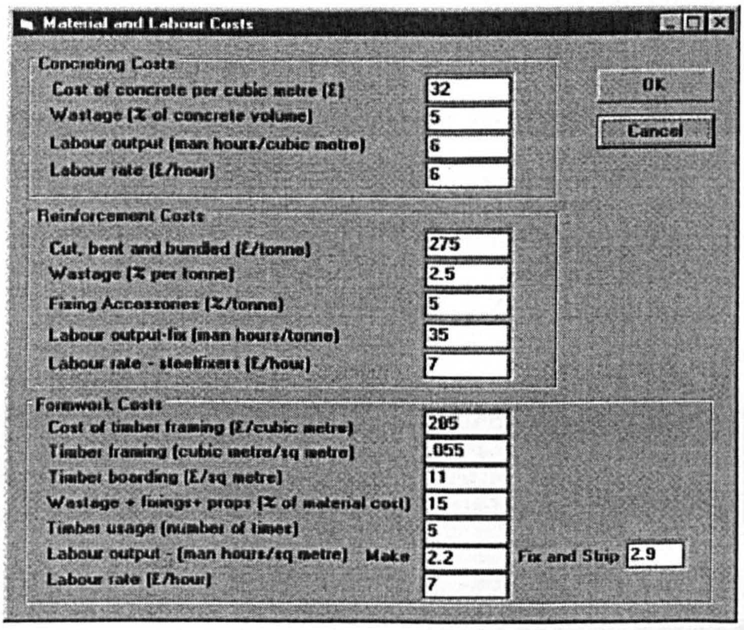
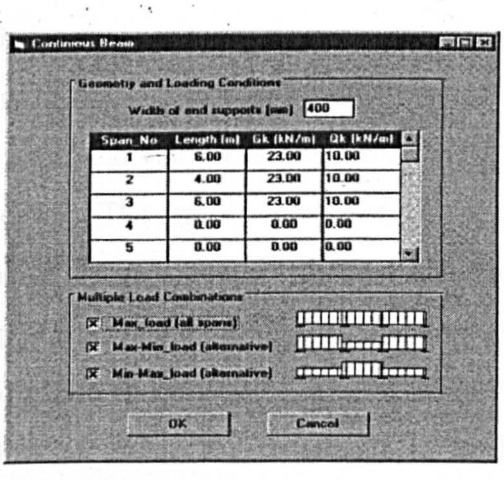


Figure 6.8 *Cost Control Form*

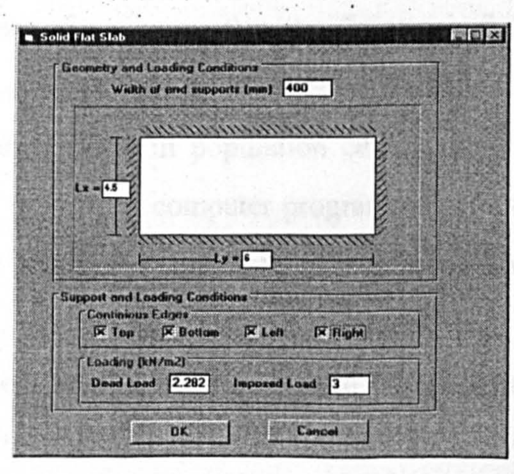
The elements of the cost objective function include the cost of concrete, reinforcement and formwork together with the costs associated with labour, making, fixing and stripping the formwork, steel fixing and material wastage. These component costs are explained in detail in the Section 6.2 for beams, slabs and frames separately. Formwork costs correspond to the number of shuttering faces, these being dependent on the type of structure.

6.3.2 *Population Representation and Initialisation*

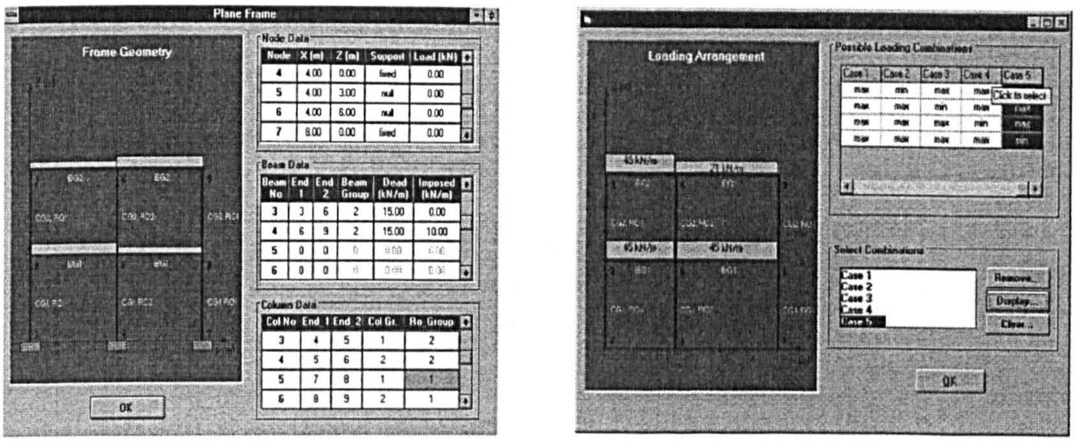
Random selection of initial population is provided according to the principles of standard GA implementation (*see Appendix* E). This is achieved by generating a required number of individuals using a random number generator, which uniformly distributes numbers in the desired range defined by the solution space. The solution space is bounded by specifying the upper and lower bounds to the design variables depending on the type of structural problem considered. The single-level binary string chromosome representation is used to encode the set of design variables in the population. The continuous design variables are approximated by their integer values, improving the algorithm efficiency whilst preserving the required level of accuracy. The coded design variables are mapped to the problem domain specified interval, as explained in *Appendix* E. Depending on the type of structure being designed, i.e. beams, slabs or frames, the structural problem formulation and hence the population representation will differ. This is controlled within the programme using the following Geometry and Loading Control Forms (see Figure 6.9).



a) RC *Continuous Beams*



b) RC *Slabs*



c) RC *Frames-Geometry* **d)** RC *Frames-Loading Combinations*

Figure 6.9 *Geometry and Loading Conditions Control Forms*

6.3.3 *Population Selection*

Ranking principle has been used for selection to determine the number of copies that a member of the population can expect to receive according to its fitness. This principle ranks the population members, allowing every chromosome to receive rank according to its fitness. The member with worst fitness receives rank 1, second worst 2 etc., ending with the fittest member that receives rank N (number of chromosomes in population). As stated by Baker (1985) and Davis (1989), the ranking scheme gives not only the maximum to average fitness normalisation, but also ensures that the fitnesses of the intermediate values are regularly spread out (*see Appendix* E). Therefore, the effect of *superfit* individuals is negligible and *overcompression* in population ceases to be a problem. The reproduction process within the developed computer programme offers a choice of direct (deterministic) selection or remainder stochastic selection without replacement, as explained by Goldberg (1989). For the latter method, the integer part of the expected number of individuals is assigned directly, with additional copies being allocated using the remainder as probability selection criteria. Furthermore, two selection strategies are implemented and evaluated; the standard evolution approach and the elitist model. In the elitist model, the best n members from the previous generation are preserved replacing the worst n individuals in the next generation. The size of population and the number of retained members for the elitist model are specified within the computer programme using the Population Control Form, as shown in Figure 6.10.

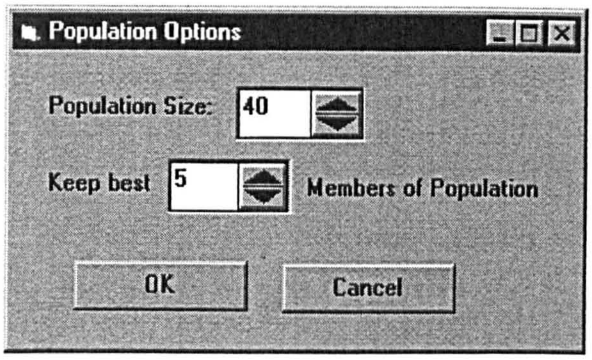


Figure 6.10 *Population Control Form*

Elitism can rapidly increase the performance of GA's as it preserves the best genetic material in the population, but great care needs to be taken in deciding how many members to retain to avoid the danger of premature forced convergence.
In this research, the probability of selection of an individual p_{si} is given by

$$p_{si} = \frac{r_i}{\sum_{i=1}^{N} r_i} \tag{6.44}$$

where r_i is the rank of the i-th individual and N is the population size.

Once the probability of selection is established, individuals are then selected by simulating the spinning of a suitably weighted roulette wheel N times. Mathematically speaking, the number of the expected copies of an individual E_{si} is given by

$$E_{si} = p_{si} N \tag{6.45}$$

Given that the rank of the individual with best (minimum) fitness is taken to be N, and the rank of the individual with worst (maximum) fitness is 1, the underlying trend of ranking is linear with the corresponding fitness function required to be minimised. For linear ranking schemes, the most common suggestion is that the fittest member is usually allocated a probability of selection of *2/N*, whilst the least fittest's probability is constrained to be zero, as outlined by Baker (1985). For the proposed probability of selection p_{si} presented by equation (6.44), the following derivation justifies the probability allocation of *2/N*.

The term $\sum_{i=1}^{N} r_i$ represents the sum of the first N integer numbers, hence

$$\sum_{i=1}^{N} r_i = 1 + 2 + \ldots.. + N = \frac{N(N+1)}{2} \tag{6.46}$$

From equation (6.44), for the individual with worst fitness whose rank is 1, the probability of selection is

$$p_{si} = \frac{1}{\sum_{i=1}^{N} r_i} = \frac{1}{\frac{N(N+1)}{2}} = \frac{2}{N(N+1)} \tag{6.47}$$

From equation (6.45), the expected number of copies is then

$$E_{si} = p_{si} N = \frac{2}{N+1} \tag{6.48}$$

For a sufficiently large size of population, i.e. $N > 40$, this value converges to zero, thereby implying the *death* of the most unfit individuals.

The probability of selecting the individual with best fitness whose rank is N will be

$$p_{sn} = \frac{N}{\sum_{i=1}^{N} r_i} = \frac{N}{\frac{N(N+1)}{2}} = \frac{2}{(N+1)} \tag{6.49}$$

The expected number of copies therefore is

$$E_{si} = p_{si} N = \frac{2N}{N+1} \tag{6.50}$$

For sufficiently large size of population, i.e. $N > 40$, this values converges towards a value of 2. Therefore, for the adopted ranking scheme, the fittest individuals will be given an opportunity to *duplicate* themselves into the mating pool of the next generation. As can be seen from equation (6.45), the expected number of copies E_{si} will not be an integer value, since probability of selection p_{si} is normally a non-integer number. Therefore, a previously explained *direct selection* and *stochastic remainder without replacement* methods for allocation of the number of copies are implemented and compared in this research.

6.3.4 *Crossover (Recombination)*

Three methods of crossover (*one-point*, *two-point* and *uniform*) are implemented in this research, and their performance is compared and discussed (*see Appendix* E). Figure 6.11 below shows the Crossover Parameters Form allowing the designer to choose different selection and crossover operators.

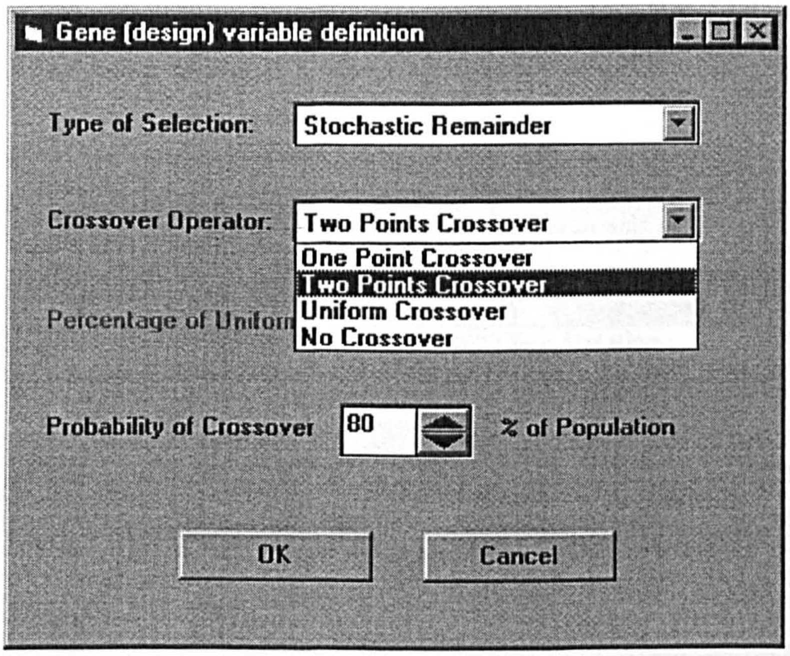


Figure 6.11 *Crossover Control Parameters Form*

Furthermore, a probability of crossover per population is introduced, giving the opportunity to some parental strings to pass the whole of their genetic material to the offspring by simple duplication. The computer programme incorporates the facility to change the type and probability of crossover at any point within the programme's execution, enabling the user to test the suitability of different operator parameters.

6.3.5 *Mutation*

Three different mutation operators are developed and implemented within the research; standard, random hill climb and directional hill climb with a choice of specifying the mutation probability per population and per gene size. The standard mutation operator performs random alteration of the allele's value, while the random hill climb method repeats this process a specified number of times retaining only the beneficial mutations. The directional hill climb mutation further explores the benefits of random mutation in a positive direction. If the fitness improves, the vector difference between the old and new string is calculated and added to the new string. This process is repeated as long as the fitness improves or the number of pre-assigned steps is achieved. These operators are specified within the developed computer programme using the Mutation Control Parameters Form, as shown in Figure 6.12.

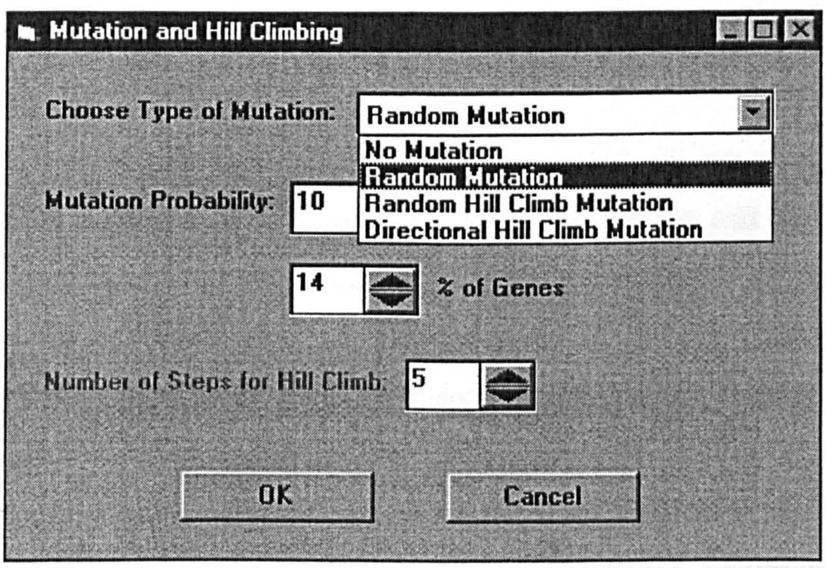


Figure 6. 12 *Mutation Control Parameters Form*

An individual for mutation is randomly chosen according to the probability of the mutation p_m. In general, every single bit of the chromosome string is susceptible to a mutation. These bits are subjected to a simulated weighted coin toss with probability of gene mutation p_{mg}, and if mutation is approved, the corresponding bit will change value. When used sparingly with reproduction and crossover operators, mutation can be seen as a safeguard against premature loss of important genetic material at a particular position. This loss could lead towards a prematurely converged population and *local* optimum problem, where mutation can represent the only means of redirecting the genetic algorithm search near the *global* optimum design space.

6.3.6 *Constraints Handling*

Standard approaches to constraint handling usually adopt a penalty-based technique, such as the *weighted penalty* approaches described by Goldberg (1989) and Jenkins (1992), (*see Appendix* E). More sophisticated approaches, which do not require extensive numerical experiments to determine suitable values of the penalty function coefficients, are also reported in the literature, such as the *Augmented Lagrangian* method, outlined by the Adeli and Cheng (1994), or the *fuzzy logic* approach given by Pearce and Cowley (1995). Whenever possible, redundant variables should be eliminated algebraically so that genetic algorithms do not have to perform unnecessary computational work.

In this research, the classical formulation of stress constraints are not required, as the equilibrium design equations are satisfied within the design process and are not treated as constraints. Deflection constraints imposed by BS8110 are satisfied by ensuring that the effective depth of beams and slabs are modified to comply with the design requirements. Lower and upper bound dimensional constraints are imposed to satisfy aesthetic and practical design considerations and these are controlled at the population reproduction stage. Only *explicit* constraints are therefore considered, such as *shear* stress constraints or *maximum and minimum reinforcement* constraints. These constraints are formulated as inequalities imposed on the continuous feasible space, and are controlled using the *improved rejection* method developed in this research. In the standard version of this method (*see Appendix* E), whenever a constraint is violated, the solution is rejected and replaced by a new population member, randomly produced from the solution space. In this research, an improved strategy to this standard approach is implemented by considering only a part of the random solution space in which another constraint violation is less likely to happen. This was achieved by recognising that part of the solution space where the depth of the structural members is either increased or decreased, depending on the nature of constraint violation. Within this reduced solution space, the new population member is randomly reproduced, replacing the one that has violated the constraint.

6.3.7 *GA Problem Formulation and Termination Conditions*

The Problem Definition Control Form shown in Figure 6.13 allows the designer to specify the necessary information related to both the structural and GA problem definition parameters. The former is displayed on the left-hand side of the form and is selected according to the structural problem formulation that is being assessed, i.e. beams, slabs or frames (*see* Figure 6.9). The latter is displayed on the right-hand side of the form and is independent of the structural problem definition.

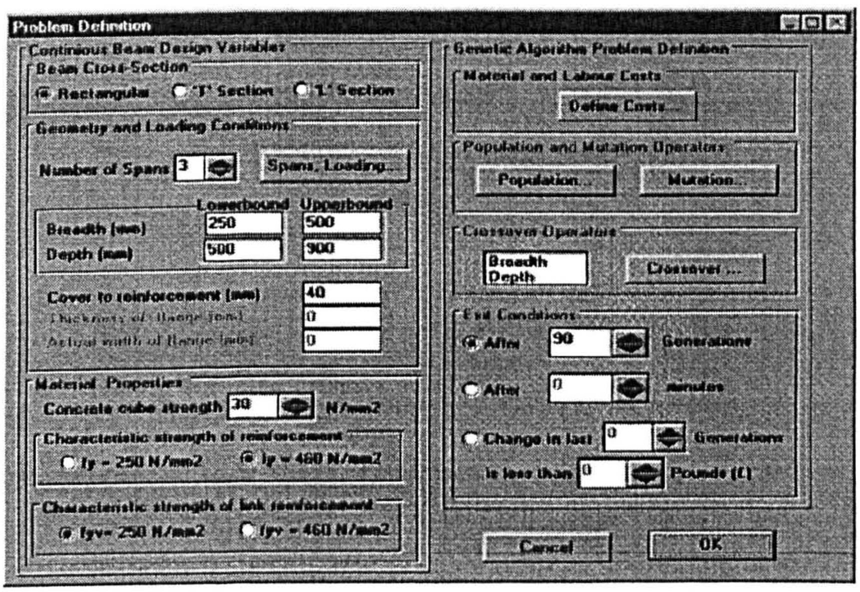


Figure 6.13 *Problem Definition Control Form*

The material and labour cost section allows the user to specify the component costs that are required for computing the fitness function. Selecting the Define Costs button displays the Cost Control Form (*see* Figure 6.8). The population and mutation operators section allows the user to specify the population and mutation control parameters.

Selecting the Population button displays the Population Control Form (*see* Figure 6.10), whilst selecting the Mutation button displays the Mutation Control Parameters Form (*see* Figure 6.12). The crossover operators section provides the user with a descriptive listing of the design variables associated with the particular structural problem, i.e. beams, slabs or frames. Selecting the Crossover button displays the Crossover Control Parameters Form (*see* Figure 6.11).

The exit conditions section offers three options for halting the GA search. The first terminates the programme after a pre-specified number of generations and assesses the quality of the solution against a problem definition. If the solution is still unacceptable, the current programme run may be continued or restarted from the beginning. The second condition limits the search time of the programme. The third condition requires that the changes in the fitness function are less than some specified small value for a predefined number of generations, as shown in Figure 6.13. Care has to be taken when using this criteria to avoid premature convergence towards a local optimum.

6.3.8 *Computer Programme Development*

The GENetic algorithm Optimum Design (GENOD) computer programme for RC skeletal structures written for this research incorporates all of the discussed principles for GA implementation. This programme has been developed using an object-orientated visual programming language utilising dynamic arrays to optimise computer memory requirements. GENOD consists of three independently developed programs for beams, slabs and frames according to their problem formulation. It offers the designer a practical and efficient approach for grouping the structural members, according to the same principles described in Section 5.2.1. Once grouped, GENOD encodes these member assignments into a complex but effective logical system that gives the designer control over the choice of design variables and optimisation search. This enables the programme to achieve both the goal of the designer and significantly improves the search efficiency in a manner that we as humans would consider intelligent.

GENOD uses a genetic algorithm search that exhibits artificial intelligence in examining large number of possibilities, continually discovering how to more efficiently improve its strategy in achieving the design goal. The proposed system performs the complex three-stage process, not only analysing and designing the structure, but also searching for the most economical design guided by the minimum cost fitness function as an objective to be achieved. In each of these stages, the computer becomes the central tool for intelligently searching and logically sorting through similar design concepts. These features establish the basis of the artificially intelligent computer design environment for the proposed design tool. Other features developed within the proposed system could be considered intelligent. For example, in the case of reinforced concrete slabs the computer uses encoded logic knowledge based rules to decide on the values of moment coefficients depending on the span ratios and supporting conditions, as shown in Figure 6.14.

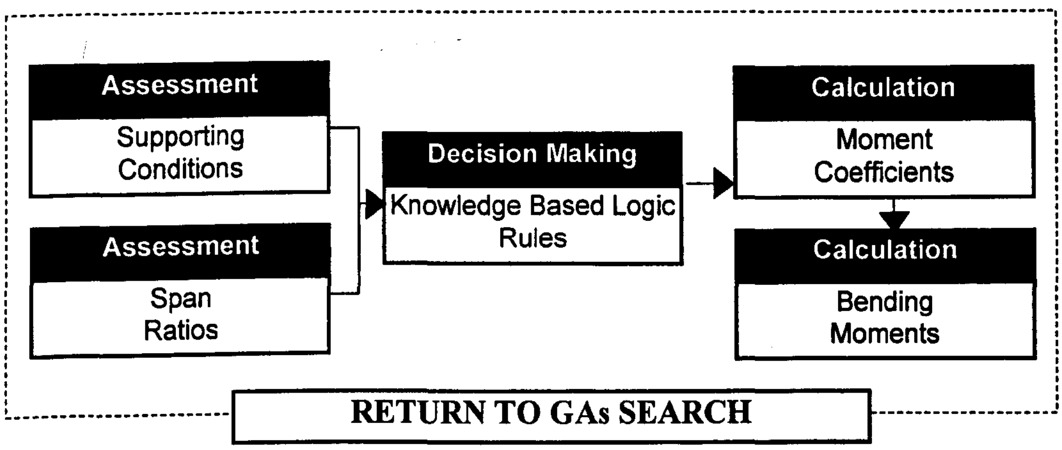


Figure 6.14 *Knowledge Based System* for *Moment Coefficient Calculation*

The analysis of reinforced concrete frames is another example where the proposed system exhibits behaviour that is similar to artificially intelligent pattern recognition, both in the current population search and through the history of GA populations. Considering that genetic algorithms perform a search from a population of individuals, there is a significant probability that for identical population members structural analysis is repeated. This computational effort is unnecessary and therefore a fast re-analysis approach has been implemented within GENOD, enabling the programme to recognise such cases so that analysis is performed only once. This approach has the ability to recognise the binary pattern of those members already analysed through the GA population history, i.e. it has ability to track down the same population members even when they belong to different generations.

Dynamic control and monitoring facilities were developed within the computer programme to improve its performance and to aid the decision-making process so that the most suitable parameter settings for a structural problem are obtained. These are discussed in the following sections.

6.3.9 *Dynamic Control Facilities*

To provide greater control and flexibility over the GA control parameters, facilities to stop, pause, continue and re-start the programme have been developed. These facilities are intended to aid the decision-making process for a suitable choice of GA control parameters where a high level of interactivity between the designer and computer programme is required. They allow the designer to dynamically control the type and probability of each GA operator and so investigate their suitability within a single programme execution. Furthermore, they provide for the constant monitoring, assessing and changing of the control parameters, speeding up the rate of convergence and allowing for fine-tuning of the optimum solution. GENOD also offers facilities to assess and monitor the current algorithm solution, corresponding costs and constraint's violation.

6.3.10 *Monitoring Facilities*

Facilities to store and compare GA graphical performances for different control parameter settings have been developed within GENOD. The ability to capture the graphical performance of GENOD for different GA settings has shown to be particularly helpful when determining the most suitable control parameters for different structural problems. The convergence history of different GA operators can be monitored, stored and their graphical performance evaluated and compared, as shown in Figure 6.15.

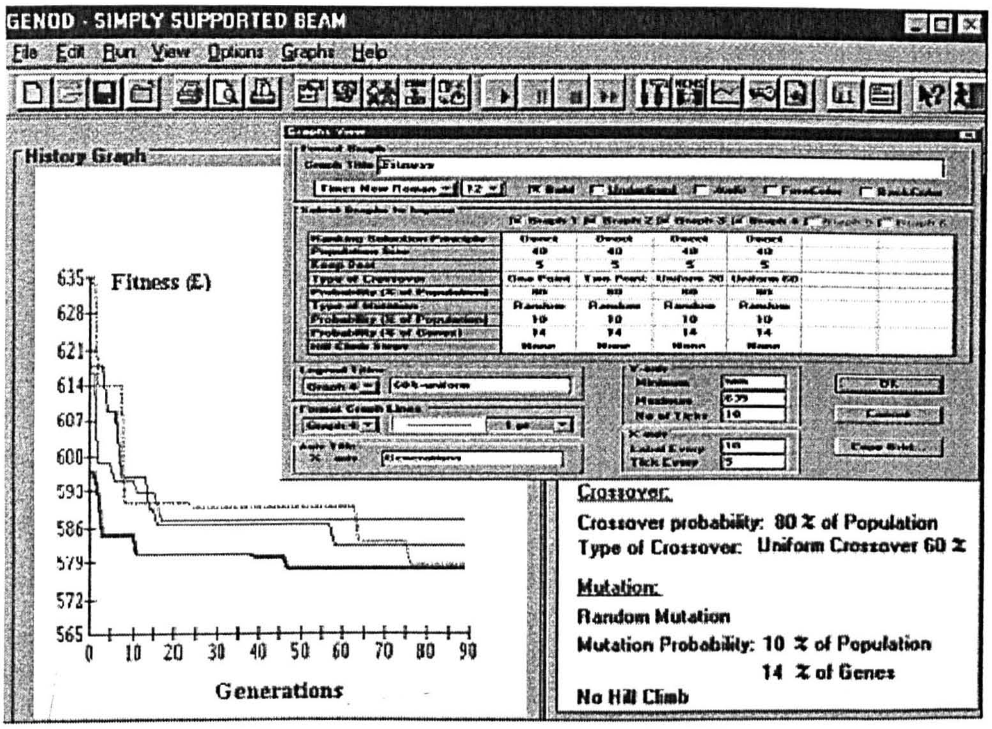


Figure 6.15 *Storing and Monitoring Control Facilities*

6.3.11 *Computational Considerations*

The procedures which control the generation and acceptance of new solutions do not require significant computational effort, in contrast to that associated with the evaluation of the fitness function. The cost fitness functions for reinforced concrete skeletal structures are complex and require repetitive analysis and design for each member generated in the population. However, for small and medium size structures the overall computing time is not critical, with optimum solutions being achieved without recourse to parallel processing or high specification computers.

6.4 Testing and Sensitivity Analysis

Numerous research studies (*see* Section 2.4.1), have shown that the performance of GA's can be enhanced by the careful selection of, and the settings associated with the control parameters. However, such enhancements are problem dependent and need to be investigated on an individual basis. Hence, in this research a variety of different GA operators were implemented and compared to determine the most suitable configuration for a given structural problem. The assessed performance objectives of the algorithm were the accuracy of the final results, convergence rate and number of function evaluations.

This research indicates that the outcome of the final optimum solution is also influenced by the choice of component costs that contribute to the fitness function. These component costs include the cost of concrete, cost of steel and the cost of formwork together with their associated labour costs. To overcome the large number of possible combinations of component costs, this research concentrated on giving a qualitative cost assessment, taking into consideration the most representative scenarios. For each type of structure, i.e. reinforced concrete beams, slabs and frames, the algorithm has been assessed under the following conditions:

(i) *Control parameters sensitivity analysis* - Testing and comparison of different GA control parameter settings for a fixed cost component combination.

(ii) *Cost sensitivity analysis* - Testing and comparison of different cost component combinations (i.e. different q values, different choices of unit costs etc.) for fixed GA control parameter settings.

The purpose of this testing and cost sensitivity analysis was to provide not only a better insight into the most suitable GA parameter settings for a given type of structure, but also to gain a better understanding of the influence of the cost components on the optimum solution.

6.4.1 *Simply Supported and Continuous Beams*

Fig. 6.16 shows the three-span continuous *T*-beam first encountered in Section 5.5.1. In this case, the upper bound depth has been increased to 900 *mm* and the characteristic strength of the shear reinforcement is 250 N/mm^2. All other values remain the same.

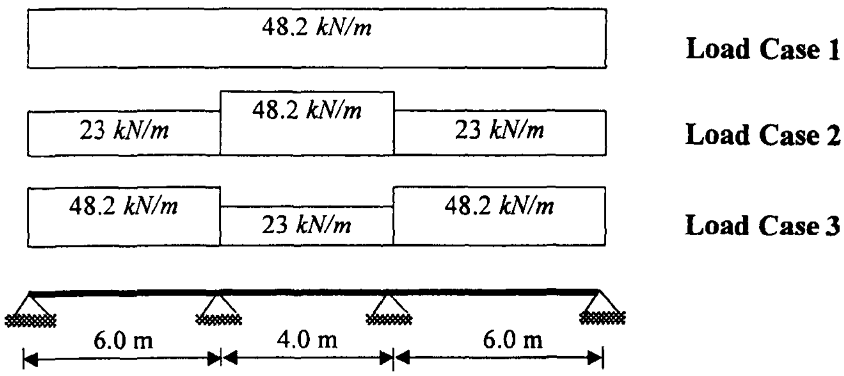


Fig. 6.16 *Three-Span Continuous T-Beam*

The costs associated with concreting, reinforcing and formworking are presented in Table 6.2.

Concreting	Rate	Reinforcing	Rate	Formworking	Rate
Cost of concrete (£/ m³)	32	Cut, bent & bundled (£/tonne)	275	Cost of timber framing (£/m³)	285
Wastage (%)	5	Wastage (%)	2.5	Timber framing (m³/ m²)	0.05
Labour (£/m³)	36	Fixing Accessories (%)	5	Cost of timber boarding (£/m²)	11
		Labour (£/m³)	245	Wastage + fixings+ props (%)	15
				Timber usage	5
				Labour Make (£/m²)	15.4
				Fix and Strip (£/m²)	20.3

Table 6.2 *Structure of Costs for Three-Span Continuos Beam*

6.4.1.1 *Sensitivity Analysis* of *GA Control Parameters*

The size of the population was fixed at 40 with the total number of generations for each run being limited to 90. The probability of crossover per population was set at 80%, with the probabilities of mutation per population and per gene size being set at 10% and 14% respectively. The percentage probability of gene mutation was set at slightly above

100/n, where n is the number of genes in the population member, to ensure that on average at least one gene mutation occurs. Figure 6.17 shows the convergence history of the minimum value of the fitness function when employing the standard evolution approach and elitist model.

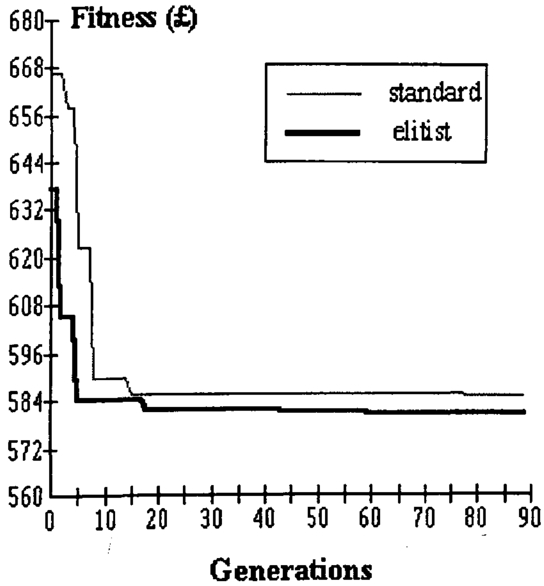


Fig 6.17 *Convergence History for Selection Methods*

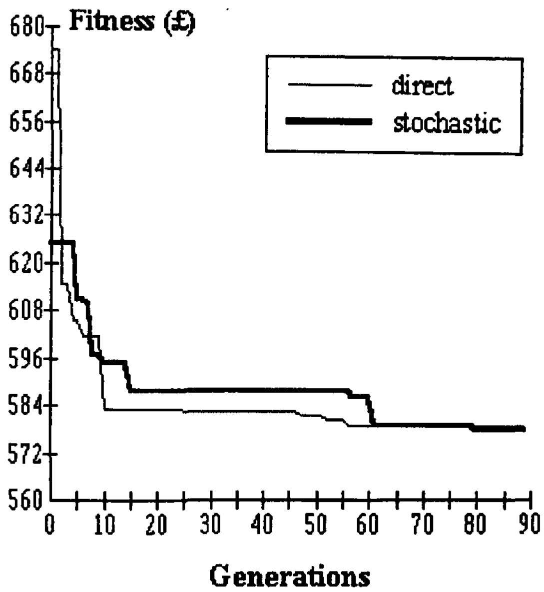


Fig 6.18 *Convergence History for Reproduction Methods*

In the elitist model, the 5 fittest members were retained for the next generation. Both selection methods used two-point crossover and random mutation. The elitist model yields better convergence tendency (that is, a higher convergence rate) than the standard evolution approach. Figure 6.18 shows the convergence history of the minimum value of the fitness function for the two different selection alternatives in reproduction; direct (deterministic) selection and remainder stochastic selection without replacement.

In both cases, elitism (best 5 members), two-point crossover and random mutation were used. No one method demonstrates a better overall convergence tendency although in this case the direct selection method shows an initially higher convergence rate.

Figure 6.19 shows the convergence history of the minimum value of the fitness function for one-point, two-point, 20%-uniform and 60%-uniform crossover.

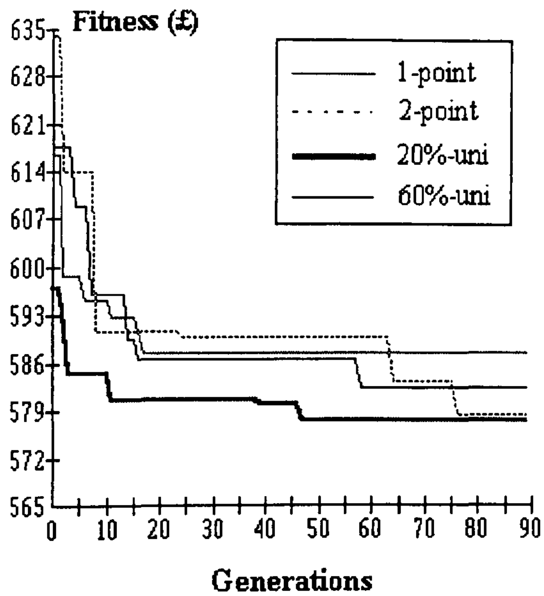


Fig 6.19 *Convergence History for Crossover Methods*

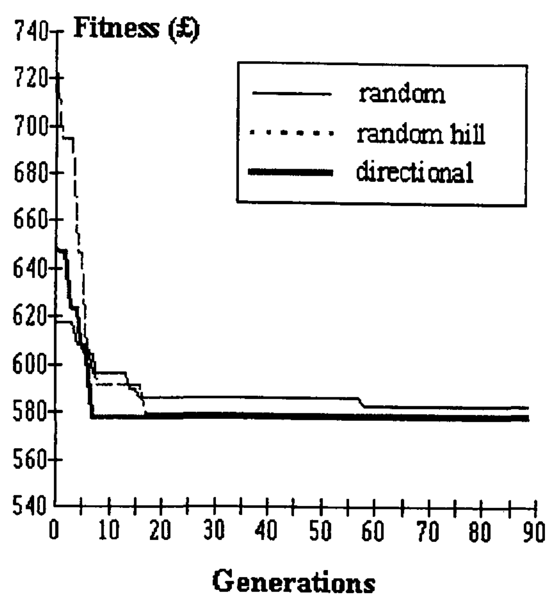


Fig 6.20 *Convergence History for Mutation Methods*

In all cases, elitism (best 5 members) and random mutation were used. The two-point and 20%- uniform crossover yield better convergence tendency, although the 60%-uniform crossover initially shows a higher convergence rate than the two-point crossover. One-point crossover has the poorest convergence tendency. Figure 6.20 shows the convergence history of the minimum value of the fitness function for random mutation, random mutation hill climb and directional hill climb. In all cases, elitism (best 5 members) and two-point crossover were used. The number of steps for both random and directional hill climbs was set to 5. The random hill climb and directional hill climb yield better convergence tendency than the random mutation method. Furthermore, the directional hill climb method shows the highest rate of convergence.

6.4.1.2 *Comparison and Testing of Results*

To test GENOD, and compare the results obtained with those reported in Section 5.5 using the SLP method and the commercial genetic algorithm software *Generator*, the three-span continuous beam in Figure 6.16 was considered, but replacing the *T*-beam with a rectangular section. Each span was allocated the same beam group with a lower bound breadth and depth of 250 *mm* and 350 *mm* respectively, and an upper bound breadth and depth of 500 *mm* and 800 *mm* respectively. The continuous beam was subjected to the three load cases as shown in Figure 6.16. The cost of the formwork, labour and wastage were not considered in this comparison, as the SLP objective function is not capable of handling such a complex cost model. The cost of concrete was taken to be *£50/m³*. The material properties and the applied loads were as those specified in the pervious example. For both GENOD and Generator the size of population was fixed at 40 with the total number of generations for each run being limited to 180. For the first 90 generations, the probability of crossover per population was set at 80%, with the probabilities of mutation per population and per gene size set at 10% and 14% respectively. For the remaining 90 generations, the probability of mutation was increased to 20% to *stimulate* the exchange of genetic material so as to *fine tune* the results. Tests were carried out for a range of q values between 25 and 95. The results achieved with GENOD compared with those obtained using *Generator* are presented in Table 6.3a.

q	GENOD				Generator
	Beam Group 1			MLC	MLC
	b_{opt} (mm)	h_{opt} (mm)	A_{smax}/bd (%)	Cost (£)	Cost (£)
25	250	412	1.68	119.80	120.32
35	250	433	1.50	132.30	134.05
45	250	439	1.46	144.60	145.18
55	250	466	1.21	155.90	156.45
65	250	482	1.06	166.70	166.90
75	250	499	0.99	176.90	175.95
85	250	513	0.91	186.50	186.20
95	250	534	0.82	196.00	197.15

Table 6.3a *GENOD and Generator Comparison - One Beam Group – Multiple Load Case*

Table 6.3a shows that GENOD gives comparable results to those obtained using *Generator* for all values of q. It was observed that these close results were obtained due to both GENOD and *Generator* performing an exhaustive search of standard design solutions that incorporates the costs of shear reinforcement.

However, minor but consistent differences in the total material costs between SLP and GENOD are evident from Table 6.3b. These differences are as a result of GENOD incorporating the costs of shear reinforcement, whilst the SLP method neglects these costs.

q	GENOD				SLP			
	Beam Group 1			MLC	Beam Group 1			MLC
	b_{opt} (mm)	h_{opt} (mm)	A_{smax}/bd (%)	Cost (£)	b_{opt} (mm)	h_{opt} (mm)	A_{smax}/bd (%)	Cost (£)
25	250	412	1.68	119.80	250	428	1.505	114.31
35	250	433	1.50	132.30	250	428	1.505	125.82
45	250	439	1.46	144.60	250	434	1.480	137.22
55	250	466	1.21	155.90	250	472	1.125	147.81
65	250	482	1.06	166.70	250	480	1.073	157.70
75	250	499	0.99	176.90	250	495	0.992	167.01
85	250	513	0.91	186.50	250	513	0.906	174.64
95	250	534	0.82	196.00	250	531	0.830	182.77

Table 6.3b *GENOD and SLP Comparison - One Beam Group – Multiple Load Case*

For a q value of 25, the optimum solution given by GENOD is that of doubly reinforced section with a tension reinforcement ratio of 1.68% and the compression reinforcement ratio of 0.23%. As the SLP method only considers singly reinforced sections, the search is constrained by the boundary between singly and doubly reinforced sections and hence the reinforcement ratio is set at its upper bound value.

6.4.1.3 *Cost Sensitivity Analysis*

The results obtained in the previous section of testing highlights one of the major limitations of the proposed SLP approach in that the contribution of the component costs to the objective function is a simplified model. For a realistic cost analysis all the component costs presented in Table 6.2 need to be considered.

Table 6.4 below presents the results of a cost analysis for a range of q values using the component costs in Table 6.2. For each q value, the optimum values are given together with the total costs of concreting, reinforcement and formworking.

q	b_opt	h_opt	A_{stmax}/bd A_{scmax}/bd	Concrete Costs	Steel Costs	Formwork Costs	Total Costs
	(mm)	(mm)	%	(£)	(£)	(£)	(£)
25	250	350	2.40/0.89	123.90	145.67	448.63	718.20
35	250	352	2.37/0.86	124.61	165.27	450.52	740.40
45	250	352	2.37/0.86	124.61	186.27	450.52	761.40
55	250	350	2.40/0.89	123.90	210.07	448.63	782.60
65	250	352	2.37/0.86	124.61	229.77	450.52	804.90
75	252	364	2.18/0.68	129.89	232.61	462.80	825.30
85	250	374	2.05/0.55	132.40	239.00	471.30	842.70
95	251	383	1.96/0.45	136.12	244.42	480.29	860.80

Table 6.4 *Cost sensitivity analysis – Test 1*

The results presented in Table 6.4 show that the formworking costs are dominant, representing an average of over 57% of the total cost of the structure. The influence of these costs on the objective function were apparent with the optimum beam breadth and depth being driven to their lower bound values, thus ensuring that the costs of formworking and concreting were kept to their minimum value. Even then, (for q>65), the depth only slightly increased from lower bound value keeping the optimum solution still within the domain of a doubly reinforced section.

Comparing these results to those obtained in Table 6.3b, it was evident that both the value and nature of the final optimum solution depended directly on the choice of the component costs for the fitness function. For example, in Table 6.3b, where only material costs were considered, the optimum solution was a singly reinforced section for all q values except for 25. In contrast, when all the structural costs were considered, the optimum solution was a doubly reinforced section for all q values (*see* Table 6.4). Furthermore, the results showed that this was dependent on the unit costs of concrete, steel and formwork. For example, when the concrete and formwork costs were reduced by 50%, it was observed that singly or doubly reinforced sections could be obtained at the optimum solution, depending on the value of q. The results of this cost analysis are presented in Table 6.5.

q	b_opt	h_opt	Astmax/bd Ascmax/bd	Concrete Costs	Steel Costs	Formwork Costs	Total Costs
	(mm)	(mm)	%	(£)	(£)	(£)	(£)
25	251	383	1.96/0.45	68.06	118.16	240.17	426.40
35	251	382	1.96/0.45	67.88	137.51	239.70	445.10
45	252	434	1.50/0.00	77.43	118.97	264.50	460.90
55	252	436	1.46/0.00	77.79	132.07	265.44	475.30
65	251	437	1.48/0.00	77.66	144.86	265.68	488.20
75	252	435	1.50/0.00	77.61	159.92	264.97	502.50
85	250	437	1.50/0.00	77.35	172.81	265.44	515.60
95	250	443	1.42/0.00	78.41	182.51	268.28	529.20

Table 6.5 *Cost sensitivity analysis – Test 2*

This research has shown the importance of formulating a realistic cost model and incorporating it within a design methodology that conforms with that specified by the relevant codes of practice. Simplifying the cost model and/or the design methodology may result in a mathematically correct optimum, but the final solution may be of limited value in the design process.

6.4.2 *Reinforced Concrete Slabs*

A simply supported slab with corners restrained against torsion was considered, with effective span lengths of 4.5*m* and 6*m* in the *x*- and *y*- directions, respectively. The slab was subjected to a uniformly distributed load of $5kN/m^2$. Standard partial factors of safety were applied. The characteristic strength of concrete and steel were set at 30 N/mm^2 and 460 N/mm^2 respectively. The costs associated with concreting, reinforcing and formworking are the same as those given in Table 6.2.

A detailed investigation was performed to find suitable settings for the GA control parameters. The investigation was conducted for different selection and reproduction methods, and crossover and mutation parameters. A cost sensitivity analysis was performed investigating the influence of different structural costs on the final solution.

6.4.2.1 *Control Parameters Sensitivity Analysis*

Using identified GA control parameter settings as for the continuous beam in Section 6.7.1.1, a sensitivity analysis was carried out. Figure 6.21 to 6.24 shows the convergence history of the minimum value of the fitness function for selection, reproduction, crossover and mutation methods, respectively.

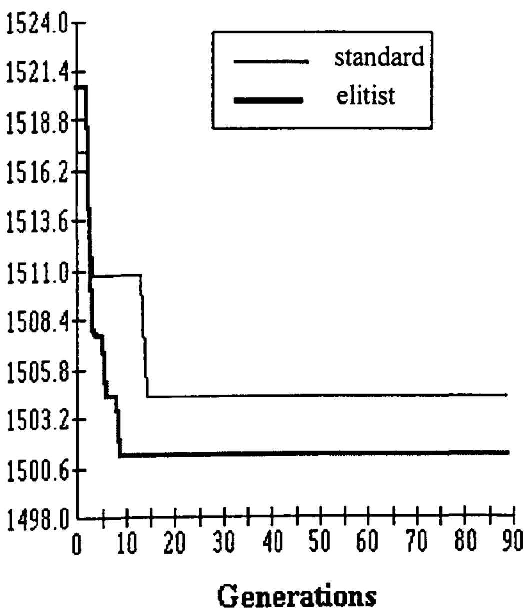


Fig. 6.21 *Convergence History for Selection Methods*

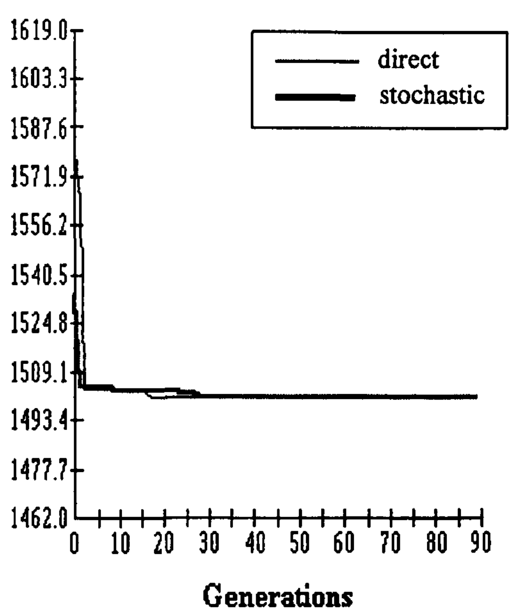


Fig. 6.22 *Convergence History for Reproduction Methods*

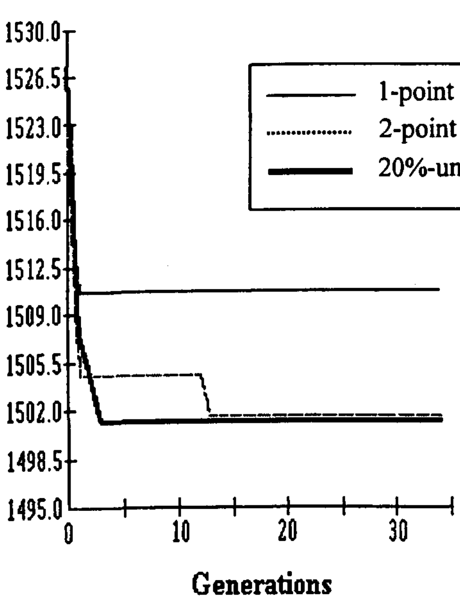


Fig. 6.23 *Convergence History for Crossover Methods*

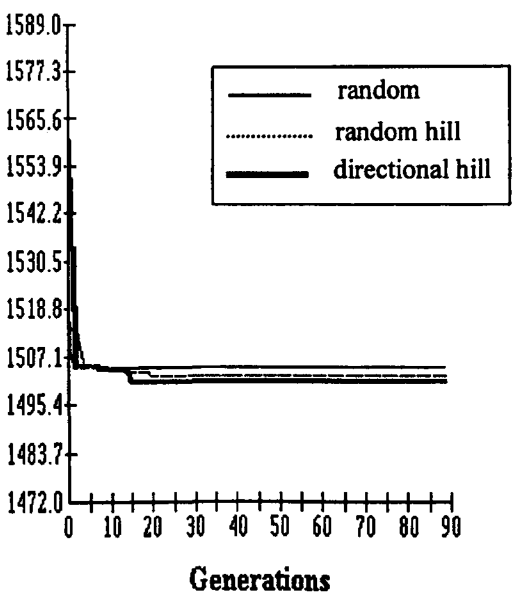


Fig. 6.24 *Convergence History for Mutation Methods*

It was observed from these tests that the behaviour of the GA control parameters was similar to that described for the continuous beams. As before, the elitist model is more efficient in directing the search towards the optimum and has a better convergence tendency than the standard evolution approach. Neither method of reproduction would appear to be more effective in achieving an optimum solution and overall performance of each method is similar to previous findings. As previously observed, 20%-uniform crossover outperformed both one-point and two-point crossover. As for the continuous beam, the directional hill mutation method demonstrated an improved convergence tendency although this has to be set against the increased computational effort associated with the method.

6.4.2.2 *Cost Sensitivity Analysis*

To both validate and test the performance of GENOD, the results of numerous tests were compared with those obtained using *Generator*. Two types of RC slabs were considered in the cost sensitivity analysis. Firstly, a simply supported slab where each edge is allowed to freely rotate, and secondly, an *encastre* slab where each edge is built in to form a monolithic construction. The examples given in this chapter are representative of all the slabs investigated. In all cases, the total cost of the structure was determined for different values of q ranging from 25 to 95.

6.4.2.3 Simply Supported Slab

The simply supported slab described in the Section 6.7.2 was considered. The slab geometry, loading, partial safety factors and characteristic strengths are as previously specified. Solutions obtained using GENOD and *Generator* are given in Table 6.6. For all values of q, the optimum solution was that corresponding to the overall depth required to satisfy the deflection constraint, *i.e.* h = 185 *mm*.

q	GENOD				*Generator*			
	Concrete Costs (£)	Steel Costs (£)	Formwork Costs (£)	Total Costs (£)	Concrete Costs (£)	Steel Costs (£)	Formwork Costs (£)	Total Costs (£)
25	404	47	1049	1500	403.79	47.67	1049	1500.46
35	404	53	1049	1506	403.79	53.76	1049	1506.55
45	404	59	1049	1512	403.79	59.85	1049	1512.64
55	404	65	1049	1518	403.79	65.95	1049	1518.74
65	404	71	1049	1524	403.79	72.04	1049	1524.83
75	404	77	1049	1530	403.79	78.13	1049	1530.92
85	404	84	1049	1537	403.79	84.22	1049	1537.01
95	404	90	1049	1543	403.79	90.31	1049	1543.10

Table 6.6 *Cost Analysis - Simply Supported Slab*

The results showed that the total costs associated with steel reinforcement were negligible when compared to the total cost of concreting. Furthermore, the cost of formworking represents over two thirds of the total costs of the slab and hence the material cost ratio q had little effect on the final solution. Reducing the thickness of the slab minimises both the costs of concreting and formworking. Although the cost of reinforcing increases as the thickness of the slab reduces, the cost of its material and labour is not significant across the range of q values to influence the optimum solution. Hence, in this example the optimum solution for all q values will be a slab with depth set to the lower bound value required to satisfy the deflection constraint.

6.4.2.4 *Encastre Slab*

To investigate further the influence of the cost components on the optimum solution, only the material costs of concrete and steel were considered, *i.e.* formworking and all labour costs were excluded. The slab geometry and material properties were as specified in the previous example with the exception that the supporting edges were considered to be built in. The imposed load was increased to 20 kN/m^2. Table 6.7 shows the results obtained using GENOD and *Generator*.

q	GENOD			*Generator*		
	Concrete Costs (£)	Steel Costs (£)	Total Costs (£)	Concrete Costs (£)	Steel Costs (£)	Total Costs (£)
25	200.20	26.60	226.80	200.20	26.68	226.88
35	200.20	37.30	237.50	200.20	37.35	237.55
45	200.20	48.00	248.20	200.20	48.02	248.22
55	200.20	58.60	258.80	200.20	58.69	258.89
65	200.20	69.30	269.50	200.20	69.36	269.56
75	200.20	80.00	280.20	200.20	80.03	280.23
85	200.20	90.60	290.80	200.20	90.70	290.90
95	200.20	101.30	301.50	200.20	101.37	301.57
140	**232.81**	**195.72**	**428.53**	**231.81**	**194.10**	**425.91**

Table 6.7 *Cost Analysis- Encastre Slab*

As with the previous example, for q values between 25 and 95, each optimum solution was reached when the slab depth was set to satisfy the deflection criteria. Although increasing the slab depth above the optimum increases the lever arm of the section, the resulting additional self-weight negates any potential reduction in the reinforcement due to an increase in the bending moments. The cost of concrete on the other hand, increases causing a rise in the total material costs. Hence, the optimum solution was achieved when the depth of the slab reached its lower bound value of 185 *mm*. However, as shown in Table 6.7 for a q value of 140, this trade-off eventually diminishes as q increases (>135). At the optimum solution, the minimum depth (h_{opt} = 220 *mm*) is greater than the lower bound value. This solution highlights the sensitivity of the final solution to the choice of the component costs. It is important to note that due to their higher self-weight content, solid slabs are less sensitive to the trade-off between steel and concrete costs than for example ribbed slabs.

6.4.3 *Reinforced Concrete Frames*

In GENOD, each population member represents a unique frame problem formulation given in the form of a chromosome string. For each population member a structural analysis and design are carried out prior to formulating the fitness function. GENOD incorporates the structural analysis procedures developed in the previous research for volume optimisation. The structural design procedures are those specified in BS8110 for beams and columns as outlined in Section 6.2. The optimum solutions obtained using GENOD were compared to those given using the sequential linear programming approach (SLP). For simple frame structures, solutions were also compared to those obtained using *Generator*. For more complex frame structures, *Generator* was unable to model the structural optimisation problems and hence could not be used in the comparative study. For these types of structures only the material costs of concrete and steel were considered to ensure compatibility between the results obtained using GENOD and the SLP method. A sensitivity analysis carried out on the GA control parameters revealed a similar convergence tendency to those obtained for continuous beams and slabs. A detailed cost sensitivity analysis was performed for a range of different values of q, and the results are reported.

6.4.3.1 *Design Example 1 One Bay - One Storey Frame*

Figure 6.25 shows a frame supporting a uniformly distributed dead load g_k of 20 *kN/m* (excluding self weight), a uniformly distributed live load q_k of 13.75 *kN/m* and two factored point loads of 1500 *kN* applied as shown. The partial safety factors for dead and live load are 1.4 and 1.6 respectively, giving a total load of 50 *kN/m*. Modulus of elasticity is 28 *kN/mm²* with characteristic material strength f_{cu}=30 *N/mm²* for the concrete and f_y=460 *N/mm²* for the steel. The cover to the reinforcement is 40*mm* both for the beams and columns.

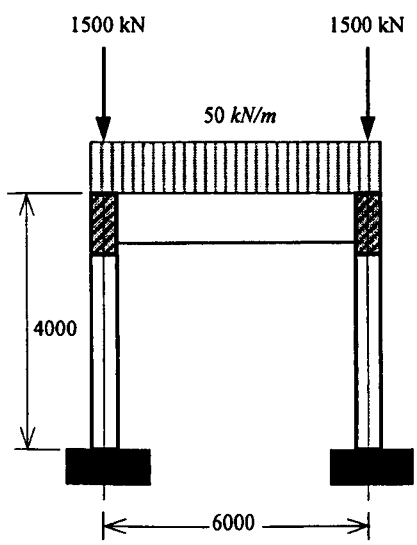


Figure 6.25 *One Bay - One Storey Frame*

The frame was tested considering one beam and one column group. The results obtained by GENOD were compared to those obtained using *Generator* and the developed SLP approach. GENOD requires the cost of concrete to be specified per unit volume (m^3), whilst the cost of steel needs to be specified per unit of weight (*tonne*). In the SLP approach however, the objective function considers both of these costs per unit volume (*see* Section 5.2.1). To allow a comparison between the SLP approach and GENOD, the cost objective function for the latter was modified to suit that of the SLP method, for all the frames investigated.

The comparison considered multiple load cases and q values ranging between 25 and 95, as shown in Table 6.8.

Material Cost Ratio	GENOD							*Generator*	SLP
	Beam Group 1			Column Group 1			MLC	MLC	MLC
q	b_{opt} (mm)	h_{opt} (mm)	A_s/bd (%)	$b_{c\,opt}$ (mm)	$h_{c\,opt}$ (mm)	A_s/bd (%)	Cost (£)	Cost (£)	Cost (£)
25	302	500	0.72	250	350	2.60	114.10	116.47	112.25
35	300	500	0.55	250	479	1.07	126.53	128.94	123.94
45	300	500	0.49	260	550	0.46	133.90	132.33	128.77
55	300	506	0.48	260	549	0.46	140.98	139.22	134.71
65	300	540	0.43	259	550	0.41	147.90	147.46	140.49
75	301	547	0.42	259	550	0.40	154.56	154.45	146.01
85	300	549	0.42	259	549	0.40	160.82	160.64	150.95
95	300	618	0.36	250	550	0.40	168.30	168.20	157.47

Table 6.8 *One Beam Group - One Column Group – Material Costs Only*

Table 6.8 shows that the results obtained are comparable for all three algorithms, with the SLP method giving the lowest costs as it does not consider shear reinforcement in its cost model. The minor cost differences observed between GENOD and *Generator* are due to the latter using an approximate calculation for determining the area of column reinforcement. It was observed that the breadths of both the beam and columns were driven to their lower bound values for any value of q. For low values of q (< 35), the depths of the beam and columns were also at their lower bound values at the optimum. However, as q increased, and hence with it the cost of the reinforcement relative to that of concrete, the depths of the structural elements increased and the percentage reinforcement ratio decreased. The columns became stiffer and more substantial reducing the member forces in the beam, which tended to be more expensive due to its higher reinforcement content. This process continued until the columns reached minimum reinforcement after which the depth of the columns remained constant and the beams adjusted their depths and reinforcement ratios accordingly to achieve a minimum cost.

To consider realistic structural costs, results were obtained using both GENOD and *Generator* that included all the component costs, as given in Table 6.2. The results are presented in Table 6.9 below.

Steel Costs (£/tonne)	GENOD							*Generator*
	Beam Group 1			Column Group 1			MLC Cost (£)	MLC Cost (£)
	b_{opt} (mm)	H_{opt} (mm)	A_s/bd (%)	$b_{c\,opt}$ (mm)	$h_{c\,opt}$ (mm)	A_s/bd (%)		
100	316	500	0.69	250	352	2.54	706.47	710.52
600	320	500	0.68	250	350	2.54	822.18	819.20

Table 6.9 *One Beam Group - One Column Group – All Costs*

Two extreme values of reinforcement costs were investigated to study their influence on the optimum solution. A comparison with the SLP method was not considered, as its objective function is not capable of handling such complex component costs. It was concluded that due to the dominant formworking and associated labour costs, both the breadths and depths of the beams and columns were driven to their lower bounds. Comparing these results to those obtained in Table 6.8 again highlights the importance of considering all component costs and how these influence the optimum solution.

6.4.3.2 *Design Example 2 Three Bay - One Storey Frame*

Figure 6.26 shows a frame first encountered in Section 5.5.2. The frame geometry, loading, partial safety factors, modulus of elasticity, and characteristic material strengths are identical to those specified in Section 5.5.2.

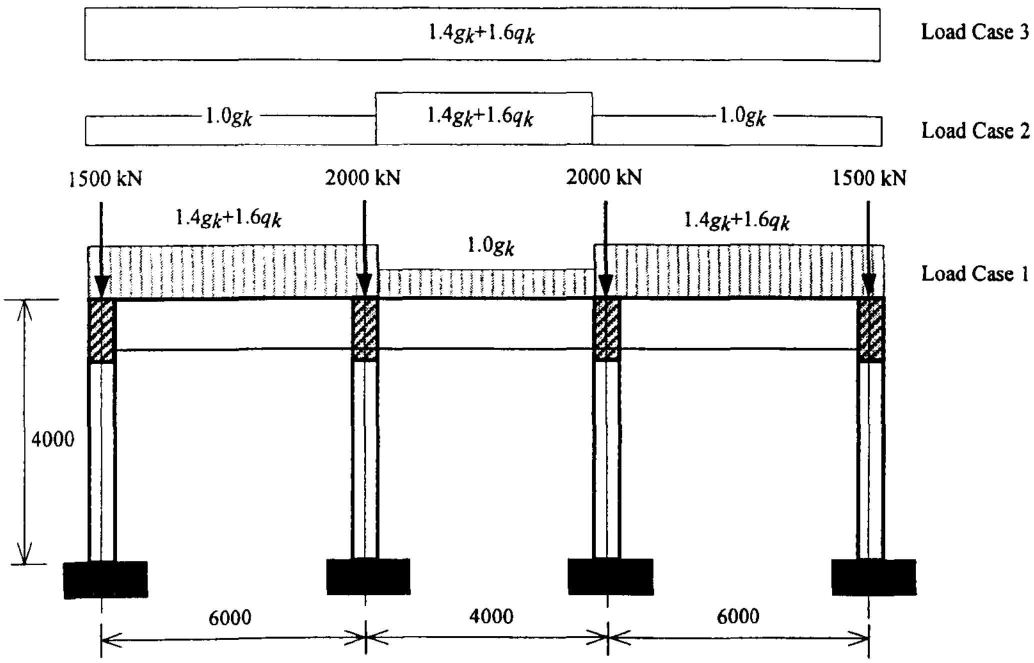


Figure 6.26 *Three Bay - One Storey Frame*

The frame was tested considering four combinations of member groups and loads; *one beam-one column* group for single and multiple load case; *two beam-two column* groups for single and multiple load case. Results for the first two combinations are given in Table 6.10, considering load case 1 as a single load case.

Material Cost Ratio	GENOD								SLP	
	Beam Group 1			Column Group 1			SLC	MLC	SLC	MLC
q	b_{opt} (mm)	h_{opt} (mm)	A_s/bd (%)	$b_{c\,opt}$ (mm)	$h_{c\,opt}$ (mm)	A_s/bd (%)	Cost (£)	Cost (£)	Cost (£)	Cost (£)
15	300	496	1.48	250	300	6.00	263.28	267.55	258.38	261.57
25	300	518	1.36	250	308	5.98	315.46	329.04	300.56	314.14
35	300	514	1.41	250	733	0.69	351.53	359.24	335.43	343.64
45	300	522	1.35	251	750	0.57	374.30	384.92	362.42	372.44
55	300	556	1.12	260	750	0.42	394.52	406.11	383.24	395.23
65	300	587	0.96	259	749	0.41	413.33	427.05	403.99	417.91
75	300	630	0.79	255	749	0.42	432.54	452.17	421.04	440.57
85	300	631	0.79	256	750	0.40	448.40	468.49	440.09	459.98
95	300	695	0.61	256	750	0.40	464.09	487.52	457.44	480.93

Table 6.10 *One Beam Group - One Column Group - Single and Multiple Load Case*

The lower and upper bound beam breadth and depths are 300, 500, 450 and 750 *mm*, respectively. The lower and upper bound beam breadth and depths are 250, 400, 300 and 800 *mm*, respectively.

As with the previous two examples, the results obtained using GENOD are comparable to those obtained using the SLP method. For all values of q, the breadth of the beams and columns were driven to their lower bound values. As q increased, and hence with it the cost of the reinforcement relative to that of concrete, the depths of the structural elements increased and the percentage reinforcement ratio decreased. For q values between 15 and 75 the reinforcement ratio in the beams reached the boundary value between a singly and doubly reinforced section, whilst the columns continued to increase their depth and reduce their reinforcement ratio. As q increased the columns became stiffer and more substantial, reducing the member forces in the beams which tended to be more expensive due to their higher reinforcement content. Table 6.11 gives the results of comparison for the two beam-two column groups combination.

q	GENOD										SLP	
	Beam Gr 1		Column Gr 1		Beam Gr 2		Column Gr 2		SLC	MLC	SLC	MLC
	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	Cost (£)	Cost (£)	Cost (£)	Cost (£)
15	507	1.50	307	4.75	366	1.98	306	6.00	252.23	259.52	248.71	253.72
25	548	1.23	309	4.08	380	1.73	324	5.80	302.10	307.86	295.92	301.99
35	596	1.11	654	0.49	388	1.36	766	0.49	337.15	346.85	328.94	335.41
45	596	1.00	674	0.41	386	1.33	776	0.40	360.05	367.23	345.79	356.61
55	618	0.88	670	0.40	407	1.14	767	0.40	377.26	386.89	364.71	377.26
65	632	0.85	671	0.40	409	1.13	767	0.40	393.95	403.56	383.61	394.95
75	694	0.65	660	0.40	410	0.89	787	0.40	410.42	421.49	401.86	416.21
85	698	0.64	670	0.40	431	1.01	777	0.40	424.15	439.54	417.26	433.23
95	700	0.64	670	0.40	446	1.09	777	0.40	440.19	455.18	432.85	449.86

Table 6.11 *Two Beam Groups - Two Column Groups - Single and Multiple Load Case*

It was observed that for this combination, the optimum solutions (for all q values) were more cost efficient designs when compared with the one beam-one column group combination. It was concluded that in this case, the GA search had a greater variety of possible section dimension combinations to balance the external forces, and hence could obtain a more cost efficient design. It was also noted that for beam group two (q values of 25 and 35), the optimum solution was a doubly reinforced section. These structural cost reductions however, need to be considered in the light of a potential increase in the formworking costs. When single (SLC) and multiple load case (MLC) are compared, it was observed that the MLC costs were consistently higher, as observed in the previous investigations. The code of practice requirement to consider only critical force envelopes effectively rules out SLC solutions, and only optimum designs from an MLC analysis should be considered.

6.4.3.3 *Design Example 3 - Two Bay - Three Storey Frame*

Figure 6.27 shows the heavily loaded frame first encountered in Section 5.5.3, having identical geometry, loading combinations and material properties as previously defined.

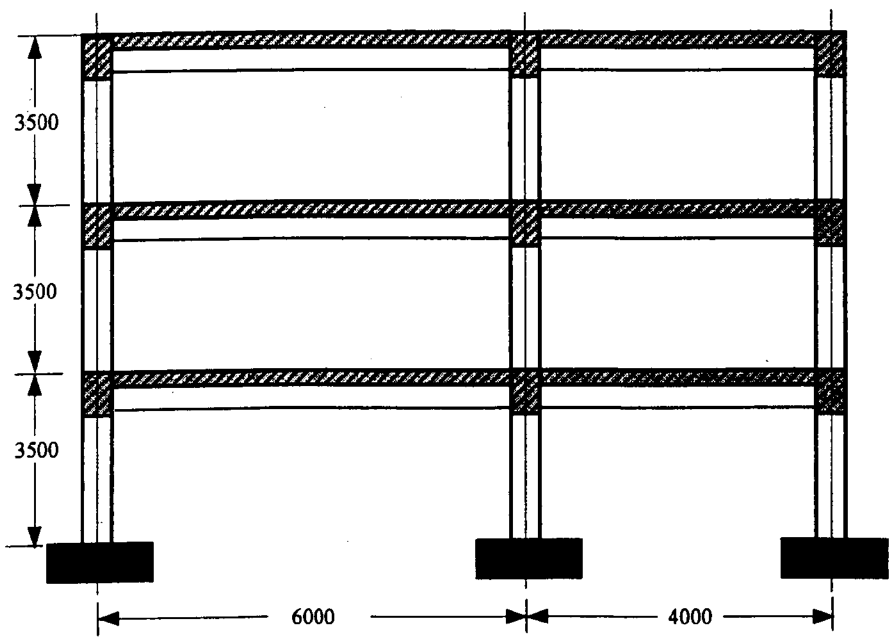


Figure 6.27 *Three Storey – Two Bay Frame*

Table 6.12 shows the results for a one beam-one column group frame for both single and multiple loading conditions. To obtain comparable results between the GA and SLP approaches, the cost fitness function utilised in GENOD had to be simplified to calculate only the total material costs, as described in Section 5.2.1.

q	GENOD								SLP	
	Beam Group 1			Column Group 1			SLC	MLC	SLC	MLC
	b_{opt} (mm)	h_{opt} (mm)	A_s/bd (%)	$b_{c\ opt}$ (mm)	$h_{c\ opt}$ (mm)	A_s/bd (%)	Cost (£)	Cost (£)	Cost (£)	Cost (£)
15	302	546	1.68	253	430	5.97	642.14	661.43	650.27	673.47
25	303	612	1.46	252	552	3.81	782.36	808.37	771.39	795.43
35	301	630	1.47	254	777	0.40	852.29	883.01	840.21	869.08
45	304	639	1.45	251	775	0.40	907.19	942.83	894.01	927.93
55	305	641	1.47	250	775	0.40	962.11	1002.6	947.82	986.77
65	302	640	1.46	253	775	0.40	1017.1	1062.4	1001.6	1045.6
75	304	644	1.46	252	780	0.40	1071.8	1122.3	1055.3	1104.5
85	301	649	1.44	250	782	0.40	1126.4	1182.1	1108.8	1163.3
95	300	637	1.46	252	783	0.40	1181.1	1237.3	1162.4	1217.5

Table 6.12 *One Beam Group - One Column Group - Single and Multiple Load Cases*

Table 6.12 shows that comparable results were obtained between the developed genetic algorithm solution and the SLP cost optimisation approach. In all but one case (q = 15), the GA solutions gave higher costs due to the inclusion of shear reinforcement costs that are not considered in the SLP problem formulation. However, despite the additional costs of the shear reinforcement, the costs of GA solution obtained for q = 15 were less than that given by the SLP approach. This apparent anomaly was due to the fact that the GA approach is capable of dealing with both singly and doubly reinforced beam sections, whilst the SLP problem formulation only considers singly reinforced beams (*see* Section 5.2.4). Due to the low unit cost of the reinforcement, the GA solution produced doubly reinforced beam sections that were cheaper than those given by the SLP approach. For the latter, the optimum solution produced beams whose reinforcement ratios were on the boundary between singly and doubly reinforced sections. For all other values of q, the breadth of the beams and columns were driven to their lower bound values, with the reinforcement ratio in the beams reaching the boundary value between a singly and doubly reinforced section. The columns continued to increase their depth until the corresponding reinforcement ratio reached its lower bound value. As with the previous frame example, as q increased the columns became stiffer and more substantial, reducing the member forces in the beams which tended to be more expensive due to their higher reinforcement content. Multiple load case solutions are again more expensive than those obtained for single load cases and is in line with findings from previous studies.

Table 6.13 shows the results for the two beam-two column groups combination. For this problem only multiple loading conditions are considered.

q	GENOD									SLP
	Beam Group 1		Col. Group 1		Beam Group 2		Col. Group 2		MLC	MLC
	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	Cost (£)	Cost (£)
15	576	1.72	315	6.00	432	1.69	475	5.93	636.53	649.50
25	631	1.46	735	0.57	483	1.42	760	0.88	791.98	782.92
35	626	1.44	780	0.44	455	1.45	794	0.43	824.57	841.49
45	623	1.46	792	0.40	448	1.46	792	0.40	900.64	889.53
55	620	1.45	795	0.41	451	1.45	798	0.40	949.31	937.17
65	622	1.45	791	0.40	454	1.46	800	0.40	1000.1	986.87
75	750	0.98	790	0.41	459	1.45	792	0.41	1043.5	1029.33
85	788	0.77	792	0.40	462	1.16	799	0.40	1085.6	1070.43
95	795	0.71	793	0.40	458	0.76	800	0.40	1127.7	1111.43

Table 6.13 *Two Beam Groups - Two Column Groups - Single and Multiple Load Case*

For each storey, beam group one and beam group two were allocated to the left and right bay spans respectively. For the columns, group one applies to the external columns and group two to the internal columns. The lower and upper bound cross-sectional design variable constraints are given in Table 6.14.

Design Variable	Beam Group 1		Beam Group 2		Col. Group 1		Col. Group 2	
	lower	*upper*	*lower*	*Upper*	*lower*	*upper*	*lower*	*upper*
Breadth (mm)	300	500	300	400	300	400	300	500
Depth (mm)	400	900	300	700	300	800	300	800
A_s/bd (%)	0.13	1.46	0.13	1.46	0.4	6.0	0.4	6.0

Table 6.14 *Lower and upper cross-sectional design variable constraints*

Table 6.13 clearly shows that the optimum solution exhibits similarities with the first member group combination with respect to the behaviour of the beams and columns. In this case however, for $q \geq 75$ the beams in the first beam group eventually increased their depths and decreased their reinforcement ratios accordingly to achieve a minimum cost. This was due to the columns reaching their upper bound depth values and effectively balancing the bending moments at the expense of an increase in the beam depths (reduction of reinforcement ratio).

In all the cases tested, the breadths of the beams and columns were always driven to their lower bounds regardless of the value of q. For low to medium values of q, it was observed that the reinforcement ratio in the beams reached the boundary value between a singly and doubly reinforced section, whilst the columns continued to increase their depth and reduce their reinforcement ratio. For increased values of q, the columns became stiffer and more substantial, reducing the member forces in the beams. This process continued until the reinforcement ratio in each column reached its minimum, after which the depth of the column remained constant and the beams adjusted their depths and reinforcement ratios accordingly to achieve a minimum cost.

When the influence of the beam and column groups on the final design solution is considered, for low values of q the differences in the minimum cost between frames with one or more member groups were less than those for higher values of q. As q increased the cost difference showed a steady increase too. Material cost optimisation for frames with two or more member groups appeared to be more cost effective than those with one group. In practice however, these cost differences would be offset against the potential increase in the formworking and labour costs.

Table 6.15 shows the results for the same frame but considering all costs associated with concreting, reinforcing and formworking, as given in Table 6.2. A cost sensitivity analysis was performed for different values of steel costs, whilst the other material and construction costs were kept constant.

It was observed that given the more realistic (complex) cost structure, the final solutions and hence their costs were significantly different to those obtained when considering the material costs only. Due to the predominant costs of formworking, the dimensions of both column groups were kept to their minimum (*i.e.* lower bounds), whilst the balancing of the external forces was performed on the expense of an increase in the depth of beam group one. Beam group two adjusted its depth and reinforcement ratios accordingly, keeping its dimensions close to its lower bound value. When the cost of steel was 50 and 100 £/tonne both beam groups were doubly reinforced sections at the optimum. For C_S = 275, only beam group two was doubly reinforced. As the cost of steel increased, the depths of the beams correspondingly increased keeping the column group dimensions at their minimum due to their higher formworking costs. This example clearly shows the importance of considering realistic structural costs and loading conditions when formulating the cost optimisation problem.

Cost of Steel	GENOD								
	Beam Group 1		Col. Group 1		Beam Group 2		Col. Group 2		MLC
C_S (£/tonne)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	h_{opt} (mm)	A_s/bd (%)	Cost (£)
50	461	2.65	306	3.18	360	3.00	301	2.58	3373.7
100	558	1.79	300	2.60	368	2.88	300	1.95	3484.4
275	636	1.46	304	2.13	408	2.31	300	1.49	3872.3
600	899	0.71	302	1.28	432	1.90	300	0.47	4455.6

Table 6.15 *Two Beam Groups - Two Column Groups - Multiple Load Case*

6.5 Conclusions

The results presented in this research illustrate the performance of the developed approach to the minimum cost design of reinforced concrete skeletal systems using genetic algorithms. They showed the proposed implementation of GA's to be a highly practical approach to the design process, capable of incorporating realistic loading conditions and limit states. The fitness function incorporates the material and labour costs associated with concreting, reinforcing and formworking. Such a computer-based design approach has the ability to not only simulate the *real world* design of skeletal systems, but also through the application of an artificially intelligent search obtain improved designs that minimise the structural costs. The approach offers a systematic, goal-orientated design process that combines analysis and design to search and sort through the similar design concepts to achieve a set objective. It was found that the ability of GA's to avoid gradient computations and rapidly search the entire feasible region independent of the starting point, provides the designer with a powerful set of tools that can be used to define structural properties that are optimal in a practical sense. The results of the studies undertaken in this research have demonstrated that a GA search can be enhanced by an appropriate choice of control parameters. Better results have been obtained using the elitist model, showing in particular an improved convergence rate when compared to the standard evolution approach. However, care needs to be taken in deciding how many members to retain to avoid the danger of premature forced convergence. It was found that uniform crossover generally achieved the best convergence rate. The random hill and directional hill climbing mutation methods have also shown advantages, mutating the genes in a beneficial manner that generally improves convergence.

The results of the cost sensitivity analysis showed the dominance of the concrete and formwork costs over the steel costs in the full costing scenario. The results obtained using GENOD were comparable to those obtained using the SLP method and *Generator*, when considering the total material costs only. The analysis of single and multiple loading conditions further reinforced the previous findings that the former does not produce practically representative optimum design solutions. Furthermore, multiple beam/column group analysis showed that more cost-effective designs are achievable than those of single beam/column groups. Finally, when considering all the structural costs, it was observed that the final solutions and hence their costs, were significantly different to those obtained when only the material costs were considered. In this context, a consideration of both the realistic structural costs and loading conditions is emphasised if the resulting structural designs are to be of a practical value.

7.

Cost Optimisation of Skeletal Systems - Substructure

This chapter investigates the minimum cost design of reinforced concrete retaining walls, as a constituent part of a skeletal system substructure. Cantilever walls were investigated as being representative of retaining structures required to resist a combination of earth and hydrostatic loading. The rationale behind the application of simulated annealing to their minimum cost design is explained. A modified approach that avoids the simple rejection of infeasible solutions and improves convergence to a minimum cost has been developed. Sensitivity analysis and detailed testing are performed and the results reported.

7.1 Introduction

After completing research relating to the optimum design of main structural components that are present in a skeletal system, the minimum cost design of retaining walls was investigated. These structures are often found as part of a system of slabs and walls in basements and other underground constructions. In addition to resisting earth loading they are also required to prevent ingress of ground water. In formulating the minimum cost design problem, it was observed that the resulting programming problem could not be formulated within an unconstrained solution space as preferred for implementation by GA's. The need to consider the overall stability of a retaining wall, i.e. sliding, overturning and maximum ground bearing pressure as a part of the design process, resulted in a constrained programming problem. Hence, the research moved on to investigating suitable optimisation (search) algorithms for such problem formulations. When GA's were considered it was observed that although their formulation can be modified to include constrained optimisation problems, the reported research work, such as that of Adeli and Cheng (1994), noted that the selection and management of the penalty coefficients requires extensive numerical experimentation and is problem dependent. Furthermore, exploring the relatively small neighbourhood around the optimum solution may require exhaustive and time consuming fine tuning of the penalty function coefficients. In this respect, published research suggests that simulated annealing (SA) or tabu search could at least offer more robust and less time consuming solutions than genetic algorithms. A further survey of published research revealed the theoretical incompleteness of tabu search and its inability to prove its search success and convergence behaviour (Glover and Taillard 1993). Simulated annealing however, offers a theoretically established, efficient and adaptive search method applicable to real-life constrained optimisation problems. Successful applications are reported, for example, Bennage and Dhingra (1995) investigated the application of simulated annealing to single and multi-objective structural optimisation problems. Their results indicate that, in several instances, simulated annealing outperforms gradient-based and discrete optimisation techniques used in the comparison.

7.2 Formulation of Structural Optimisation Problem

The fundamental requirement of the retaining wall design is that is capable of holding retained material in place, without any significant movement arising from deflection, overturning or sliding. These walls may be classified into the three basic types; gravity, counterfort and cantilever walls. Although the structural action of each type is fundamentally different, very similar techniques are used in their designs. In this research, only cantilever retaining walls are considered. However, the formulation of the structural optimisation problem could be readily adapted to all types of retaining structures. A cantilever wall is designed as a vertical beam rigidly connected to a large base often relying on the weight of backfill to provide stability (*see* Figure 7.1). The addition of a heel beam is very common, providing more effective resistance against sliding of the structure.

7.2.1 *Analysis and Design*

This research incorporates the procedures for analysis and design outlined by BS8110, divided into three main stages:

(i) *Stability analysis* - ultimate limit state

(ii) *Bearing pressure analysis* - serviceability limit state

(iii) *Member design and detailing* - ultimate and serviceability limit states

In certain cases, additional stages of design must be considered, i.e. failure by slip, total and differential settlement (tilt).

7.2.1.1 *Stability Analysis*

At ultimate limit state a retaining wall is required to be stable in terms of resistance to overturning and sliding. To guard against a stability failure, it is common to apply factors of safety as outlined in BS 8110. The factor of safety against sliding γ_s, should be a minimum of 1.5 if only cohesion or base friction is considered, or 2 if the passive resistance in front of the toe is also considered. The factor of safety against overturning γ_o is usually a minimum of 2. When considering cantilever walls, the weight of the backfill and heel beam resistance force H_p have to be accounted for in the stability analysis (see Figure 7.1). The worst conditions for stability are when the self-weight of the wall G_k and the vertical load from the backfill V_k are a minimum; therefore the corresponding partial safety factors are taken to be 1.0. The final form of the stability requirements can be expressed as follows

$$1.0G_k x + 1.0V_k q \geq \gamma_o H_k y \qquad \text{for overturning} \qquad (7.1)$$

$$\mu\left(1.0G_k + 1.0V_k\right) + H_p \geq \gamma_s H_k \qquad \text{for sliding} \qquad (7.2)$$

where H_k is the horizontal force of the backfill and μ is the coefficient of friction.

7.2.1.2 *Bearing Pressure Analysis*

The base of a retaining wall also acts as its foundation, and hence the bearing pressures underneath are assessed on the basis of the serviceability limit state as outlined in BS 8110. The base is subject to the combined effect of an eccentric vertical load coupled with an overturning moment as shown in Fig. 7.1. It is assumed that the effective eccentricity of the resultant vertical force lies within the 'middle third' of the base with the extreme bearing pressures being expressed as

$$p_{1,2} = \frac{N}{D} \pm \frac{6M}{D^2} \qquad (7.3)$$

where N is the resultant vertical load on the base, M is the moment about the centre line of the base and D is the length of the base.

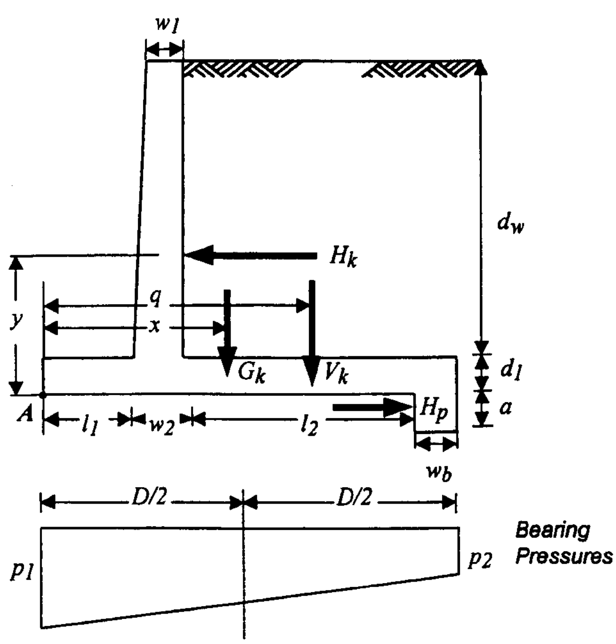


Figure 7.1 *Cantilever Wall – Forces, Bearing Pressures and Design Variables*

7.2.2 *Design Variables*

Considering the wall geometry shown in Figure 7.1, together with the limit state requirements, this research took as its design variables the width of the stem at the top (w_1), width of the stem at the bottom (w_2), depth of the base (d_1), length of the toe (l_1), length of the heel (l_2), height of the heel beam (a) and width of the heel beam (w_b). The height of the stem d_w is set constant to reflect practical construction requirements. The size of the solution space (i.e. lower and upper bound on the variables) is defined by the designer to encompass limit state requirements and aesthetic considerations.

7.2.3 Objective Function

A practical approach for assessing the structural costs is adopted in this research, allowing for the evaluation of both material and labour costs. The proposed cost function includes the costs of concrete, steel and formwork together with their associated labour costs. In addition, the construction costs associated with making, fixing and striping formwork, steel fixing and material wastage, are also included. The total cost of the reinforcement is apportioned between that required to resist the ultimate forces, and the secondary steel necessary to resist cracking. Formwork costs apply only to the vertical faces of the wall as it is assumed that the base will be concreted directly into the excavated shape of the foundation. Taking account of all these costs the fitness function can be shown to be

$$Z = Z_c + Z_s + Z_f \tag{7.4}$$

where Z_c, Z_s and Z_f are the total cost of concreting, reinforcing and formworking respectively. Furthermore, the costs of concreting can be broken down into their individual elements and represented in the form

$$Z_c = \left[C_c\left(1 + w_{fc}\right) + C_{cl}\right]V_w \tag{7.5}$$

where C_c is the cost of concrete per unit volume, w_{fc} is the wastage allowance factor, C_{cl} is the cost of labour per unit volume of concrete and V_w is the volume of concrete per unit length of retaining wall.

Similarly, the cost of steel can be represented as

$$Z_s = \left[C_s\left(1 + w_{fs} + f_{fs}\right) + C_{sl}\right]\left(Ws_l + Ws_c + Ws_d\right) \tag{7.6}$$

where C_S is the cost of steel per unit weight, W_{ls} and W_{ds} are the weight of longitudinal structural and distribution reinforcement respectively, W_{CS} is the weight of compression face reinforcement in the wall to resist surface cracking, w_{fs} is the wastage allowance factor, f_{fs} is the steel fixing allowance factor and C_{sl} is the cost of labour per unit weight of steel.

Finally, the cost of formwork can be shown to be

$$Z_f = \left[\left(T_f C_{tf} + C_{tb}\right)\left(1 + w_{fp}\right) / T_u + C_{lm} / T_u + C_{lfs}\right] A_{fw} \tag{7.7}$$

where C_{tf} is the cost of timber framing per unit volume, C_{tb} is the cost of timber boarding per unit area, T_f is the volume of timber framing per unit area of timber boarding, T_u is the timber usage factor, C_{lm} and C_{lfs} are the labour costs to make and fix, and strip per unit area of timber respectively, and A_{fw} is the area of the formwork.

7.3 Implementation of SA Algorithm

The structure of the SA algorithm implemented in this research is in principle the one outlined in *Appendix* G (*see* Figure G.1). However, the algorithm has been modified to improve its flexibility, efficiency and convergence rate. Furthermore, a probabilistic weight estimate (PWE) approach has been developed to handle constraints, avoiding the simple rejection of infeasible solutions. To provide greater control and flexibility over the simulated annealing algorithm, a facility to stop, pause and continue the computation process has been developed and incorporated within the main user-interface (*see* Figure 7.2). Additional facilities are also provided to continuously assess and monitor the current algorithm solution, its corresponding costs and constraints violation. Figure 7.2 shows a typical example of the output from the computer programme developed in this research, providing a graphical representation of the fitness function and system temperature together with information related to the algorithm's control parameter settings.

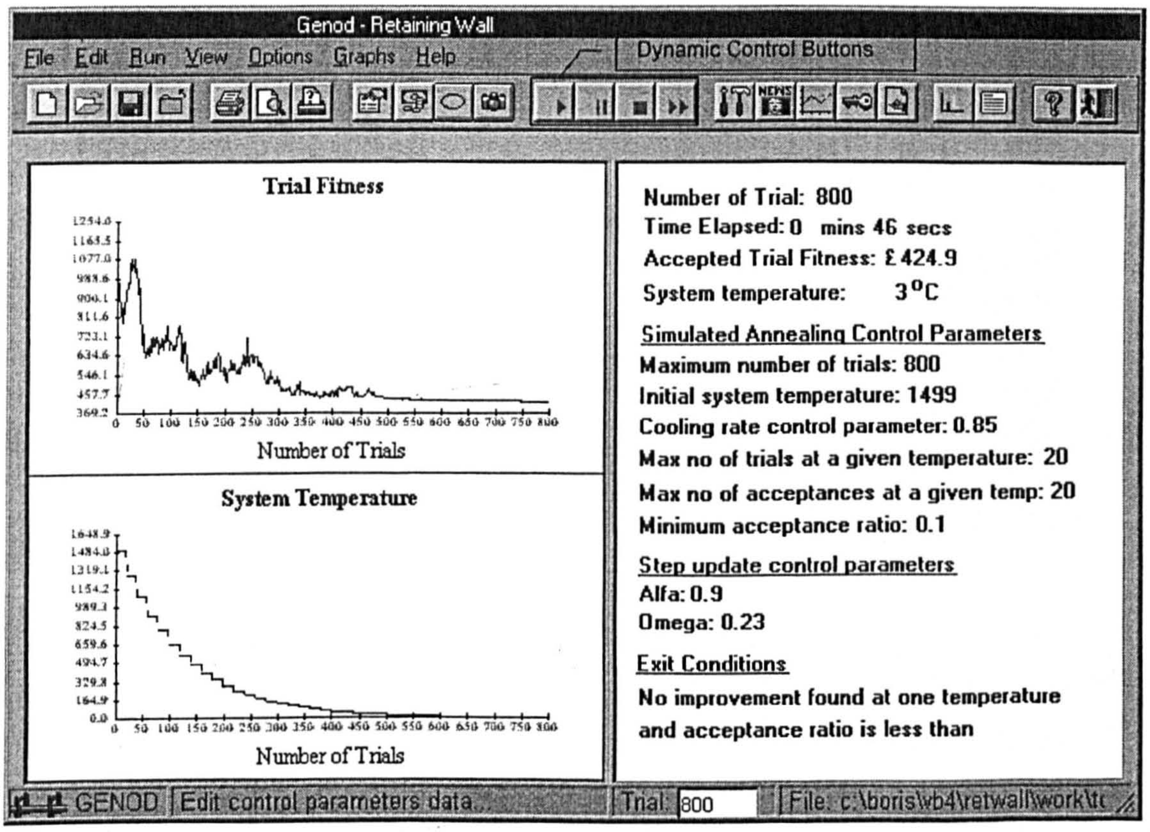


Figure 7.2 *Main I/O Program Control Form*

7.3.1 Solution Generation

In the investigations, the initial starting point is produced either randomly or estimated by conducting a random pilot survey of *N*-solutions (*see Appendix* G, Section G.2).

Estimating the initial starting point has, in general, proven to give a greater convergence rate, requiring less iterations to arrive in the neighbourhood of the optimum solution. The solution space is bounded by upper and lower bound side constraints on the design variables, specified within the programme using the Wall Geometry Control Form, as shown in Figure 7.3.

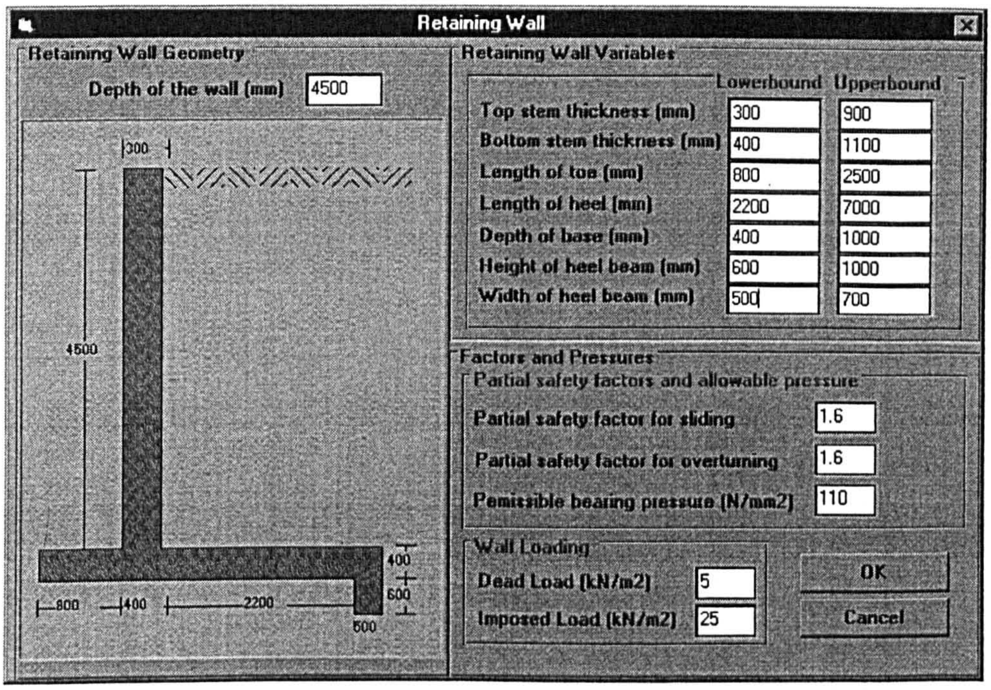


Figure 7.3 *Wall Geometry Control Form*

Current solutions are generated according to the strategy suggested by Parks (1990), presented by equations (G.3) and (G.4) (*see Appendix* G). The main advantage of this approach is that it does not require refreshing the step covariance matrix S every time the system temperature has changed, thus significantly reducing the computational effort. Furthermore, matrix D which measures the maximum change allowed in each variable does not need adjustment as the system temperature changes.

7.3.2 Initial Temperature

For the computer programme developed in the research, the initial temperature of the system (T_0) can be either user-defined or estimated automatically by conducting a random pilot survey of the solution space, as discussed in Section G.5 (*see Appendix* G). As proposed by Kirkpatrick et al. (1982), a suitable initial temperature will result in an average increase of acceptance probability $\aleph_o$=0.8.

The initial temperature is specified within the Simulated Annealing Control Parameters Form as shown in Figure 7.4. To use the random pilot survey estimate, the initial temperature is set to zero.

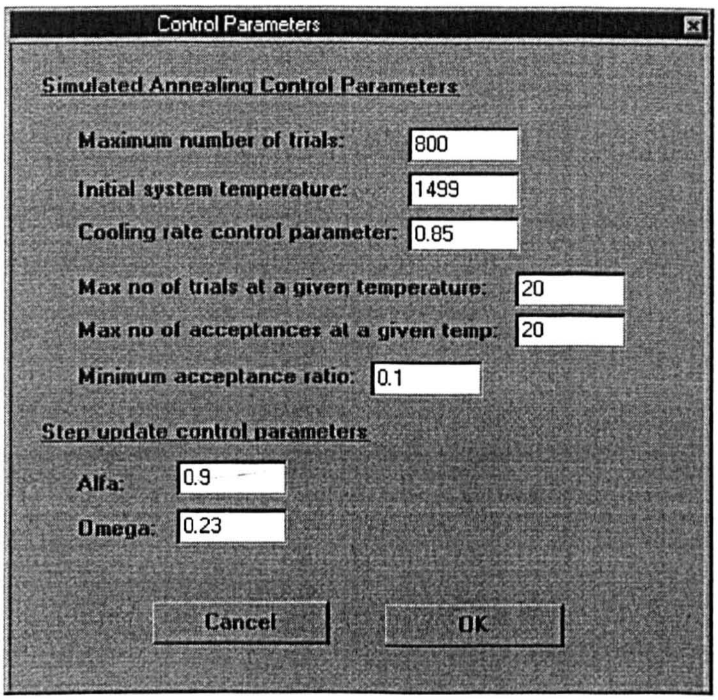


Figure 7.4. *Simulated Annealing Control Parameters Form*

7.3.3 Exit Conditions and Final Temperature

In this research, the search is halted when no improvement has been found combined with the acceptance ratio falling below a specified value. This determines the final temperature of the annealing schedule and hence the stopping criterion of the algorithm. Additionally, two more conditions to halt the search are incorporated; namely a specified maximum number of trials or a limit on the search time, as shown in Figure 7.5. Hence, the final temperature can be indirectly determined by fixing the number of temperature values (number of iterations), or by fixing the maximum search time for the algorithm. Although conceptually simple, these exit conditions are often employed in heuristic search algorithms with the most appropriate exit condition being selected based on the knowledge of the performance of a particular structural optimisation problem.

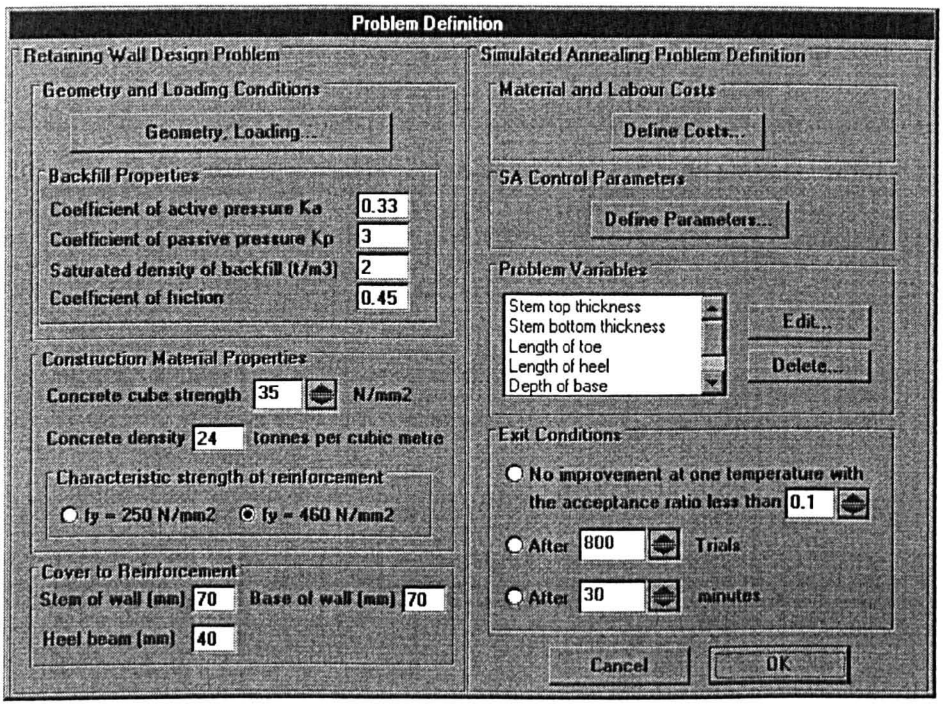


Figure 7.5 *Problem Definition Control Form*

7.3.4 Annealing Schedule

An exponential annealing schedule proposed by Kirkpatrick (1984) is adopted by the research, using a cooling parameter α to control the temperature decrement. To control this decrement a minimum number of transitions that should be accepted at each temperature together with the minimum acceptance ratio are established within the algorithm. The temperature decrement is given by equation (G.7) in *Appendix* G, where the cooling rate parameter is specified in the Simulated Annealing Control Parameter Form as shown in Figure 7.4.

7.3.5 Constraints Handling

Cantilever retaining walls are constrained structural problems being required to satisfy stability conditions and ground bearing pressure restrictions. The limitations associated with the simple rejection of infeasible solutions or the standard penalty coefficient methods for handling constraints have already been discussed. In this research, a probabilistic weight estimate (PWE) approach to constraint handling has been developed that automatically updates the penalty values depending on the magnitude of constraint violation.

Three types of constraints are considered, i.e. sliding, overturning and maximum ground bearing pressure. The original cost objective function $f(x)$ is augmented into an unconstrained objective function $f_A(x)$ to give

$$f_A(x_1, x_2, \ldots x_n) = f(x_1, x_2, \ldots x_n) + \frac{1}{T}\sum_i w_i \; g_i^+(x_1, x_2, \ldots x_n) \qquad i = s, o, bp \tag{7.8}$$

where

$$g_i^+(x_1, x_2, \ldots x_n) = \max(0, c_i) \tag{7.8a}$$

The coefficients w_i and c_i are the non-negative penalty weights and magnitudes of constraint violation respectively, for sliding (s), overturning (o) and bearing pressure (bp). The latter values are equal to zero in the case of non-violation. Due to the inverse dependence of the augmented objective function on the system temperature T, the search is intensively biased towards the feasible space as it progresses. The estimate of the weight coefficients in this approach can be shown to be

$$w_i = \frac{T_1}{c_i} \ln\left(\frac{1}{p_{acc,i}}\right) \tag{7.9}$$

where T_1 is the initial system temperature and $p_{acc,i}$ is the probability of acceptance of the violated solution dependant on the ratio of magnitude of constraint violation g_i to a maximum constraint violation g_{imax}. The latter is initially estimated when all dimensions of the wall are at their lower bounds, and automatically updated if the algorithm comes across a solution with greater magnitude of violation. In this approach it is assumed that the estimated penalties will in general result in an increase of the objective function. They will also be dependant on the magnitude of the constraint violation, allowing solutions with minor violations to be accepted in the early and intermediate stages of the search, and therefore improving the quality of the solution space surrounding the constraint boundaries. However, as the search approaches its final *cooling* stage, the penalties increase to such a level that only feasible solutions are accepted.

7.3.6 *Computational Considerations*

This research used an object-orientated visual programming language and dynamic arrays to optimise computer memory requirements. The procedures which control generation and acceptance of new solutions do not require significant computing effort, and so the computational cost of implementing the algorithm is almost invariably dominated by that associated with the evaluation of the objective function. Similar to most practical problems in structural optimisation, the objective function for retaining walls is complex and requires repetitive analysis and design, every time a new trial solution is generated. However, due to the fact that the number of design variables is small the overall computing time was found not to be critical. Computational time was fast with optimum solutions being achieved without recourse to parallel processing or high specification computers.

Facilities that allow the user to stop, pause and continue the computation process have been developed and incorporated within the programme. These facilities allow for the constant monitoring, assessing and changing of parameter settings, hence speeding up the rate of convergence and allowing for fine-tuning of the optimum solution. Furthermore, detailed information on the simulated annealing parameter settings can be obtained (*see* Figure 7.2). Figure 7.6 shows an example of a structure report that offers detailed feedback at any point in the annealing cycle, including information on the final *frozen* state of the optimum solution. This report also gives a cost breakdown for the final solution, optimum reinforcement for the stem, heel, toe and heel beam and information about constraint violation.

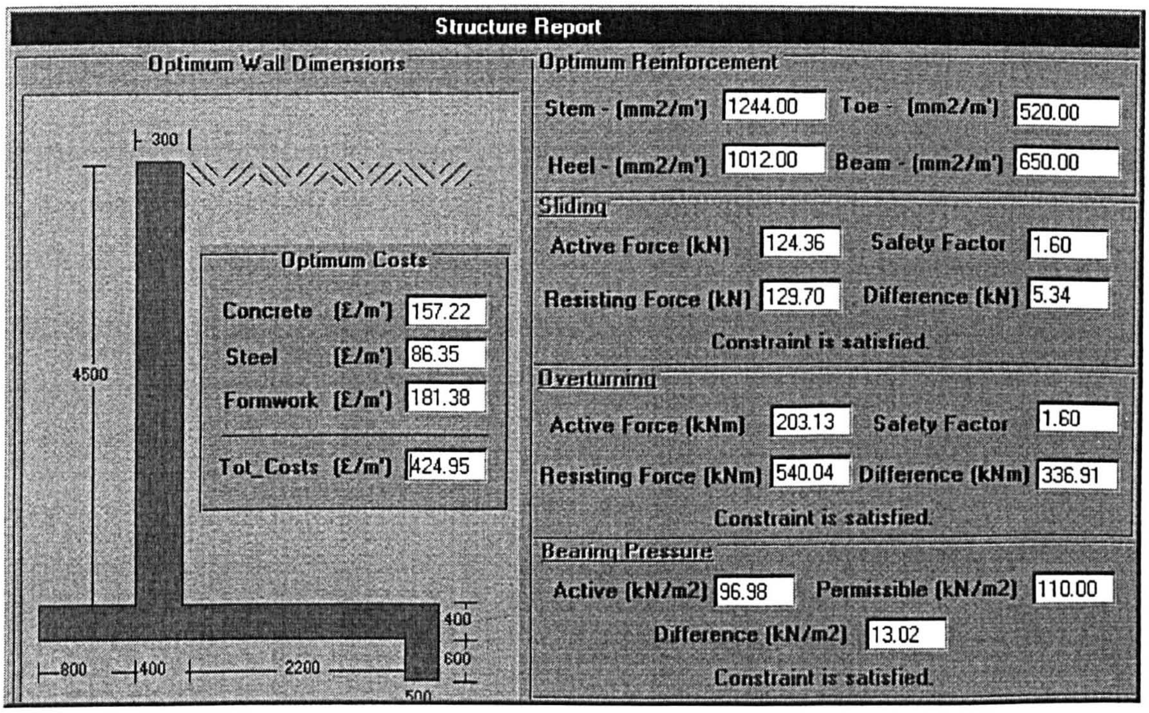


Figure 7.6 *Status and Structure Report Form*

The ability to capture the graphical performance for different algorithm settings is particularly helpful when determining the most suitable control parameter values for different retaining wall problems.

7.4 Design Examples

A literature survey (*see* Section 2.4.3) has shown that the performance of the SA search can be enhanced by an appropriate choice of the control parameters, and that they are problem dependent. Hence, an investigation was performed to find suitable settings for cantilevered retaining structures. The investigation was conducted for the selection of the initial starting point (random or estimated), step size distribution, choice of the initial temperature and the annealing speed.

Figure 7.7 shows a reinforced concrete cantilever retaining wall with depth of the stem d_w=4500*mm*, supporting a granular material with saturated density of 2000 kg/m^3. The coefficients of active and passive pressures are taken to be 0.33, and 3.0 respectively. Coefficient of friction is 0.45 and the partial safety factors against sliding and overturning are both 1.6. Upper and lower bounds on the wall dimensions are given in Table 7.1. The characteristic strength of the concrete is 35 N/mm^2, and the characteristic strength of the steel is 460 N/mm^2. Allowable bearing pressure for the base of the retaining wall is 110 kN/mm^2.

Dimension	Lower Bound	Upper Bound
w_1	400	1100
w_2	300	900
l_1	800	2500
l_2	2200	7000
d_1	400	1000
a	600	1000
w_b	500	700

Table 7.1 *Wall Dimensions (mm)*

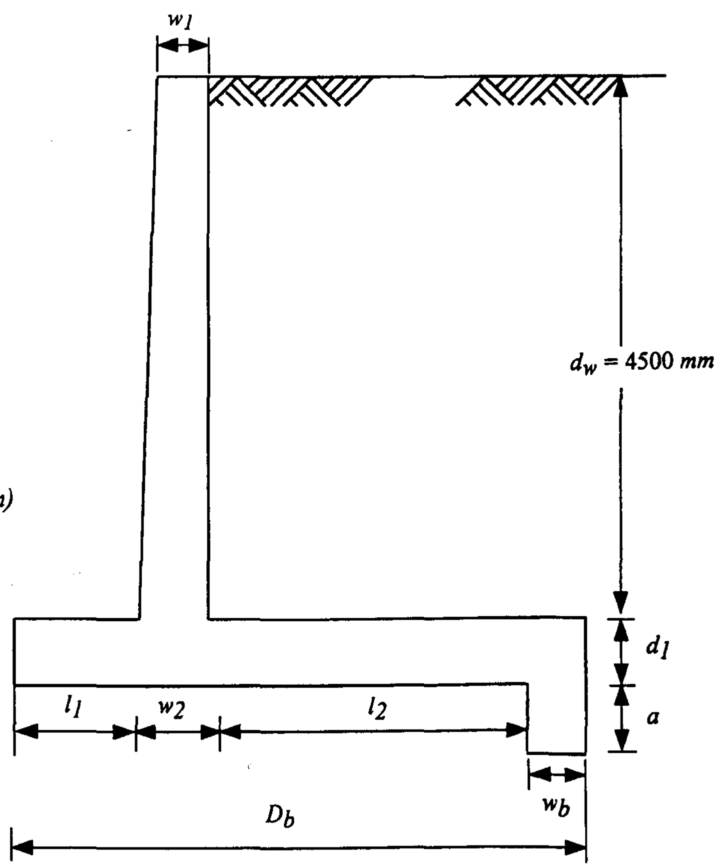


Figure 7.7 *Design Example - Cantilever Retaining Wall*

The costs of associated with concreting, reinforcing and formworking are given in Table 7.2. The results presented are representative of all the cases investigated in this research.

Concreting	Rate
Cost of concrete (£/ m^3)	32
Wastage (%)	5
Labour (£/m^3)	15

Reinforcing	Rate
Cut, bent & bundled (£/tonne)	275
Wastage (%)	2.5
Fixing Accessories (%)	5
Labour (£/m^3)	245

Formworking	Rate
Cost of timber framing (£/m^3)	285
Timber framing (m^3/ m^2)	0.05
Cost of timber boarding (£/m^2)	11
Wastage + fixings+ props (%)	15
Timber usage	7
Labour Make (£/m^2)	14
Fix and Strip (£/m^2)	14

Table 7.2 *Costs associated with Concreting, Reinforcing and Formworking*

7.4.1 Choice of Initial Solution

Figure 7.8 shows an example of an investigation into the choice of the initial starting point using both the random and estimation approaches. The latter was obtained by conducting a random pilot survey of *N*-solutions (*N*=100), and accepting that which gave the minimum value of the objective function. The estimate is based on the principle that for the initial temperature all changes in the objective function should be accepted.

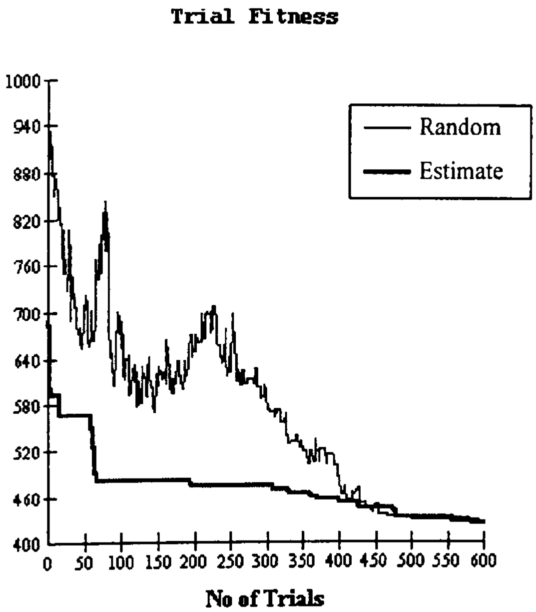


Figure 7.8 *Convergence History for Different Choices of Initial Solution*

Control Parameters	Initial Solution	
	Random	Estimate
Maximum Number of Trials	600	600
Initial System Temperature (°C)	800	800
Cooling Rate Parameter	0.85	0.85
Max_No of Trials at a Given Temperature	20	20
Max_No of Acceptances at a Given Temperature	20	20
Step Update – Alfa	0.9	0.9
Step Update – Omega	0.23	0.23
Final System Temperature (°C)	4	1
Minimum Fitness (£/m)	426.4	424.9

Table 7.3 *Control Parameters Setting*

To compare the convergence rate for both approaches, the settings of the other control parameters are identical in both cases, as shown in Table 7.3. The estimate trial fitness graph has been obtained by plotting the best fitnesses up to the current trial, whilst the random trial fitness graph illustrates the typical characteristics exhibited by the simulated annealing algorithm. Although the final solutions are almost identical, it was observed (*see* Figure 7.8) that the approach based on estimating the initial starting point showed an improved convergence tendency (i.e. higher convergence rate).

7.4.2 Choice of Step Size

Further investigations were conducted to compare different choices of step size, being an important and problem dependent issue that influences new trial solutions and the structure of the neighbourhood. Figure 7.9 shows an example of this investigation. Relaxed bounds in general contributed towards exploring a variety of possible configurations in the neighbourhood, moving the search in a random and more efficient

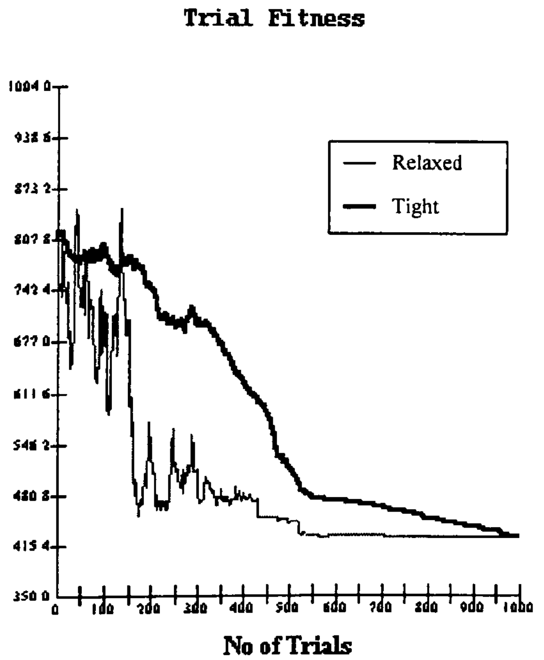


Figure 7.9 *Convergence History for Different Choices of Step Size*

Control Parameters	Step Size	
	Tight	Relaxed
Maximum Number of Trials	1000	1000
Initial System Temperature (°C)	2000	2000
Cooling Rate Parameter	0.85	0.85
Max_No of Trials at a Given Temperature	20	20
Max_No of Acceptances at a Given Temperature	20	20
Step Update Alfa	0.9	0.9
Step Update Omega	0.23	0.23
Final System Temperature (°C)	3	1
Minimum Fitness (£/m)	426.29	425.10

Table 7.4 *Control Parameters Setting*

manner towards the final destination of the optimum solution neighbourhood. Tight bounds resulted in a time consuming and less efficient search, requiring more iterations to achieve a similar optimum solution. To compare their efficiency the setting of control parameters are identical, as shown in Table 7.4.

It has been noted that a higher initial temperature is required to encourage convergence of the algorithm, arriving in the neighbourhood of the optimum solution only when a sufficiently long annealing schedule compensates for the tedious and time-consuming small movements between the neighbourhoods of the consecutive trial solutions. However, it is important to emphasise that the choice of step size is problem dependent and requires a thorough investigation for each type of structural problem.

7.4.3 *Choice of Initial Temperature*

Figure 7.10 shows an example of an investigation considering three characteristic cases for the initial system temperature, that is high, estimated and low. The setting of the control parameters is given in Table 7.5.

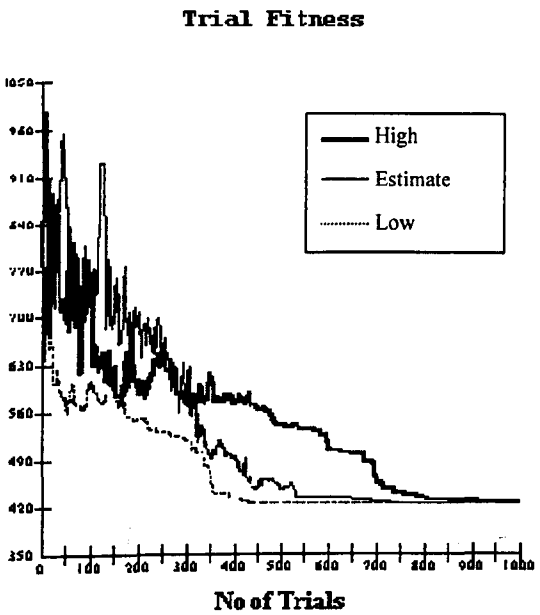


Figure 7.10 *Convergence History for Different Choices of Initial Temperature*

Control Parameters	Initial Temperature		
	High	Estimate	Low
Maximum Number of Trials	1000	1000	1000
Initial System Temperature (°C)	2000	1024.2	400
Cooling Rate Parameter	0.85	0.85	0.85
Max_No of Trials at a Given Temperature	20	20	20
Max_No of Acceptances at a Given Temperature	20	20	20
Step Update – Alfa	0.9	0.9	0.9
Step Update – Omega	0.23	0.23	0.23
Final System Temperature (°C)	3	2	1
Minimum Fitness (£/m)	426.54	424.90	425.35

Table 7.5 *Control Parameters Setting*

In the first case, the selection of a high initial system temperature resulted in an inefficient search taking an excessive number of iterations for the system to *freeze* inside the neighbourhood of the optimum solution. In the last case, it was noted that a low initial system temperature may result in premature convergence to a local minimum, as the system never had a chance to *melt* appropriately. However, an optimum solution still may be achieved if the algorithm is allowed to perform additional iterations (*see* Figure 7.10). The stability of the system and the convergence characteristics are generally better behaved when the initial system temperature is estimated from a random pilot survey of N-solutions based on the Kirkpatrick's rule (*see Appendix* G, Section G.5).

7.4.4 *Annealing speed*

Figure 7.11 shows an example of an investigation into the effects of annealing speed on convergence rates. Three characteristic cooling rates are considered and the results are given in Table 7.6.

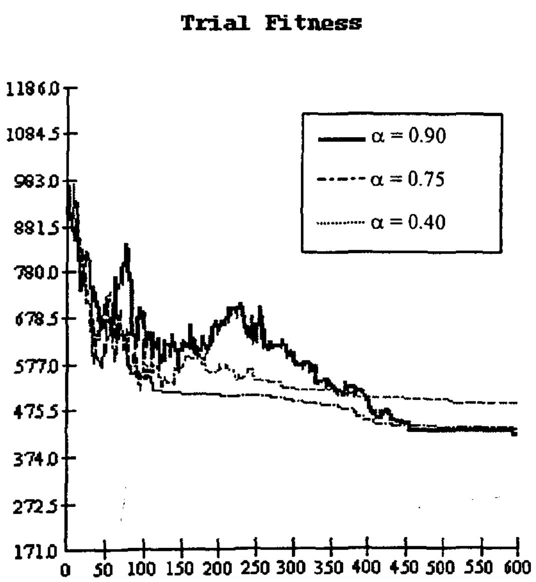


Figure 7.11 *Convergence History for Different Choices of Cooling Parameter*

Control Parameters	Cooling Rate		
	Slow	Medium	Fast
Maximum Number of Trials	600	600	600
Initial System Temperature (°C)	1000	1000	1000
Cooling Rate Parameter	0.90	0.75	0.40
Max_No of Trials at a Given Temperature	20	20	20
Max_No of Acceptances at a Given Temperature	20	20	20
Step Update – Alfa	0.9	0.9	0.9
Step Update – Omega	0.23	0.23	0.23
Final System Temperature (°C)	4	1	1
Minimum Fitness (£/m)	424.90	437.60	503.30

Table 7.6 *Control Parameters Setting*

Results from the tests suggested that the optimum solution and system stability were generally improved with a slower cooling rate (*i.e.* high values of α), at the expense, of course, of greater computational effort. For higher cooling rates (i.e. lower values of α), the algorithm often became trapped at a local minimum, not being able to find improvement in the solution due to the extremely fast cooling speed. Setting the cooling rates between these values (i.e. α = 0.75), showed a similar behaviour to the higher cooling rates, although convergence to the optimum solution could be achieved by performing additional iterations. Hence, it was observed that the performance of the algorithm depends more on the relative cooling rate than the absolute temperature reductions.

7.4.5 Constraint Handling

Further investigations were conducted to compare the simple rejection method with the PWE-based approach developed in this research. Figure 7.12 shows an example of this investigation using both approaches. To compare their efficiency the setting of the control parameters is identical in both cases, as shown in Table 7.7.

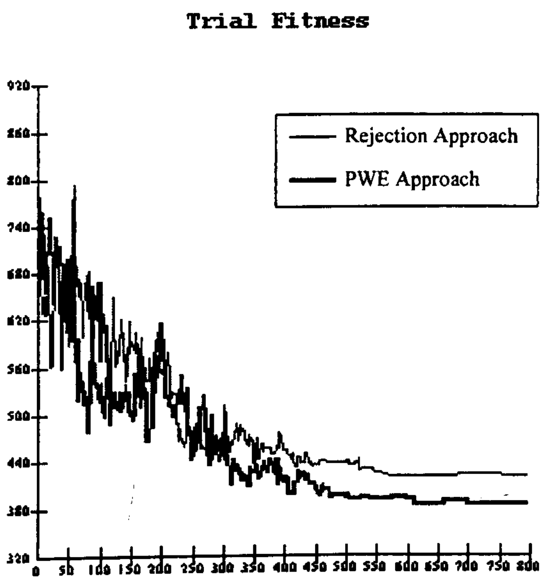


Figure 7.12 *Convergence History for Different Choices of Constraint Handling*

Control Parameters	Approach	
	Rejection	PWE
Maximum Number of Trials	800	800
Initial System Temperature (°C)	1200	1200
Cooling Rate Parameter	0.85	0.85
Max_No of Trials at a Given Temperature	20	20
Max_No of Acceptances at a Given Temperature	20	20
Step Update – Alfa	0.9	0.9
Step Update – Omega	0.23	0.23
Final System Temperature (°C)	3	2
Minimum Fitness (£/m)	420.10	388.30

Table 7.7 *Control Parameters Setting*

The simple rejection approach guarantees the search of the feasible region, arriving to the neighbourhood of the optimum solution if the solution space is constrained only by inequalities and has no disjointed features. However, this approach does not take account of the magnitude of constraint violation and hence rejects any solution that violates the constraints. The in-efficiency of this approach is evident when the neighbourhood of the optimum solution surrounds the intersection of constraint boundaries, as shown in Figure 7.12. The PWE based approach on the other hand, allowed solutions with minor constraint violations to be accepted in the early and intermediate stages of the search, hence improving the quality of the solution space which surrounds the constraint boundaries. However, as the search approached its final *cooling* stage, estimated penalties increased to such an extent that only feasible solutions were accepted. When compared to the simple rejection approach, the PWE-based technique had a greater search flexibility and efficiency in exploring the neighbourhood of the solutions on the constraint boundaries. It also automatically estimates and updates weight coefficients, hence avoiding time consuming and repetitive numerical experimentation required by the ordinary penalty approaches.

7.4.6 Cost Sensitivity Analysis

Having carefully considered the selection and choice of the problem dependent control parameters when performing a simulated annealing search, a cost sensitivity analysis was carried out to assess the behaviour of the final solution for different choices of the component costs. As stated earlier, the adopted cost objective function includes the cost of concrete, cost of steel and the cost of formwork together with their associated labour costs. Since the wall stem is of fixed height, the associated formwork costs are constant throughout the search, and hence have no influence on the optimum solution. The optimum solution therefore, depends on the balance between the costs of concreting (C_c) and reinforcing (C_s). Hence, to compare these cost contributions, a cost sensitivity analysis with respect to the ratio q (C_s/ C_c) has been performed.

Figure 7.13 shows an example of the cost contribution made by each material for different values of q ranging between 25 and 95.

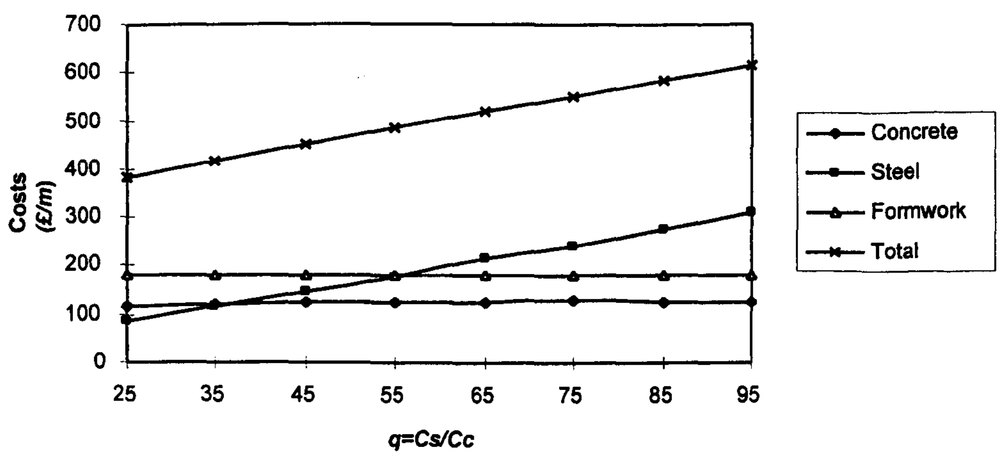


Figure 7.13 *Retaining wall cost analysis for different values of q*

The settings of the SA control parameters were kept constant in this comparison to isolate the influence of different values of q on the optimum solution. Upper and lower bounds on the dimensional variables were set loose in the constrained region, allowing the algorithm to search an increased number of possible feasible and non-feasible configurations. It was observed that the cost of concrete tends to be constant for any value of q, corresponding to a wall geometry where the design variables are driven towards their lower bounds until the permissible bearing and sliding constraints became critical. This was also observed from Table 7.8, where the factor of safety for sliding and overturning, and the actual ground bearing pressure are evaluated at the optimum solution for each value of q. Due to the geometry of the wall and the given loading conditions, the overturning constraint was found to be not critical.

Cost Ratio	Sliding FOS = 1.60			Overturning FOS = 1.60			Bearing Pressure	
q	Active Force (kN)	Resisting Force (kN)	Actual FOS	Active Moment (kNm)	Resisting Moment (kNm)	Actual FOS	Actual Pressure (kN/m²)	Permissible Pressure (kN/m²)
25	119.3	122.9	1.65	190.9	481.9	4.04	109.3	110
35	119.3	119.5	1.60	190.9	463.5	3.88	108.8	110
45	119.3	119.9	1.61	190.9	462.7	3.88	109.1	110
55	119.3	119.8	1.61	190.9	463.4	3.88	108.7	110
65	119.3	119.7	1.61	190.9	470.8	3.95	106.1	110
75	119.3	122.2	1.64	190.9	420.5	3.52	108.6	110
85	119.3	119.8	1.61	190.9	410.7	3.44	109.1	110
95	119.3	121.5	1.63	190.9	450.4	3.77	110.6	110

Table 7.8 *Constraints assessment for a different values of q*

Table 7.8 shows that the bearing pressure and sliding constraints however were critical for each q value, with the optimum solutions being located at their intersection. It was observed that the search of the design space was performed along the feasible design boundary until it reached the intersection with the other critical constraint. This confirms the ability of the developed algorithm to avoid potential local optimums along the single constraint interface by further exploring the design space in the search for improved solutions.

Table 7.9 presents the optimum values for the design variables considering the identical range of q values as given in Table 7.8.

Dimension Variables	Lower Bound	Upper Bound	Cost of steel to cost of concrete ratio q (C_s/C_c)							
			25	35	45	55	65	75	85	95
(mm)	*(mm)*	*(mm)*	*(mm)*	*(mm)*	*(mm)*	*(mm)*	*(mm)*	*(mm)*	*(mm)*	*(mm)*
w_1	250	900	250	250	250	250	250	250	250	250
w_2	300	1100	300	338	363	381	327	352	356	376
l_1	500	2500	603	628	615	613	657	615	691	608
l_2	1000	7000	2209	2204	2189	2175	2216	2205	1955	2140
d_1	300	1000	300	300	300	300	300	300	300	300
a	500	1000	500	500	518	513	500	520	772	614
w_b	400	700	400	400	400	400	400	400	400	400

Table 7.9 *Retaining wall optimum solutions for different values of q*

Tables 7.8 and 7.9 show that to satisfy the critical sliding constraint, the length of the base (l_1+w_2+l_2) and the height of the heel beam a were adjusted accordingly. For all values of q, the actual ground bearing pressure reached or was close to the permissible value. The width of the stem at the top w_1, the depth of the base d_1 and the width of the heel beam w_b were driven to their lower bounds for all values of q.

It was also noted that the majority of the steel provided in the retaining wall was attributed to the main reinforcement in the stem and to the distribution reinforcement in the whole of the wall. Since this reinforcement is proportional to the cross-sectional area of the wall, the dimensional variables were driven towards their lower bound values until the permissible bearing and sliding constraints became critical.

7.5 Conclusions

The presented results illustrate the performance of a constrained simulated annealing algorithm applied to the minimum cost design of reinforced concrete cantilever retaining walls. The proposed implementation of the algorithm is a highly practical approach to the design process, incorporating realistic loading conditions and limit states, together with material and labour costs associated with concreting, reinforcing and formworking. The ability of simulated annealing algorithms to avoid gradient computations and rapidly search the feasible region independent of the initial starting point was considered to be a very robust feature. However, the investigations conducted have shown that the method of selecting the initial starting point generally affects the convergence rate of the algorithm. Furthermore, the research has obtained good results using the estimated initial temperature based on conducting a random survey of *N*-solutions, showing in particular an improved rate of convergence. This rate was also affected by the choice of step size, with relaxed bounds generally contributing towards a more efficient search. Tight bounds, on the other hand, resulted in time-consuming and less efficient searches requiring long annealing schedules to compensate for tediously small movements between the successive iterations. Research investigations into the effects of the cooling rate on the adopted annealing scheme indicated that the performance of the algorithm depends more on the relative cooling rate than absolute temperature reductions. The developed PWE-based approach for constraint handling exhibited a promising superiority over the simple rejection approach, having a greater search flexibility and efficiency in exploring the neighbourhood of the solutions on the constraints boundaries. When compared to the ordinary penalty approaches that require time consuming and repetitive numerical experimentation, its ability to automatically estimate and update weight coefficients was considered to be a significant advantage.

The conclusions from the cost sensitivity analysis performed in this research indicated that the optimum solution lies on the intersection of the critical constraints with the design variables being driven towards their lower bounds until the constraints boundaries were reached. Furthermore, this investigation has revealed the algorithm's ability to pinpoint the multiple constraints intersection solutions, hence avoiding premature local optimums on a single constraint boundary.

8. Conclusions and Further Work

This chapter provides conclusions from the research work undertaken and presented in this thesis. Recommendations for further research are given for possible future developments.

8.1 Introduction

Although modern structural optimisation research was evidently both rapid and productive in its developments over the past three decades, its insufficient penetration in professional design practice has been recognised, especially in the field of realistic reinforced concrete structural systems. Published research in this field is scarce and is mainly found in isolated papers and publications, with the vast majority of published work concentrating on steel structures. Given the broadness and complexity of optimisation techniques, practising designers are faced with difficult decisions regarding their choice, suitability and relevance to the design of the structural systems under realistic loading conditions. Furthermore, designers often lack confidence and underlying knowledge of the optimisation theory required for a competent application of these methods. Recent surveys, such as that carried out by Cohn (1995), have reported that efficient practical applications have been limited only to highly specialised companies with *in-house* developed computer optimisation software. The vast majority of design offices still use conventional approaches to the design of reinforced concrete structures as discussed in Section 1.1, based on the repetitive check analysis and without any specific objectives that guide such designs. As stated by Cohn (1995), it seems reasonable to conclude that they will accept these optimisation tools only when they find them to be either highly beneficial or indispensable, and naturally, relevant to their problems.

Therefore, the research presented in this thesis was carried out with specific reference to the choice of objective function, non-linear programming techniques, design constraints and assemblage of the optimal design problem, being cognisant of practical design methodology and its implementation in the design office. Taking this into account, optimisation implementation theory of realistic reinforced concrete skeletal systems and general computer programs for the automatic optimum (improved) design were developed and their performance discussed.

8.2 Research Contributions

The *problem-seeks-optimum design* approach adopted in this research has resulted in the development of novel cost objective functions and corresponding critical design constraints, utilising design procedures and analytical methods familiar to the structural engineer. A computer-based approach was developed that not only simulates the real world design of skeletal systems, but also offers a systematic, goal-orientated design process. It combines both structural analysis and design to search and sort through similar design concepts guided by a set objective.

The developed structural problem formulations incorporate design equations and procedures specified in BS8110, complying with the codes of practice in a way that is intuitive to the designer. Different optimisation techniques are consequently investigated, assessing their suitability when applied to the practical optimum design of a range of problems. The reasoning behind the application of these methods is given and their performance discussed, with particular reference to their suitability given the structural problem formulation and its objective function. The research showed that structural optimisation is both feasible and is a natural extension of the mutually inclusive but currently separate activities of structural analysis and design. It concluded that the validity of structural optimisation was directly dependent on the balance between the structural problem formulation, the implemented algorithm and the physical reality of the underlying structural problem.

Taking this into the consideration, the research contributions are summarised within the following sections.

8.2.1 *Structural Problem Formulation*

Objective function: A practice-orientated approach to the optimum design of skeletal structural systems both on the elemental and structural level, resulted in the continual re-appraisal and development of the objective function and problem formulation in accordance with the stated aims and objectives of the research. The formulation of the volume objective function as a stepping stone towards the development of a more sophisticated material cost objective function was established early within the research. Invaluable insight was gained both in the functionality of the basic member grouping approach within the multi-level optimisation environment and in the overall performance of the applied mathematical non-linear programming techniques. Using this knowledge, the objective function was further developed to account for the material costs of the structure that more truly reflects the goal of both the client and designer. The resulting objective function incorporated the practical assessment of the material costs, taking into account topology, loading arrangements and curtailment of the reinforcement. Its formulation reflects a novel approach to multi-level cost optimisation by grouping the structural elements in a manner that mirrors design office practice. The results obtained using the implemented mathematical programming approach were encouraging, with the optimisation algorithm showing respectable performance and stability. However, when additional construction costs were considered, the resulting complexity of the objective function together with the discontinuity of the design equations revealed the limitations of the mathematical programming approach. In that context, the investigations conducted in the application of modern heuristic methods showed a promising potential in overcoming these limitations. Genetic algorithms and simulated annealing were chosen and implemented in this research for the reasons previously discussed in Sections 6.1 and 7.1, and their performance is summarised within Section 8.2.2.

The application of these methods allowed the research to further develop the objective functions and implement a practical and realistic estimation of the total structural costs, including additional construction costs associated with the formwork and labour. The resulting objective functions were developed both on the elemental level for beams, columns, slabs and retaining walls, and on the structural level for skeletal systems.

Design variables: Particular attention has been paid to the design variables and their assemblage in the objective functions and the corresponding design constraints and equations. The design variables identified reflected a conscious decision to map them against practical design office procedures. Being dependant on the type of structural system assessed, the nature of the objective function and the adopted member grouping approach, the method for identifying the design variables had a direct influence on both the problem formulation and the performance of the implemented optimisation algorithm. For the volume optimisation, where only the cross-sectional dimensions where chosen as design variables, it was observed that although the resulting optimisation process was efficient and mathematically stable, the objective function was not directly sensitive to the material costs of a structure. Instead, it was the value of the upper bound reinforcement ratio as non design variable that had a direct influence on the total cost of a volume optimised structure. Using this knowledge, a cost objective function was developed to include the reinforcement ratios as design variables. The resulting objective function gave a clear means of representing the significant difference in the unit costs of steel as compared to those of concrete. The developed approach to multi-level cost optimisation through the grouping of the structural elements also had a direct influence on the number of design variables, these being significantly reduced due to the variables being allocated to the beam/column groups rather than to the individual structural elements. In this context, the results from earlier investigations on the minimum cost design of main structural components were invaluable. Particularly important was the identification of the design variables and understanding their behaviour within complex skeletal structural systems.

Multiple loading arrangements: The research results showed that optimum solutions for single load cases, whilst being mathematically correct, do not satisfy the design requirements as specified in BS8110, where the worst case scenario for the loading arrangements should be considered. As part of the research into cost optimisation, a method for determining the critical member forces from multiple load case analyses was developed, and incorporated within the multi-level optimisation environment. This allowed the developed software to optimise only the critical structural members within each beam/column group, and then intelligently assign the cross-sectional design variables for the remaining structural elements within the same group. Critical forces are determined by assessing both the shear and bending stresses for beams, and the combined axial and bending stresses for the columns. This approach was incorporated within the software with only a minor increase in the required computational time. The results however, showed that a consideration of realistic loading conditions is important if the optimum solution obtained is to be representative of a real structure.

Design constraints and equations: For the minimum volume problem formulation where mathematical optimisation was applied, the relevant design constraints were represented in their classical formulation as a mixture of equalities and inequalities, with the stress constraints being critical within the optimisation process. The theory and principles behind the removal of the stiffness equality constraints were developed, implemented and its performance investigated. This approach simplified the problem formulation, hence encouraging the convergence to the optimum to be both mathematically stable and rapid. The bending constraints for beams were derived both for singly and doubly reinforced sections, and their behaviour was investigated through the application of the Lagrangian Multiplier Method (*see* Section 3.3). Column stress design constraints for the volume optimisation were derived using mathematical modelling techniques with the assumption of a near linear behaviour for the column design graphs provided in BS8110. Deflection constraints were included by limiting the span/effective depth ratios as specified in BS8110. The lower and upper bound dimensional constraints are assigned by the designer, taking account of practical buildability, aesthetic requirements, and the minimum/maximum reinforcement ratios specified in BS8110.

For the cost optimisation method, the stress constraints were considered in the form of equilibrium equality equations. The dimensional, deflection and additional reinforcement ratio constraints were also incorporated and comply with BS8110. The column design equations developed for the volume optimisation were impractical due to their cumbersome derivations. Hence, different approaches were investigated resulting in the development of the *least square* and *revised* methods for the column design equations for approximating the area of reinforcement. These provided good approximations over a wide range for each column design chart, although on a few occasions the level of error was unacceptable. However, when combined within an iterative procedure that modifies the gradient of the area reinforcement ratio contours at every global iteration, the methods were reliable and satisfactory.

For the structural problem formulations solved using genetic algorithms, consideration was given to the assessment of additional construction costs, resulting in a more complex objective function. Equilibrium equality design equations derived from the earlier research were modified and applied, thus ensuring an unconstrained problem formulation more suited for solution using GA's. However, further improvements in the problem formulation were achieved both for beams and for columns. The optimisation process was now capable of handling both singly and doubly reinforced beam sections, since the discontinuity between the design equations was not a limitation for GA's, in contrast to the mathematical programming approach. Furthermore, the advantage of GA's in not requiring any gradient information to perform an optimisation process allowed for the development and implementation of the exact solution to the column design equations within the computer programme. This solution avoided both the problems of accuracy associated with the approximate design equations and the application of an iterative procedure for balancing the design equations that was time consuming and sometimes ill-conditioned. *Hard* constraints, such as deflections, shear stresses and minimum/maximum reinforcement ratio constraints were handled by an *improved rejection* method developed in this research. In this method, the population member that violates the constraint is replaced by another, considering a random increase in the depth of the structural member where the constraint violation was originally committed.

Retaining walls were considered as part of the skeletal system's substructure. In the problem formulation a probabilistic weight estimate (PWE) based approach to constraint handling was developed to model the design requirements for ensuring the stability of the wall and that allowable ground bearing pressures are not exceeded. This approach exhibited a promising superiority over the simple rejection approach, having a greater search flexibility and efficiency when exploring the neighbourhood of the solutions on constraint boundaries. When compared to ordinary penalty approaches its advantage in efficiency was evident due to the automatic update of weight coefficients that for the former method requires time consuming and repetitive extensive numerical experimentation.

8.2.2 *Optimisation Methods*

Choice of the optimisation method: The optimisation methods implemented in this research were carefully selected to match the requirements posed by the particular structural problem formulation and the complexity of the objective function. It was not intended in this research to perform a detailed comparison of the individual methods regarding their performance, but to assess their suitability, advantages and limitations for the structural problems being considered. In this context, it was concluded that for both volume and material cost optimisation, the implemented SLP method showed itself to be a powerful optimisation tool, exhibiting advantages in its robustness, efficiency and mathematical stability. However, limitations in its applicability were encountered when considering additional construction costs and the discontinuous design equations. Coupled with this, difficulties were also encountered in obtaining the derivates required for constructing the gradient vector. Due to the increased complexity of the objective functions and their associated problem formulation, the SLP method became progressively impractical to implement. In addition, the algorithm's sensitivity to the initial starting point and the lack of the convergence in certain cases, prompted further research to identify more suitable optimisation techniques. A comprehensive literature review and the experience gained in use of *Generator* suggested that heuristic methods, namely genetic algorithms and simulated annealing, offered a promising horizon for investigation. Their potential for solving problems that are unsuitable for traditional mathematical programming methods make them particularly attractive. Their ability to overcome the limitations of the SLP method and provide near-optimum solutions should the algorithm stop short of reaching the global optimum, led to further cost optimisation investigations. Furthermore, the *blindness* of heuristic methods to the nature of applied structural problems, their ability to avoid derivates and linearisation errors, and their efficiency in dealing with discontinuous design equations made them particularly attractive to the types of reinforced concrete structures and loading conditions being investigated.

Implementation of the optimisation methods: The implementation of each optimisation method is discussed taking into account the structural problem formulation and its associated objective function.

Minimum material costs – Elemental level: Using a simplified cost model and problem formulation, the Lagrangian Multiplier Method proved to be an effective optimisation technique for simple structural beams. The developed approach offers the designer closed solutions for both singly and doubly reinforced beam sections, without recourse to prior knowledge of mathematical optimisation. The approach provided a valuable insight into the minimum cost design of beams within more complex skeletal structures. It was also successfully employed for estimating the upper bound reinforcement ratios when assessing the cost sensitivity of solutions obtained for volume optimised structures.

Minimum volume and material costs – Superstructure: A modified approximation programming algorithm based on Sequential Linear Programming was proposed both for the minimum volume and material cost design of skeletal structural systems. A novel approach for removing non-critical stiffness constraints was developed and implemented, with the resulting constraint set being considerably simplified. The method was tested regarding its sensitivity to the number of design variables, moving limit values and the choice of initial design point. Testing showed that the algorithm is both mathematically stable and robust, and is an efficient optimisation tool.

Minimum material and construction costs – Superstructure: The proposed implementation of genetic algorithms is a highly practical approach to the design process, capable of assessing both realistic structural problem formulations and loading conditions. The ability of GA's to avoid gradient calculations and search the feasible region independent of the starting point provided a very suitable alternative to the limitations encountered with the mathematical programming approach. The developed computer-based design approach not only simulates the *real world* design of skeletal systems, but takes advantage of the artificially intelligent search offered by GA's to obtain improved designs so as to minimise the total structural costs. The results of numerous studies undertaken in this research showed that the performance of the GA search can be enhanced by an appropriate choice of control parameters. Good results have been obtained using the elitist model that showed higher convergence rate than the standard evolution approach. Uniform crossover generally achieved the best convergence rate, although its sensitivity to the percentage of exchanged genetic material was clearly recognised. Both the random hill and directional hill climbing mutation methods have shown advantages over the standard method, mutating the genes in a beneficial manner that generally improves convergence. However, this needs to be considered in light of the increase in the total number of function evaluations required for these methods. An *improved rejection* method for *hard* constraints handling and the *fast re-analysis* approach developed in this research have both shown to improve the algorithm performance, avoiding the unnecessary computational effort in a manner that we as humans would consider intelligent.

Minimum material and construction costs – Substructure: Published research indicated that the application of simulated annealing could lead to a more computer efficient optimisation approach when small to medium size constrained structural problems are considered. Simulated annealing is by its nature a one-point random search method and as such has an advantage over the GA's random population search, when the number of design variables for a given structural problem is considered. The nature of the problem formulation for the retaining walls studied in this research are well suited to solution using SA. Similar to genetic algorithms, the ability of simulated annealing algorithms to avoid gradient computations and rapidly search the feasible region independent of the initial starting point was considered to be a very robust feature. However, the investigations conducted in this research indicated that although the algorithm search is independent of the initial starting point, the method of its selection generally affected the convergence rate of the algorithm. When the suitability of SA control parameters were considered, the research obtained good results using the estimated initial temperature based on conducting a random survey of N-solutions, showing in particular an improved rate of convergence. The effects of the cooling rate on the adopted annealing scheme indicated that the performance of the algorithm depended more on the relative cooling rate than absolute temperature reductions. The probabilistic weight estimate (PWE) approach was developed in this research as a novel approach to constraints handling, showing itself to be superior to the *simple rejection* and standard penalty approaches. This approach demonstrated its ability to robustly deal with *hard* constraints that have to be satisfied if feasible design solutions are to be obtained. The ability of the implemented algorithm to pinpoint the multiple constraint intersections, and hence avoid potential local optimums on the single constraint boundaries was considered to be a significant advantage.

8.2.3 Cost Sensitivity Analysis

The early research investigations on the minimum cost design of reinforced concrete beams identified three distinct optimum solutions dependent on whether the beam was singly, boundary or doubly reinforced. The validity of these optimum solutions for a particular beam problem was directly dependent on the chosen value of cost of steel to cost of concrete ratio q. As q increased, and with it the cost of steel relative to those of concrete, it was observed that the minimum material costs were achieved through a reduction of the percentage reinforcement ratios coupled with a corresponding increase in the effective depths of the sections. Parametric design curves and tables have been developed in this research to guide the designer in the selection of these optimum reinforcement ratios in a manner that is both intuitive and practice orientated.

On the structural level, and for minimum material costs, the research has shown that optimising a skeletal structure subject to a single load case does not give the true minimum cost of a structure. Furthermore, it was concluded that multiple beam and column group design gives in general more cost efficient solutions, although this needs to be considered in light of a potential increase in formworking costs. A novel approach in the use of volume optimisation for estimating the minimum costs of a structure was developed, and in that context the potential of the Lagrangian Multiplier solution (*see* Section 3.3) was clearly recognised.

A cost sensitivity analysis of those structural problems that considered both material and additional construction costs reinforced further the findings relating to multiple load cases and the use of beam/column groupings. In addition, it was observed that optimum solutions were also directly dependent on the choice of the additional construction costs, such as formworking and labour costs. Changes in these unit component costs resulted in significant differences in the final optimum designs. The cost sensitivity analysis of the cantilever retaining walls indicated that the optimum solution lies on the intersection of the critical constraints with the design variables being driven towards their lower bounds until those constraints boundaries were reached.

8.2.4 *Computer Programmes Development*

Based on the developed implementation theory, the research has produced general computer programs for the automatic optimum (improved) design of realistic, rigidly jointed reinforced concrete skeletal structures using ultimate limit state theory, as embodied in BS8110. These practice orientated programs allow for the inclusion of further relevant design constraints and additional costs, if so required. Their development followed chronologically and in parallel with that of the implementation theory for each optimisation method. Hence, the programs that incorporate SLP, GA's and SA algorithms were developed, tested and results compared throughout the research. For the minimum volume and material cost optimisation approaches, where the SLP algorithm was implemented, the structure of each program was clearly defined, with a two-phase switch for a cyclical structural analysis and optimisation procedure identified. These computer programmes incorporate a number of subroutines, that are classified into four groups according to their function, namely; control routines, ancillary subroutines, element and speciality subroutines. This modular structure allows the programme to be extended through the inclusion of additional subroutines or libraries, making future developments easier to code and implement.

When implementing both the genetic and simulated annealing algorithms, the research developed a more sophisticated computer design environment taking advantage of the artificially intelligent search capabilities offered by both algorithms. The programmes were designed around the structural problem formulations on both the elemental and structural level. For each level the design stages were identified so as to simulate the *real world* design of skeletal systems and provide the designer with a user-friendly interface. This approach offered a systematic and goal-orientated design process, allowing the optimisation algorithms to intelligently search and sort through the similar design concepts to achieve a set objective. The supporting facilities developed for dynamically monitoring the performance of each algorithm and for editing the control parameter settings, significantly improved the control over the performance of each optimisation method. Some of the other developed features exhibited an artificially intelligent behaviour, such as the *fast re-analysis* approach that uses the binary pattern recognition to determine already analysed members of the population and hence avoid the unnecessary and time consuming structural analyses repetitions. This approach is capable of recognising identical members not only in the current population but also through the entire GA's evolution history (Ceranic and Fryer 2000). To develop such a complex computer design environment, an object-orientated visual programming language was used together with dynamic arrays to optimise the computer memory requirements. The programmes were implemented within the Windows environment, offering both a user-friendly interface and interactivity capabilities that are now expected by the professional designer.

8.3 Future Work

The research work presented in this thesis was concerned with developing practice orientated optimisation implementation theory for the optimum design of realistic reinforced concrete skeletal structural systems. Detailed investigations in the structural problem formulation for beams, slabs, 2D skeletal frames and cantilever retaining walls have been presented and discussed. The reasoning behind the selection of a suitable optimisation technique for the solution of each problem has been given. However, further research is recommended to enhance the adopted structural problem formulation approaches, their implementation theory and the resulting computer-based design environment.

Structural Problem Formulation: The algorithms developed in this research have not incorporated all of the possible design constraints and structural problem formulations that may be required by current design practice for particular structural problems. However, they have been developed to allow for the inclusion of any additional constraints and for the modification of the problem formulations, if so required. On the elemental level, a further research to improve the developed cost objective function for beams is suggested, taking into consideration practical lengths, cut-off points and minimum spacing requirements of the reinforcement, as specified by BS8110. For the columns the design equations could be further developed to incorporate unbraced columns, taking into account the design of *slender* columns. On the structural level, investigations are suggested on how to incorporate non-structural, yet often costly construction elements, such as vertical movement joints and shear keys for retaining walls. Furthermore, future work that includes additional constraints, such as total and differential settlement, different distributions of ground bearing pressures and a full slip-circle analysis is suggested for retaining walls.

On the global level, further research is proposed to investigate the multi-objective optimum design of complex 3D reinforced concrete structures considering slabs as plate elements forming a constituent part of a rigidly jointed skeletal system. As noted from design office practice, the relevance of realistic designs often cannot be satisfied by a single objective, and hence the multiobjective approach is suggested. It is also recommended to analyse the slabs using finite element method and incorporating it within the joint displacement method for space frames. The ability to analyse such complex structures would require further research to explore ways to reduce the exhaustive computational requirements that would be associated with a heuristic search for the optimum solution. In this context, the potential of micro-GA's has been recognised and is suggested as a suitable optimisation method, whereby the computational effort can be significantly reduced by considering relatively small population sizes (Woon and Querin 1999).

Optimisation methods: Further developments of more efficient operators and control parameters to improve the performance of genetic and simulated annealing algorithms is suggested. A more comprehensive assessment and performance monitoring of the developed PWE approach to the constraints handling in simulated annealing is required for different types of retaining structures.

Further research into hybrid strategies is proposed, such as assimilating genetic algorithms and simulated annealing into a co-operative approach. Such an approach has the potential for offering a robust and efficient algorithm when applied to highly dimensional constrained problems where a population search of the feasible region would exhibit significant advantages.

When 3D structures are considered, an investigation into the potential of the learning capabilities of neural networks for approximate finite element analysis is proposed, with the objective of reducing time consuming and costly structural analyses. This could have a significant impact on the performance of the optimisation process where the structural analysis is an integrated and repetitive task.

Automated computer based design toolkit: The development of a fully integrated computer aided reinforced concrete design tool is suggested for future research. The objective is to incorporate all the investigated optimisation approaches into an automated structural design tool for skeletal structural systems. This design tool should have the flexibility to include the developments from the proposed further research, together with any additional requirements. This would be an important step forward in offering a structural engineer a more efficient and economical design tool for reinforced concrete structures, than those currently available in design offices.

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Appendix A Implementation of SLP Method

Since the partial derivations are evaluated at the initial design point x^o, they can be taken as constants c_j and w_{ij} over a given range. Furthermore, if the following substitution $\Delta x_j = (x_j - x_j^o)$ is made, then the problem can be rearranged in the form of

Optimise objective function

$$y = y^0 + \sum_{j=1}^{n} c_j \, \Delta x_j \qquad j = 1, \ldots\ldots n \qquad \text{(A.1)}$$

subject to the constraints

$$g_i = g_i^0 + \sum_{j=1}^{n} w_{ij} \, \Delta x_j \begin{Bmatrix} \le \\ - \\ = \end{Bmatrix} b_i \qquad i = 1, \ldots\ldots m \qquad \text{(A.2)}$$

with moving limits imposed as

$$k_{1p} - x_j^o \le \Delta x_j \le k_{2p} - x_j^o \qquad p = 1, \ldots\ldots n \qquad \text{(A.3)}$$

We now have a linear programming problem of the usual form if Δx_j were restricted to be non-negative, as it is required by the theory of linear programming. This is simply accomplished by replacing Δx_j with two sets of variables. We let

$\Delta^{+}x = \Delta x$ when $\Delta x > 0$ and

$\Delta^{-}x = - \Delta x$ when $\Delta x < 0$

Finally, allowing for the above analysed replacements, an objective function is given by

$$y = y^0 + \sum_{j=1}^{n} c_j \, \Delta^{+} x_j - \sum_{j=1}^{n} c_j \, \Delta^{-} x_j \qquad j = 1, \ldots\ldots n \qquad \text{(A.4)}$$

subject to constraints of the form

$$g_i = g_i^0 + \sum_{j=1}^{n} w_{ij}\, \Delta^+ x_j - \sum_{j=1}^{n} w_{ij}\, \Delta^- x_j \begin{Bmatrix} \le \\ - \\ = \end{Bmatrix} b_i \qquad i = 1, \ldots\ldots m \tag{A.5}$$

and with moving limits redefined as

$$p_j\ \Delta^+ x_j\ + q_j\ \Delta^- x_j\ \le k_{3j} \tag{A.6}$$

where $p_j = \max\left[1, k_{3j} \Big/ \left(k_{2j} - x_j^0\right)\right]$ and $q_j = \max\left[1, k_{3j} \Big/ \left(x_j^0 - k_{1j}\right)\right]$, $j = 1, \ldots, n$.

This formulation ensures that the maximum movement distance of any x_j is not greater than k_{3j}. Furthermore, if a variable is near its upper or lower limit, large values of p_j and q_j are generated to keep the variable from exceeding the limit, while allowing movement away from the limit at the same time.

Appendix B Two-Phase Simplex Method

The method is divided in two phases. In the first phase, the system of constraints is first augmented by artificial variables which are constrained to be non-negative, as follows:-

$$
\begin{array}{llllll}
a_{11}x_1 + a_{12}x_2 + \ldots & + a_{1n}x_n + x_{n+1} & & & = b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots & + a_{2n}x_n & + x_{n+2} & & = b_2 \\
\ldots & & & & \\
\ldots & & & & \\
a_{m1}x_1 + a_{m2}x_2 + \ldots & + a_{mn}x_n + & \ldots & + x_{n+m} & = b_m
\end{array} \qquad \text{(B.1)}
$$

Having created the artificial variables in order to produce a set of equality constraints, they must now be removed from the Simplex table. Their removal is accomplished not by considering at first the original objective function, but by minimising the infeasibility form W defined by

$$W = x_{n+1} + x_{n+2} + \ldots + x_{n+m} \qquad \text{(B.2)}$$

To eliminate the artificial variables from the system we will subtract the first equation in system (B.1) from equation (B.2), then the second, third, and so on. The infeasibility form then becomes

$$W - \sum_{i=1}^{m} b_i = -x_1 \sum_{i=1}^{m} a_{i1} - x_2 \sum_{i=1}^{m} a_{i2} - \ldots - x_n \sum_{i=1}^{m} a_{in} \qquad \text{(B.3)}$$

Rearranging equation (B.3) to yield

$$-W = x_1 \sum_{i=1}^{m} a_{i1} + x_2 \sum_{i=1}^{m} a_{i2} + \ldots + x_n \sum_{i=1}^{m} a_{in} - \sum_{i=1}^{m} b_i \qquad \text{(B.4)}$$

Minimising the function is same as maximising its negative, hence we have

$$\max(-W) = \min W = \min\left(x_1 \sum_{i=1}^{m} a_{i1} + x_2 \sum_{i=1}^{m} a_{i2} + \quad . \quad . \quad . \quad + x_n \sum_{i=1}^{m} a_{in} - \sum_{i=1}^{m} b_i \right) \qquad \text{(B.5)}$$

Phase *I*, representing the minimisation of the infeasibility form, is terminated if one of two situations are reached

1) The value of $W = 0$ and all artificial variables have been removed from the Simplex table. In this case the conditions for obtaining a feasible solution have been reached, and we can proceed with Phase *II*. In this phase, the original objective function is optimised using standard methodology of the Simplex method.
2) The value of $W \neq 0$ and one or more of the artificial variables are refusing to leave the Simplex table. This is signified by the coefficients of x_i in the equation (B.5) becoming less or equals zero. This indicates that the problem is infeasible and the algorithm will be terminated.

It is important to notice that the removing of the artificial variables is essential. If the artificial variables refuse to leave the table, a feasible answer cannot be obtained. The author has found that non-feasible solutions can in general be overcome by changing the combination of move limits, selecting a new initial design point, or by relaxing the upper bound restrictions.

Appendix C Least Squares and Revised Method for Determining r_α

C.1 *Least Squares Method*

As shown in Chapter 5, for the compression failure zone ($K \geq 1$), the proposed stress factorised value of the reinforcement ratio α is presented by

$$\alpha = \left[\frac{N}{bhf_{cu}} - 0.45\right]\frac{1}{0.87} + r_\alpha \frac{M}{bh^2 f_{cu}} \tag{C.1}$$

Let

$$y_i = \left[\frac{N}{bhf_{cu}} - 0.45\right]\frac{1}{0.87}$$

and

$$x_i = \frac{M}{bh^2 f_{cu}} \tag{C.2}$$

Equation (C.1) can now be expressed as

$$\alpha_i = y_i + r_\alpha x_i \tag{C.3}$$

Using equation (C.3) the sum of the squares of the errors S can be expressed as

$$S = \sum_{i=1}^{n} \left(\alpha_i - y_i - r_\alpha x_i\right)^2 \tag{C.4}$$

This function has a stationary point when

$$\frac{\partial S}{\partial r} = -2\sum_{i=1}^{n} x_i\left(\alpha_i - y_i - r_\alpha x_i\right) = 0 \tag{C.5}$$

Furthermore, this stationary point is the minimum of the function, as we have

$$\frac{\partial^2 S}{\partial r^2} = 2x_i^2 > 0 \tag{C.5a}$$

Rearranging equation (C.5) gives

$$\sum_{i=1}^{n} x_i y_i + r_a \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i \alpha_i \tag{C.6}$$

Solving for r_a gives

$$r_\alpha = \frac{\sum_{i=1}^{n} x_i \alpha_i - \sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} \tag{C.7}$$

C.2 *Revised Method*

The following equation for the estimate of the initial value r_a is proposed

$$r_\alpha = r_g + \left[r_p - r_g\right] a_1^3 \frac{1}{\left(N / bhf_{cu}\right)^3} \frac{a_2}{\left(M / bh^2 f_{cu}\right)} \tag{C.8}$$

where r_g is the global value of r determined using the principle of least squares and r_p is the value of r at a point P located at the intersection of the K=1 line and zero reinforcement contour, as shown in Figure C.1. Furthermore, a_1 and a_2 are the values of N/bhf_{cu} and M/bh^2f_{cu} at the point P, respectively.

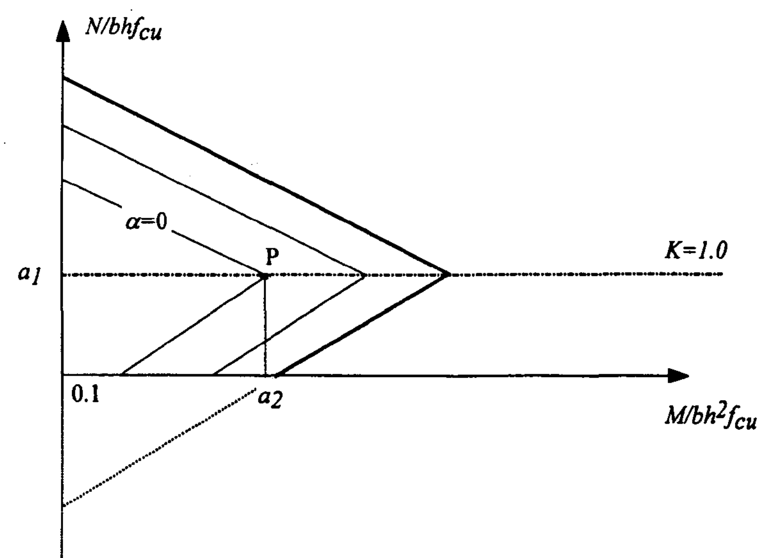


Figure C.1 *Standard Column Design Graph*

Expanding equation (C.8) and substituting $n = N/bhf_{cu}$ and $m = M/bh^2f_{cu}$, gives

$$r_\alpha = r_g\left(1 - a_1^3 a_2 \frac{1}{n^3}\frac{1}{m}\right) + a_1^3 a_2 r_p \frac{1}{n^3}\frac{1}{m} \tag{C.9}$$

Substituting n and m into equation (C.1) simplifies the expression for α to give

$$\alpha = [n - 0.45]\frac{1}{0.87} + r_\alpha m \tag{C.10}$$

Substituting equation (C.9) into equation (C.10) gives

$$\alpha = [n - 0.45]\frac{1}{0.87} + r_g\left(m - a_1^3 a_2 \frac{1}{n^3}\right) + a_1^3 a_2 r_p \frac{1}{n^3} \tag{C.11}$$

The value of r_g is now estimated using the principle of least squares, with the following substitutions

$$y_i = \left[\frac{N}{bhf_{cu}} - 0.45\right]\frac{1}{0.87}; \quad x_i = \frac{M}{bh^2 f_{cu}}; \quad z_i = a_1^3 a_2 r_p \frac{1}{n^3} \tag{C.12}$$

The sum of the squares of the errors S is then given by

$$S = \sum_{i=1}^{n}\left(\alpha_i - y_i - r_g x_i - z_i\right)^2 \tag{C.13}$$

The function has a stationary point when

$$\frac{\partial S}{\partial r} = -2\sum_{i=1}^{n} x_i\left(\alpha_i - y_i - r_g x_i - z_i\right) = 0 \tag{C.14}$$

Hence, the final estimate of the initial value of r is given by

$$r_g = \frac{\sum_{i=1}^{n} x_i\alpha_i - \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i z_i}{\sum_{i=1}^{n} x_i^2} \tag{C.15}$$

Appendix D Cost Optimisation Sensitivity Study

Design Example 1 *Three Span Continuos Beam*

Figure D.1 shows the three span continuos beam encountered in section 5.5.1. The length of each span and the corresponding loads (excluding self-weight) are indicated in the figure. The partial safety factors for imposed and dead load are 1.6 and 1.4 respectively with a minimum partial safety factor of 1.0.

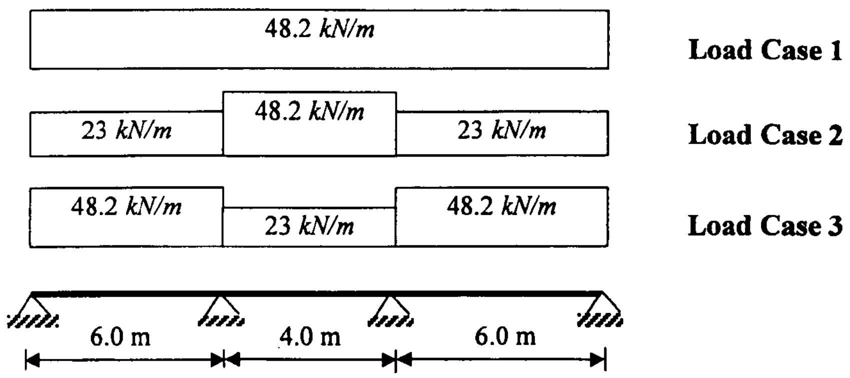


Figure D.1 *Three-Span Continuous Beam*

The lower and upper bounds of breadth and overall depth are given as 250 *mm* and 500 *mm,* and 350 *mm* and 800 *mm* respectively. Cover to reinforcement is 50 *mm* and the cost of concrete is £50/m^3. Characteristic concrete and steel strengths are f_{cu} =30 N/mm^2 and f_y =460 N/mm^2, with the modulus of elasticity for concrete taken to be 28 kN/mm^2.

Case 1 ***Initial design point close to the optimum – tight and loose move limits***

An initial design point close to the optimum combined with tight moving limits was tested in the first instance. The breadth and depth of the beams were 300 and 500 *mm* respectively, with the move limits set to be 0.1. The value of q was set to 65. The results are shown in Table D.1 below

Iteration No.	Move Limits	b (mm)	h (mm)	Reinforcement Ratio (%)	Costs (£)
7	0.1	250	437	1.344	155.73
8	**0.1**	**250**	**480.3**	**1.073**	**153.667**
9	0.1	250	528	0.782	156.02

Table D.1 *Initial design point close to optimum with tight move limits*

The optimum was achieved at the 8-*th* iteration with the program running for a total of 12 iterations. Using the same initial design point but with loose moving limits of 0.8 the optimum was achieved at the 12-*th* iteration with the program running for a total of 20 iterations. It was observed that in this instance, the use of loose move limits required more global iterations to achieve an optimum solution due to increased linearisation errors associated with the larger design space.

Case 2 ***Initial design point far from the optimum – tight move limits***

To study the convergence behaviour of the algorithm an initial design point far from the optimum was considered. The initial breadth and depth of the beams was set to be 450 and 750*mm* respectively, with the lower and upper bounds left unchanged. To restrict the effects of the linearisation errors, the move limits were set to 0.1. The results are shown in Table D.2 below

Iteration No.	Move Limit	b (mm)	h (mm)	Reinforcement Ratio (%)	Costs (£)
19	0.1	250	434	1.368	155.95
20	**0.1**	**250**	**478.3**	**1.091**	**153.61**
21	0.1	250	525	0.793	155.79

Table D.2 *Initial design point far from optimum with tight move limits*

The optimum was achieved at the 20-*th* iteration with the program running for a total of 26 iterations. Although the algorithm achieved an optimum solution, it was at the expense of a significant increase in the total number of global iterations required.

Case 3 ***Multiple Load Cases***

The continuous beam is subjected to three load cases as shown in Figure D.1. *Load Case* 1 produces maximum hogging moments at the supports whilst *Load Case* 3 produces maximum sagging moments in the middle of each end span. The results of a comparison between load Case 3 (SLC) and multiple load cases (MLC) are presented in the Table D.3 below.

q	SLC h opt (mm)	MLC h opt (mm)	SLC Costs (£)	MLC Costs (£)	Cost Difference %
25	427.76	427.76	112.49	114.31	1.592
35	427.76	427.76	123.36	125.82	1.955
45	429.03	438.76	134.62	137.22	1.895
55	471.93	456.76	144.41	147.81	2.300
65	480.30	475.50	153.67	157.64	2.518
75	494.83	495.00	162.60	167.01	2.641
85	512.64	521.14	171.14	174.64	2.004
95	530.90	539.01	179.30	182.77	1.899

Table D.3 *Cost comparison between SLC and MLC analyses*

Due to the magnitude of the member forces being similar for the single load case and the multiple load case critical envelope, the percentage cost difference is small. However, when the costs associated with *Load Case 2* were compared with the corresponding MLC costs, the differences became quite significant. This study highlights the importance of considering realistic loading conditions on a structure.

Case 4 ***Multiple Beam Groups***

The continuous beam shown in Figure D.1 was further investigated by subdividing the individual spans into two beam groups (*BG*2). These results were then compared to those obtained with one beam group (*BG*1). For the two beam groups, the external beams were allocated to the first beam group with the same upper and lower bound dimensions as in Case 1. The internal beam was allocated to the second beam group with the lower and upper bounds for breadth and depth given as 250 *mm* and 400 *mm*, and 300 *mm* and 700 *mm*, respectively. Table D.4 shows the results of the comparison.

q	Cc_{BG1} Concrete (£)	Cs_{BG1} Steel (£)	C_{BG1} Total (£)	Cc_{BG2} Concrete (£)	Cs_{BG2} Steel (£)	C_{BG2} Total (£)	$\frac{C_{BG1} - C_{BG2}}{C_{BG2}}$ %
25	85.55	26.94	112.49	84.18	25.00	109.18	3.0
35	85.55	37.81	123.36	84.18	35.00	119.18	3.5
45	85.81	48.81	134.62	85.13	43.72	128.85	4.5
55	94.39	50.02	144.41	87.77	50.28	138.05	4.6
65	96.06	57.61	153.67	91.90	54.61	146.51	4.9
75	98.97	63.63	162.60	95.05	59.48	154.53	5.2
85	102.53	68.61	171.14	95.86	66.65	162.51	5.3
95	106.18	73.12	179.30	101.30	68.07	169.37	5.9

Table D.4 *Cost comparison - one / two beam groups*

It was observed that for the low q values the percentage difference in the total costs between a one and two beam group solutions were less than those for higher values of q. As q increased, the cost difference between the two solutions showed a steady increase, from 3% to 5.9%. For all values of q, the total material cost for two beam groups is less than that of one beam group. However, in practice these cost differences have to be considered in the light of a potential increase in the formworking costs.

Design Example 2 *Three Bay - One Storey Frame*

Figure D.2 shows the heavily loaded industrial frame encountered in Section 5.5.2. The frame geometry, loading, partial safety factors, characteristic material strengths and unit material costs are as previously specified.

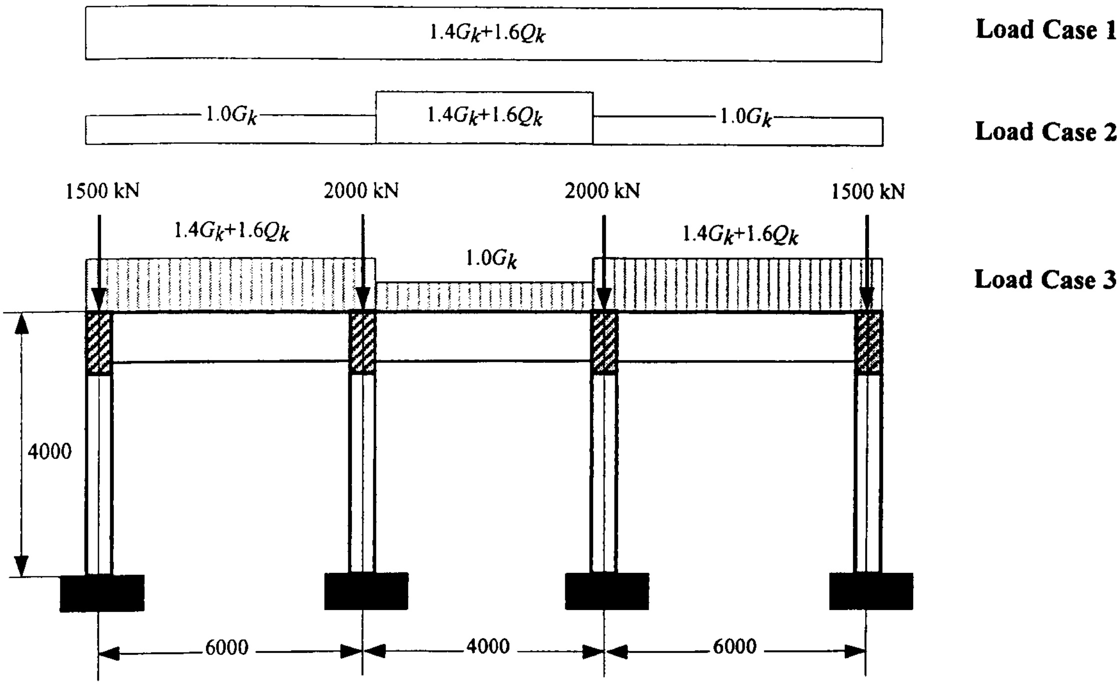


Figure D.3 *Three Bay - One Storey Frame*

Case 1 ***Initial design point close to the optimum***

An initial design point close to the optimum was tested, with b=300 *mm* and h=500 *mm* for the beam group and b=250 *mm* and h=700 *mm* for the column group. Tight moving limits (0.1) were chosen both for the beams and columns. The q value was set to be 45. Results are presented in Table D.5 below

Iteration No.	Beam Design			Column Design			Total Costs
	b (mm)	h (mm)	ρ_{opt} (%)	b (mm)	h (mm)	ρ_{opt} (%)	(£)
6	300	502	1.50	250	727	0.41	362.72
7	**300**	**503**	**1.49**	**250**	**738**	**0.4**	**362.42**
8	300	503	1.49	250	740	0.4	362.71

Table D.5 *Initial design point close to optimum with tight move limits*

The optimum was achieved at the 7-*th* iteration with the program running for a total of 9 iterations. However, when loose move limits (0.8) were applied, the optimum was achieved at the 17-*th* iteration with the program running for a total of 25 iterations. This convergence behaviour was similar to that observed for the continuous beam, with loose move limits allowing linearisation errors to increase. The use of loose move limits required in this instance searching a larger portion of the feasible region and consequently more iterations were required to achieve an optimum solution. Dynamically reducing the value of the move limits at each global iteration helped to improve convergence and reduce linearisation errors.

Case 2 ***Initial design point far from optimum***

To study the convergence behaviour of the algorithm an initial design point far from the optimum was considered. The initial depths of the beams and columns were set to 750 *mm* and 350 *mm* respectively, with the breadths kept the same as in the previous case. The lower and upper bounds for the design variables were left unchanged. The move limits were set at 0.1. The results are shown in Table D.6 below.

Iteration No.	Beam Design			Column Design			Total Costs
	b (mm)	h (mm)	ρ_{opt} (%)	b (mm)	h (mm)	ρ_{opt} (%)	(£)
27	300	502	1.50	250	727	0.41	362.72
28	**300**	**503**	**1.49**	**250**	**738**	**0.40**	**362.42**
29	300	503	1.49	250	740	0.40	362.71

Table D.6 *Initial design point far from the optimum with tight move limits*

The optimum was achieved at the 28-*th* iteration with the program running for a total of 35 iterations. Although the algorithm was capable of obtaining the optimum solution, it was at the expense of a significant increase in the total number of global iterations. Considering that these design examples are representative of all the continuous beam and structural frames investigated, a low convergence failure rate was recorded. However, it was observed that the efficiency of the cost optimisation algorithm was directly dependent on the choice of the initial design point and move limit settings.

Appendix E Genetic Algorithms

Genetic algorithms are stochastic global search and optimisation methods based on the mechanics of natural selection and genetic processes of biological organisms. Simulating the evolution principles of natural reproduction by applying selection, crossover and mutation operators to the population of possible solutions, new generations are produced containing higher proportion of the characteristics possessed by the fit members of the previous generation. Genetic algorithms systematically modify tentative solutions of a design problem scanning through the feasible population, and producing new offspring generations of improved fitness with respect to the preceding parental population. The problem of feasibility of the optimum design becomes *insignificant*, as stopping the algorithm short of reaching a real optimum still ensures a possible near-optimum solution.

E.1 Introduction

In 1859 Charles Darwin (1809-82) published a controversial book known under a shorter title as *The origin of the species*. He suggested a continual development of a species and observed a great deal of variation within population. This led him to deduce a famous *survival of the fittest* theory in which he saw an evolution as the natural selection of the offsprings with inherited variations. Following the Gregor Mendela (1822-84) examination of plant hybrids, Walter Sutton (1877-1916) discoveries about the chromosome structure and Hugo de Varis (1848-1935) theory of mutation for discontinuous variation, the foundations for the study of genetics were laid. As a consequence, in the 1960's a number of biologists have attempted to perform simulations on the genetic systems, as described by Goldberg (1989), not recognising the potentials of nature's search algorithm for application in artificial systems. It was Holland (1962), who first established genetic algorithms on a sound theoretical basis, clearly recognising the analogy between the principle of natural selection and the general optimisation in the artificial setting. Holland recognised the fundamental role of unnatural selection as an '*artificial survival of the fittest*', unambiguously endorsing a population rather than individual approach to the search in his adaptive artificial systems theory. During the second part of the 1960's and first part of the 1970's, Holland and his students developed the means of representing non-biological complex structures and operators to improve these structures, in much the same form as reproduction, crossover and mutation are known today. Finally, the important theory of schemata was developed around the turn of the decade (Holland, 1968, 1975), providing a mathematical tool for the explanation of the similarity templates for given string classes. This work has rigorously laid down the basic principles of GA's, further described and developed by many authors, such as Goldberg (1989), Davis (1987, 1991), Grefenstette (1986) and Michalewicz (1992). The nature of GA's differs fundamentally from traditional mathematical optimisation algorithms, as follows:

- Throughout the search, GA's perform operations with the *coding* of the design variables, rather than with the variables themselves
- GA's use probabilistic transition rules, not a deterministic set applied to the procedures
- GA's employ a population-to-population search, rather than searching from one individual solution to another.
- To perform a search, GA's require the objective function information only, not the derivates or any other additional knowledge.

These features, in particular the last two, make GA's a powerful tool for the optimisation of structural problems. Many traditional optimisation methods use a point-to-point movement method based on predetermined transition rules; hence running a high risk of locating false peaks in the multimodal search spaces. In contrast, GA's perform a search from a rich population of points simultaneously climbing many peaks in parallel. Thus the probability of finding a false peak is significantly reduced when compared to those methods which search point-to-point. Furthermore, the randomness of the employed search, the *blindness* of GA's to the nature of the applied structural problems, the ability to avoid derivates and linearisation errors, and their robustness in achieving a feasible solution makes them particularly attractive.

Broadly speaking, genetic algorithms are part of the larger class of evolutionary algorithms (EAs), which also include evolutionary programming (EP), evolutionary strategies (ESs) and genetic programming (GP). Often they are also classified within the wider group of modern heuristic methods, which also include simulated annealing (SA), tabu search (TS), biological growth techniques (BG) and other hybrids thereof.

E.2 Structure of Standard Genetic Algorithms

Figure E.1 shows the basic structure of the standard genetic algorithms.

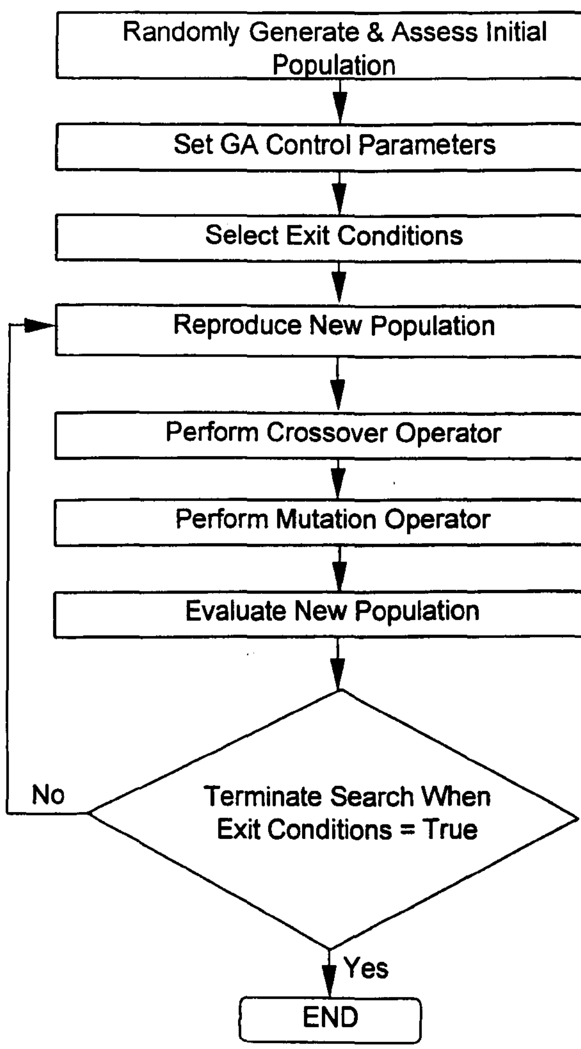


Figure E.1 *Structure of Standard Genetic Algorithms*

In order to implement this algorithm for a particular problem, a certain number of decisions must be made. These can be divided into two types; generic and problem specific. The generic decisions are mainly concerned with the setting of the GA control parameters, such as population size, crossover and mutation types together with their settings, and the conditions under which the algorithm will be declared to have reached the vicinity of the optimum solution. The problem specific decisions involve the definition of the solution space and its neighbourhood structure, the formulation of the objective function and the way in which an initial population is generated.

E.3 Population Representation and Initialisation

The most commonly used form of encoding the set of design variables for a given problem is that of a single-level binary string chromosome representation. Whilst integer and decision type of variables are easily encoded in this form, the representation of continuous design variables or control variables for combinatorial optimisation problems are not so simple. For continuous design variables, approximation with equivalent integer variables is often the most practical solution, having to compromise between accuracy and execution time. Different techniques of mapping the coded variables to the problem domain specified interval are suggested in literature as explained by Goldberg (1989). However, for some problem domains it is argued that the binary representation is in fact deceptive, obscuring the nature of the search (Bramlette 1991). Furthermore, some authors such as Wright (1991) report that the real value encoding offers a number of advantages over binary encoding for some problem domains and specific structures of phenotype. These advantages are summarised through an improved efficiency, as there is no need for constant conversion and mapping of chromosomes, less memory requirements, no loss of accuracy by discretisation to binary code and greater freedom in the use of different genetic operators. An example of this is combinatorial optimisation problems which not only requires problem specific solution encoding, but also problem specific operators to manipulate the chromosome strings.

Having decided on the representation of the problem, the next step in the implementation of the standard GA is to create an initial population. This is usually achieved by generating the required number of population members randomly within the range predefined by the desired solution space. Some authors use a controlled random initial population generation, such as Bramlette (1991) who employs random initialisation per each individual and chooses the one with best performance. Other authors, such as Grefenstette (1987) and Whitley et al. (1991) make use of an understanding of the problem beforehand, or the knowledge based system, to *seed* the initial population with superfit members known to be somewhere in the vicinity of the global optimum solution.

E.4 Population Selection

Within the GA algorithm, population selection is the process based on the natural principle of *survival of the fittest*. It involves determining the number of copies that a particular parental solution receives during the reproduction phase, and thus, the number of the corresponding offsprings. In the standard GA implementation, the most common selection schemes are fitness scaling, fitness ranking and tournament selection. The fitness scaling procedure involves defining the selection probability p_{si} of the i-th solution according to the formula:

$$p_{si} = \frac{f_i}{\sum_{i=1}^{N} f_i} \tag{E.1}$$

where f_i is the fitness of the i-th population member and N is the population size.

If the objective function is to be minimised then the probability of selection given by Equation (E.1) is modified to give:

$$p_{si} = 1 - \frac{f_i}{\sum_{i=1}^{N} f_i} \tag{E.2}$$

Once a measure of the performance is established, individuals are then selected by simulating the spinning of a suitably weighted roulette wheel N times. In the early stages of the search, a few *superfit* solutions will dominate the selection process therefore forcing premature convergence. Various schemes have been suggested to overcome this problem, such as *linear scaling, sigma (σ) truncation* or *power law scaling*. For example, in the linear scaling method, the relationship between the scaled fitness f' and the raw fitness f is established as follows:

$$f' = af + b \tag{E.3}$$

where the coefficients a and b are chosen such that the average scaled fitness f'_{avg} is equal to the raw average fitness f_{avg} , and the maximum scaled fitness f'_{max} is specified as multiple of (usually 2) the raw average fitness f_{avg}. Even so, linear scaling should be used with caution as it is possible for negative values of the scaled fitness to be introduced. Furthermore, in the case where just one, or very few *superfit* individuals are present, *overcompression* becomes a problem, as most of population will have scaled fitnesses clustered closely about 1. Therefore, fitness scaling solves the problem of premature convergence, but at the expense of flattening a fitness function. In the case of *overcompression* this leads to a slow convergence or even a drift away from the optimum. On the other hand, the ranking scheme not only gives the maximum to average fitness normalisation, but also ensures that the fitnesses of the intermediate values are regularly spread out. Therefore, the effect of *supefit* individuals is negligible and overcompression ceases to be a problem. Ranking fitness schemes are described in literature in different ways, for example, Baker (1985) suggests that the probability of the selection should be simply made a linear function of the corresponding solutions rank within the population. In this case, the best solution is usually allocated a probability of the selection of *2/N*, whilst the worst solution probability is then constrained to be a zero. Davis (1989), on the other hand, suggests an exponential ranking scheme.

The tournament selection technique forms a mating pool without the intermediate stages of fitness remaping. The simplest variant of this method is a binary tournament selection, in which pairs of individuals are randomly chosen from the population and the individual with the highest fitness is copied to the mating pool. Both individuals are then replaced in the original population and whole process repeated until the mating pool is full. Longer tournaments can be also introduced, where the best n randomly chosen individuals are copied to the pool. To avoid *selection pressure* some authors use a probabilistic tournament selection, in which better individuals win the tournament with some probability p. As reported by Goldberg (1989), other methods of the selection have been investigated, such as steady-state and proportionate selection.

An *elitist* model or *elitism* has been intensively researched in the recent years, based on the original work by De Yong (1975). He noticed in the weighted roulette wheel selection that there is no guarantee of the best solution being copied into the mating pool, although it will posses the greatest probability to do so. Therefore, in this approach, the best single or n solutions from the previous population are retained in the current one. De Yong concluded that elitism greatly improves the local search at the expense of the global perspective. Hence, care has to be taken when using this method as choosing too many individuals to be preserved may lead to premature convergence. Numerous schemes which introduce different levels of determinism in the selection process have also been investigated, as outlined by Goldberg (1989). They are mostly associated with the practical matter of reducing the stochastic error from the roulette wheel selection.

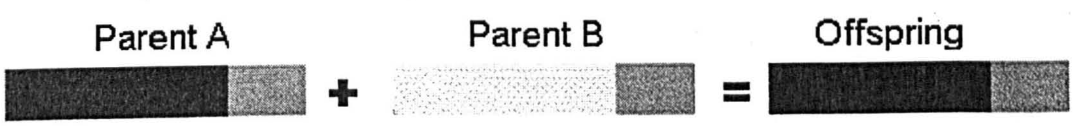
If the probabilities of the selection p_{si} are calculated then the expected number of copies E_i for i-th individual can be obtained from:

$$E_i = p_{si}\ N \tag{E.4}$$

The probability of selection p_{si} is in general a fractional number, and therefore E_i is not an integer number. The simplest approach to this problem is to round the number of the copies to the nearest integer. However, different and more sophisticated approaches are elaborated and explained by Goldberg (1989). For example, in the case of the stochastic remainder without replacement scheme the integer part of the expected number of copies is assigned directly, whilst additional copies are allocated using the remainder as probability selection criteria. For example, a string with $E_i = 1.6$ would certainly receive one copy and another one with probability of 0.6. This process then continues per each individual until the mating pool is full.

E.5 Population Crossover

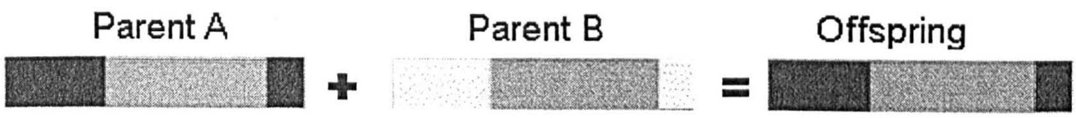
Selection procedures do not introduce any new genetic material in the population, they solely decide on the formation of a mating pool. Crossover is the operator mainly responsible for the introduction of new genetic material allowing offsprings to share some features from both parents. Crossover techniques are commonly classified in the literature as one-point, two-point, multiple, uniform and problem specific crossover. This operator is not usually applied to *all* pairs of individuals selected from the mating pool, instead, a random choice is made depending on the predetermined probability of the crossover. This gives a chance to some parental strings to pass the whole of the genetic material to the offspring by simple duplication. One-point crossover is the simplest form of this operator, in which after a random selection of the parental pair, offsprings are produced by parents exchanging the *head* and *tail* genetic material determined by the randomly selected crossover point (locus) on a string, see Figure E.2 below.



11001011+11011111 = 11001111

Figure E.2 *One-Point Crossover*

For two-point or multiple crossover, two or more randomly selected loci are introduced, and parental genetic material between these loci is exchanged forming two new offsprings. Figure E.3 below shows the exchange of genetic material in the case of two-point crossover.



11001011 + 11011111 = 11011111

Figure F.3 *Two-Point Crossover*

De Yong (1975) reported on the use of multipoint crossover operators which exchange more than one substring, and found that the GA performance degraded increasingly with the number of increased cross points. He then observed that with increased number of cross points, fewer good schemata of the parents can be preserved resulting in the *exploration* (introducing a new features through the random shuffle), rather that *exploitation* (using good features of a parental structure).

Uniform crossover is radically different to single and multi-point crossover in that it generalises their scheme to make every locus a potential crossover point, as shown in Figure E.4.

Crossover Mask	1 0 0 1 0 1 1 1 0 0
Parent 1	1 0 1 0 0 0 1 1 1 0
Offspring 1	1 1 0 0 0 0 1 1 1 1
Parent 2	0 1 0 1 0 1 0 0 1 1

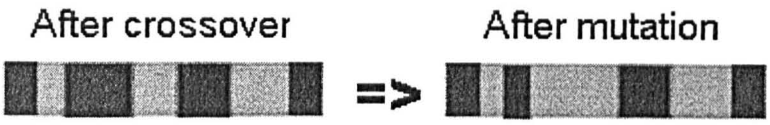
Figure E.4 *Uniform Crossover*

A *crossover mask* of the same length as the parental chromosomes is generated, with the parity of the bits in the mask indicating which parent will supply the offspring with its genetic material. Where there is 1 in the crossover mask, the gene is copied from the first parent, and where there is a 0 in the mask, a gene is copied from the second parent. The process is then repeated using the inverse of the mask, or simply by swapping the parents, to produce a second offspring.

Other variations of crossover have been developed and reported in the literature, such as *shuffle* and *reduced surrogate* crossover. In the first case, as explained by Caruana et al. (1989), hidden positional bias is removed by randomly shuffling bits of the parental strings before exchange. Bits are then exchanged with a pre-selected single cross point, and then unshuffled within the resulting offsprings. For the latter case, a *reduced surrogate* operator implements crossover which always produces new variables whenever possible, as explained by Booker (1987). Usually, this is achieved by restricting the location of the crossover point such that these points only occur where genes differ. The efficiency of crossover can also be improved by introducing dynamic probability of the crossover, which depends on either some statistical measure of the crossover performance in the previous generations, or on the predetermined probability distribution. Finally, for some practical applications, problem-specific solution representation and crossover operators have been developed to improve the performance of the GAs, as recently reviewed by Davis (1989).

E.6 Population Mutation

Usually considered as a background operator, mutation is randomly applied with a low probability of producing new genetic material by single random allele's alteration. The role of mutation is often seen as a safeguard against premature loss of important genetic material caused by the action of selection and crossover, hence maintaining diversity within the population. For instance, if every population member has 0 as the value of a particular gene, then no amount of crossover will produce an offspring with a 1 at that gene position. In standard GA's an individual for mutation is randomly chosen according to the probability of mutation p_m. In general, every single bit of this string is susceptible to a mutation. These bits are subjected to a simulated weighted coin toss with probability of gene mutation p_{mg}, and if mutation is approved, the corresponding bit will change value. For example, Figure E.5 below shows the single bit exchange that randomly takes place.



11001001 => 10001001

Figure E.5 *Random Mutation*

With non-binary representation, mutation is achieved by either perturbing the gene value or by random selection of the new values within the allowed range. Wright (1991) and Janikov and Michalewicz (1991) demonstrate how real-coded GAs can have the advantage of higher mutation rates when applied to the complex combinatorial optimisation problems, yielding significantly better solutions than the standard approach with binary coding. Many other variations on mutation are proposed, such as dynamic probability which decreases as the population converges, as explained by Foggarty (1989), *trade* mutation directed on the weaker genes proposed by Lucasius and Kateman et al. (1992), or *reorder* mutation which swaps the position of genes to increase diversity in the solution space, proposed by the same authors. Furthermore, other different techniques analysing beneficial or knowledge-augmented mutation operators are reported in the literature, and well reviewed by Goldberg (1989).

E.7 Advanced Operators

The foregoing operators are the essence of any GA's implementation. However, as natural genetics presents much more complex phenomena, many authors have explored alternative representations and operators, in an attempt to improve upon the robustness of the GA's when applied to the certain problems. These operators and techniques are well reviewed by Goldberg (1989), including *diploidy* and *dominance*, *niche* exploitation and speciation, *migration*, knowledge-augmented and problem-specific genetic operators. *Diploidy* and *dominance* are considered as low-level operators that introduce solutions represented by (several) pairs of chromosomes. The decoding of these chromosomes then depends on which bits are dominant or recessive. Such an approach allows alternative solutions to be held in *abeyance,* which can prove to be useful in certain types of problems. On the other hand, *niche* exploitation and speciation are introduced on a higher level, viewing *niche* as an organism's job or role in the environment and *species* as a class of organism with common characteristics. The concept of the *niche* exploitation and speciation is to maintain diversity in multimodal problems by introducing *stable* sub-populations of strings (*species*) serving different sub-domains of a objective function (*niches*). In order to do so, alternative selection and recombination rules are elaborated. Finally, the knowledge-augmented or intelligent control operators are explained, such as techniques which *mate* similar individuals as long as *family fitness* continues to improve, or *greedy* operators which are heavily dependent on the knowledge of the underlying problem.

E.8 Population Assessment and Constraint Handling

Since every member of the population has to be supplied with fitness function information, it is in the interest of the overall computational effort to perform these objective function evaluations efficiently. Furthermore, as most of realistic applications are constrained in some manner, basic guidelines for constraint handling are required. For continuos and well-defined solution space constrained only by equalities, the infeasible solutions can be simply *rejected* and replaced by new randomly generated individuals. If these conditions are not met, which is a common case for most practical problems with a mixture of equality and inequality constraints imposed on the disjointed solution space, then some form of penalty function approach may be used. A suitable form of the resulting augmented objective function $f_a(x)$ is:

$$f_A(x) = f(x) + G_n^{\;k}\, w^T c_v(x) \tag{E.5}$$

where w is a vector of non-negative penalty coefficients, vector c_v quantifies the magnitudes of any constraint violations and G_n is the number of current generation with

k as suitable exponent. The dependence of the penalty on the number of the current generation will bias the search increasingly heavy towards a feasible space as it progresses. However, it is important to note that the values of the penalty coefficients require extensive numerical experimentation. Furthermore, these values are also dependent on the type of the optimisation problem.

E.9 Control Parameters

Assuming that the basic features of the selection procedure are determined, the convergence rate and efficiency of the GA search will depend on the values of the control parameters. In a standard GA setting these parameters are:

- the population size, N
- the crossover probability p_c
- the mutation probability p_m

Obviously, when more advanced operators and schemes of selection are employed, the additional control parameters may be required to be predetermined. The choice of these parameters will be highly problem-dependant and therefore should be a matter of thorough investigation and experimentation. In general, a population size should not be smaller than 25, with a high probability of crossover and usually low mutation rate.

E.2.10 Termination of the GA

Since the GA is a stochastic search method, it is difficult to formally specify convergence criteria and exit conditions. The conventional termination criteria are not applicable, as the fitness of a population may remain static for a number of generations before the neighbourhood of a best solution is detected. A common practice is to terminate the GA after a pre-specified number of generations and assess the quality of the solution against the problem definition. If the solution is still unacceptable, a GA run may be continued or restarted from the beginning. An alternative stopping criteria is to limit the search time of the programme, or in some cases when the fitness approaches a certain specified value. Finally, the termination criteria in which changes in a fitness function are less than some specified small value for a predefined number of generations can be used. Care has to be taken when using this criterion to avoid premature convergence towards a local optimum.

Appendix F Column Design Equations – Exact Solution

Case 1 (f_1 = -400; f_2 = -400)

For this case the value of m is zero, and hence equation (6.30) is reduced to

$$\frac{M}{bh^2 f_{cu}} = -0.1809\left(\frac{x}{h}\right)^2 + 0.201\left(\frac{x}{h}\right) \tag{F.1}$$

Solving this quadratic equation, x/h is obtained and α calculated from

$$\alpha = \frac{1}{n}\left(\frac{N}{bhf_{cu}} - 0.402\frac{x}{h}\right) \tag{F.2}$$

The reinforcement ratio ρ_{sc} is then obtained from equation (6.29).

Case 2 (-400 < f_1 < 400; f_2 = -400)

For this case the value of m/n is derived from equation 6.32a to be

$$\frac{m}{n} = \left(\frac{d}{h} - 0.5\right)\left[1 + \frac{1.143x/h}{0.429x/h + (d/h - 1)}\right] \tag{F.3}$$

Substituting this value into equation (6.30), a cubic equation with unknown x/h is obtained as

$$(x/h)^3 + K_{12}(x/h)^2 + K_{22}(x/h) + K_{32} = 0 \tag{F.4}$$

where factors K_{12}, K_{22} *and* K_{32} are functions of N/bhf_{cu}, M/bh^2f_{cu} and d/h.

Solving this cubic equation using the Newton-Ralphson method yields the value of x/h. The reinforcement ratio ρ_{sc} is then obtained from equation (6.29).

Case 3 (f_1 = 400; f_2 = -400)

For this case the value of n is zero, and the unknown x/h is directly derived from equation (6.27) as

$$\frac{x}{h} = \frac{1}{0.402}\left(\frac{N}{bhf_{cu}}\right) \tag{F.5}$$

α is calculated from equation (6.28) as follows

$$\alpha = \frac{1}{m}\left[\frac{M}{bh^2 f_{cu}} - 0.402\frac{x}{h}\left(0.5 - 0.45\frac{x}{h}\right)\right] \tag{F.6}$$

The reinforcement ratio ρ_{sc} is then obtained from equation (6.29).

Case 4 ($f_1 = 400$; $-400 < f_2 < 400$)

The procedure is same as in Case 2 with the expression for m/n derived from equation (6.32a) as

$$\frac{m}{n} = \left(\frac{d}{h} - 0.5\right)\left(\frac{-0.429x/h + d/h}{1.571x/h - d/h}\right) \tag{F.7}$$

Following the procedure given in Case 2, a cubic equation with unknown x/h is obtained as

$$(x/h)^3 + K_{14}(x/h)^2 + K_{24}(x/h) + K_{34} = 0 \tag{F.8}$$

where factors K_{14}, K_{24} *and* K_{34} are functions of N/bhf_{cu}, M/bh^2f_{cu} and d/h.

Solving the cubic equation, both α and the reinforcement ratio ρ_{sc} are obtained as explained in Case 2. The solution is valid for any value of x/h that is less than 1.111. When x/h exceeds this value, the concrete stress block covers the whole section and hence there is no moment from the stress block. This special case is valid for a partial range of x/h applicable to Case 4 and for the whole range of x/h values in Case 5. The solution to this case is explained in the following section.

Case 5 ($f_1 = 400$; $f_2 \leq 400$)

When x/h exceeds 1.111, the equation (6.27) and (6.28) become

$$\frac{N}{bhf_{cu}} = 0.4466 + \alpha$$

$$\frac{M}{bh^2 f_{cu}} = m\alpha \tag{F.9}$$

where

$$n = (1+1.15f_2/f_y)/2.3$$
$$m = (2d/h - 1)(1-1.15f_2/f_y)/4.6 \tag{F.10}$$

By multiplying the second equation in (F.10) by n/m and deducting it from the first equation, we derive

$$\frac{N}{bhf_{cu}} - \frac{n}{m}\frac{M}{bh^2 f_{cu}} = 0.4466 \tag{F.11}$$

The ratio n/m is then calculated to be

$$\frac{n}{m} = \frac{(1+0.0025f_2)}{(d/h-0.5)(1-0.0025f_2)} \tag{F.12}$$

Substituting this expression in equation (F.11) and solving for f_2 we obtain the unknown value of the stress in bottom reinforcement. The values of n and m now can be calculated and α is obtained from

$$\alpha = \frac{1}{n}\left(\frac{N}{bhf_{cu}} - 0.4466\right) \tag{F.13}$$

Appendix G Simulated Annealing

Simulated annealing is a stochastic relaxation technique which is based on the analogy to the physical process of annealing a metal. Molten metal cooled rapidly will often solidify into a phase that is not the lowest energy state for the final temperature. However, cooled slowly it stands a far better chance of finding the lowest energy phase, as thermal motion over longer periods of time allows the system to explore many more configurations.

G.1 Introduction

The inspiration for simulated annealing is the *law of thermodynamics* which states that at temperature, T, the probability P of an increase in energy of magnitude, δE, is given by

$$P[\delta E] = \exp\left(\frac{-\delta E}{kT}\right) \tag{G.1}$$

where k is the physical constant known as *Boltzmann's constant.*

This equation can be used in simulation of a system that is cooling until it converges to a steady, "frozen" state. Having generated a perturbation from the current state, the resulting energy change is assessed and the new system is directly accepted if the energy has decreased. However, if the energy has increased, the new system is accepted according to the probability given in equation (G.1). This cycle is then repeated for a fixed number of iterations after which the temperature is decremented. The same number of cycles is repeated for the new lower temperature, and this whole process is then repeated until the system freezes into its steady final state.

The solution to a general optimisation process can be associated with this system states behaviour. The cost of a structure corresponds to the concept of energy and moving to any new set of design variables corresponds to a change of state. Simulated annealing randomly generates new configurations by sampling from the probability distribution of the system. It employs a random search which not only accepts changes that decrease the objective function, but also changes that increase it. The latter are accepted with probability

$$p = \exp\left(\frac{-\delta f}{T}\right) \tag{G.2}$$

where δf is the increase in the objective function and T is the system temperature. The expression in equation (G.2) is also known as *Boltzmann's probability distribution.*
This feature of simulated annealing algorithms is considered to be their major advantage, making them less susceptible to the premature convergence towards a local optimum.

G.2 Solution Generation and Evaluation

Implementing simulated annealing requires a representation of the problem variables, a definition of the solution space and its neighbourhood structure, a choice of acceptance (sampling) probability, the structure of the objective function and the nature of the random solution generator. It is important to adopt an efficient strategy in producing new trial solutions whilst considering a problem dependant representation of the design variables and the structure of the solution neighbourhood. For problems with continuous variables a number of authors, such as Vanderbilt and Louie (1984), propose methods which generate new trial solutions on the random principle employing the matrix which controls step size distribution. A drawback of these methods is that they require the constant updating of a covariance matrix by solving a system of equations. This can be a substantial computational overhead especially for problems with high dimensionality.
Parks (1990) suggested a more efficient strategy that generates solutions according to the formula:

$$\boldsymbol{x}_{i+1} = \boldsymbol{x}_i + \boldsymbol{Du} \tag{G.3}$$

where $\boldsymbol{u}$ is a vector of random numbers in the range (-1,1) and $\boldsymbol{D}$ is the diagonal matrix which defines the maximum change allowed in each variable. The value of $\boldsymbol{D}$ is then updated after each successful trial according to the formulae:

$$\boldsymbol{D}_{i+1} = (1-\alpha)\boldsymbol{D}_i + \alpha\omega\boldsymbol{R} \tag{G.4}$$

where $\boldsymbol{R}$ is a diagonal matrix with elements consisted of the magnitudes of the successful changes made to each control variable, and α is the damping constant which controls the rate at which information from $\boldsymbol{R}$ is folded into $\boldsymbol{D}$ with weighting ω. This procedure is responsible for tuning the maximum step size associated with each control variable towards a value giving acceptable changes. The probability p of accepting an increase in objective function f is given by:

$$p = \exp\left(\frac{-\delta f}{T\bar{d}}\right) \tag{G.5}$$

where $\bar{d}$ is the average step size, so that $\delta f / \bar{d}$ is a measure of the effectiveness of the change made. As the size of step taken is considered in calculating p, $\boldsymbol{D}$ does not need to be adjusted when the temperature of the system $\boldsymbol{T}$ is changed.

G.3 Structure of Simulated Annealing Algorithm

Figure G.1 shows the basic structure of the standard simulated annealing algorithm. To implement this algorithm for a particular problem a certain number of generic and problem specific decisions must be made.

The generic decisions are mainly concerned with controlling the temperature of the system including the determination of its initial value, the temperature decrement function, the number of iterations at the current temperature and the conditions under which the system will be declared 'frozen'. The problem specific decisions involve the definition of the solution space and its neighbourhood structure, the formulation of the cost objective function and the way in which an initial solution is generated.

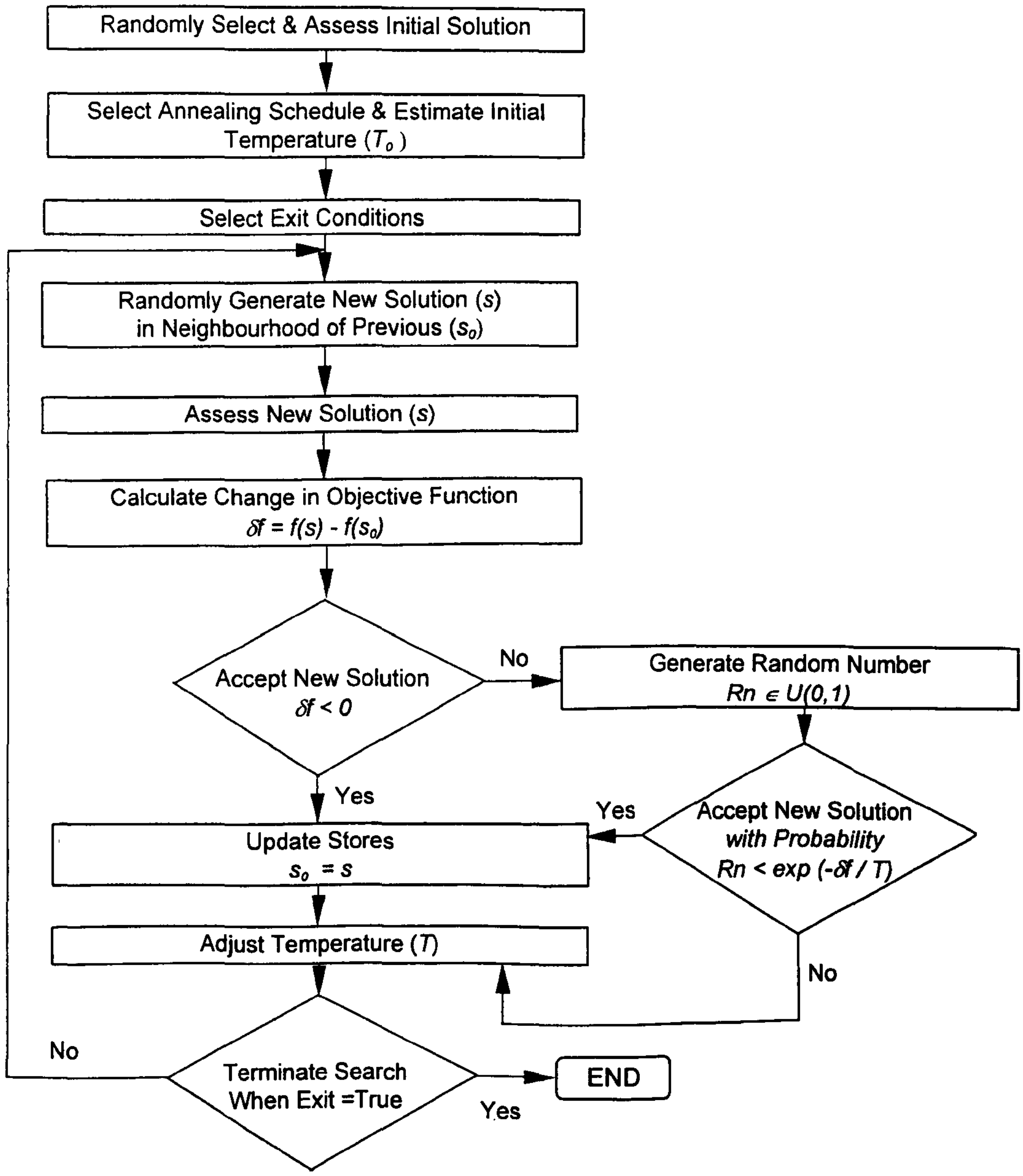


Figure G.1 *Structure of Simulated Annealing Algorithm*

G.4 *Annealing Schedule*

A suitable annealing schedule is one in which the initial temperature (T_0) should be high enough to 'melt' the system completely. Cooling should be sufficiently slow to allow the system to explore an adequate amount of possible configurations to find the lowest energy phase. The standard implementation of the simulated annealing algorithm is one proposed by Kirkpatrick et al. (1982), in which homogeneous Markov chains of finite length are generated at decreasing temperatures (T_k).

G.5 *Initial Temperature*

The initial temperature of the system is determined in such a way that virtually all transitions are accepted at that temperature. Kirkpatrick et al. (1982) propose an empirical rule suggesting that a suitable initial temperature is one that results in an average increase acceptance probability of $\aleph$=0.8. The value of T_0 will clearly depend on the scaling of f, hence it will be problem specific. It can be estimated by conducting an initial random search in which all increases of objective function $\overline{\partial f}^{+}$ are accepted, as given by

$$T_0 = -\frac{\overline{\partial f}^{+}}{\ln(\aleph_0)} \tag{G.6}$$

This idea is further refined by a number of authors. For example, Jonson et al. (1987) determine T_0 by calculating the average increase in the cost for a number of random transitions, whilst White (1984) uses an approach based on the configuration density function.

G.6 *Final Temperature*

In simple implementations of the SA algorithm the final temperature can be determined by fixing the number of temperature values to be used, as stated by Nahar et al. (1985). A convenient stop criterion could also be the total number of solutions to be generated. Alternatively, the search can be halted when it ceases to make progress, as outlined by Jonson et al. (1987). They define the lack of progress as being when no improvement is found in an entire Markov chain at one temperature, combined with the acceptance ratio falling below a given (small) value η_{min}. More elaborate cooling schedules determine the final temperature using other sophisticated approaches, such as extrapolation of the average costs of configurations over a number of consecutive Markov chains, outlined by Aarts and Van Laarhoven (1985), or using iterative improvements approach explained by Huang et al. (1986).

G.7 *Length of Markov Chains*

The simplest choice for the length of the Markov chain L_k is a value which depends (polynomially) on the size of the problem. Thus L_k is independent of the current value of the control parameter T_k. This choice is made by many authors, such as Bonomi and Lutton (1984) and Burkand and Rendl (1984). More elaborate proposals for the length of the Markov chain are based on the argument that a minimum number of transitions should be accepted at each temperature T_k. However, as temperature approaches zero, transitions are accepted with decreased probability and thus L_k eventually approaches infinity. Consequently, the length of the Markov chain is limited by some constant, and in practice an algorithm is terminated after L_k transitions or η_{min} acceptances, whichever comes first. Rules of this type are proposed by a number of authors, such as Kirkpatrick et al. (1982), Jonson et al. (1987) and Leong et al. (1985). Other approaches are also considered, such as one by Nahar et al. (1985), determining the length of Markov chains by limiting the number of rejected transitions.

G.8 *Decrementing the Temperature*

A frequently used decrement rule, first proposed by Kirkpatrick et al. (1982), is given by

$$T_{k+1} = \alpha\, T_k \tag{G.7}$$

where α is the constant close to, but smaller than 1. This exponential cooling scheme is widely used by other authors, such as Jonson et al. (1987), Bonomi and Lutton (1984), Burkard and Rendl (1984) and Leong et al. (1985). A variety of other approaches has been explored by different authors. For example, Huang et al. (1986) based their decrement rule on the average cost values of consecutive Markov chains, whilst Randelman and Grest (1986) explore benefits of linear cooling schemes in which T is reduced after every L trials. Many researchers have proposed more elaborate schemes, dealing with variable decrement of the control parameter, statistical measures of the algorithm's current performance or deriving their schemes partially based on experimental observations. These methods are well reviewed by Van Laarhoven and Aarts (1987).

G.9 *Constraints Handling*

Simulated annealing is in principle a non-constrained optimisation solver and therefore an efficient approach to constraint handling is required. In many cases, simple rejection of any proposed changes which violate constraints can be successfully incorporated into the algorithm, resulting only in the search of the feasible domain. However, this simple approach has serious limitations not being applicable to equality constraints and disjoint feasible space environments. A more efficient approach reported in the literature (*see* Section 2.4.3), is to transform the original constrained problem into an unconstrained one, by constructing an augmented objective function which incorporates any constraint violation as a penalty to the original function

$$f_A(x) = f(x) + \frac{1}{T} w^T c_v(x) \tag{G.8}$$

where w is a vector of non-negative penalty coefficients and the vector c_v quantifies the magnitudes of any constraint violations.

The inverse dependence of the penalty on temperature will bias the search increasingly heavily towards a feasible space as it progresses. It is important to note that the values of the penalty coefficients are determined after extensive numerical experimentation and are problem dependant.