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Application of Caputo–Fabrizio operator to suppress the Aedes Aegypti mosquitoes via Wolbachia: An LMI approach

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Abstract

The aim of this paper is to establish the stability results based on the approach of Linear Matrix Inequality (LMI) for the addressed mathematical model using Caputo–Fabrizio operator (CF operator). Firstly, we extend some existing results of Caputo fractional derivative in the literature to a new fractional order operator without using singular kernel which was introduced by Caputo and Fabrizio. Secondly, we have created a mathematical model to increase Cytoplasmic Incompatibility (CI) in Aedes Aegypti mosquitoes by releasing Wolbachia infected mosquitoes. By this, we can suppress the population density of A.Aegypti mosquitoes and can control most common mosquito-borne diseases such as Dengue, Zika fever, Chikungunya, Yellow fever and so on. Our main aim in this paper is to examine the behaviours of Caputo–Fabrizio operator over the logistic growth equation of a population system then, prove the existence and uniqueness of the solution for the considered mathematical model using CF operator. Also, we check the α-exponential stability results for the system via linear matrix inequality technique. Finally a numerical example is provided to check the behaviour of the CF operator on the population system by incorporating the real world data available in the known literature.

Keywords: Caputo–Fabrizio operator; Wolbachia; Aedes aegypti; Mosquito borne disease control

1. Introduction

Fractional order derivatives are extended from the integer order derivative to get the congenital characteristics and memory property of some more complicated problems in scientific and engineering fields and also in fractional order derivative we can use arbitrary order. Due to these properties the fractional order became more stronger and useful than that of the integer order derivatives, please see Refs. [23,37,38,42]. In recent years, researchers take

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more interest to solve the initial and boundary value problems via fractional order derivatives. At first the fractional order derivative is proposed by Riemann–Liouville [42] after that many people proposed a type of fractional order derivatives like Caputo and Grunwald–Letnikov [37]. In recent years, there are some new fractional order operators proposed by Caputo and Fabrizio without non-singular kernel [9] and Atangana and Baleanu with non-local and non singular kernel [5]. The new Caputo–Fabrizio operator (or CF operator) is distinct from the classical Caputo derivative in two aspects. One is, it only partially depends on the past and another one is, it is linearly increasing and diverging. One can refer the article [29] to get lots of results and information of CF operator. Also it is more useful to describe the real phenomena [6]. Because of these wide range of applications of CF operator we chose this particular operator throughout this paper. In the existing literature [30,39,41], the stability results of various types of neural network systems (integer and fractional order) were solved via linear matrix inequality techniques. In our work, we aimed to introduce the LMI based stability results into the fractional order dynamical systems with CF operator.

Our main aim is to check the behaviours of this CF operator on population systems. The characterization of fractional derivative over a logistic growth equation is derived in [14]. In [26,28], using the CF operator the stability results of fractional order system with delay and without delay are derived through Laplace transform and matrix theory. They proved up to the asymptotic stability results of a CF operator. In our paper, we proved that the CF operator is α-exponentially stable. In [12,22], some real world problems like pine wilt disease model and cancer treatment model are solved and the existence and uniqueness of the solutions via CF operator were analysed.

In another part of the paper, we mainly focused to find the optimal control technique for mosquito borne diseases. Mosquito borne diseases represent the vertical transmission of bacteria and viruses from mosquitoes to human while taking a blood meal. Mosquito borne diseases such as Dengue, Chikungunya, Yellow fever, Zika virus, Japanese encephalitis etc., cause over one million deaths per annum [7,10,17,18]. The primary vector for most of the mosquito borne diseases is Aedes Aegypti and recently Aedes albopictus also added as a secondary vector. More than that, Dengue causes 20000 deaths all over the world [16,19,24,35,48,49].

In recent years, there are several articles about to control vectors by genetic modifications. For instance, the authors of [4,8,15,20,21,32,43,44] discussed some biological control methods to replace the wild mosquitoes by releasing genetically modified mosquitoes. Those biological control methods are, sterilization of male mosquitoes, genetic modifications and Wolbachia release (to reduce the reproduction) see [36]. There are some other methods to control mosquito borne diseases. For example, bed nets, mosquito repellents, chemical insecticides, mosquito traps, and so on. For instance, in [3,31,33], the authors tried some other type of control agents like, bed nets, mosquito repellents, indoor residual spray, condoms during sex, by medically treating infected human, quarantine, make modifications in feeding behaviours of a vector and so on. Our main aim is to control the mosquito borne diseases via biological control. In our work, the Wolbachia pipientis an endosymbiotic bacterium is used to stop the vertical and horizontal transmissions of viruses. This method is practically done by group of people in Australia called World Mosquito Program (see [1]). The world mosquito program is first established in Australia in the year 2011. And this method is implemented in 12 countries including India. In [13], the authors considered the Wolbachia bacteria as a biological control agent to increase CI. More than that, Wolbachia has a special quality that, it can block the virus particles inside the salivary gland itself. Because of these properties, Wolbachia can be used as a biological control to eradicate mosquito-borne diseases. Supriatna et al. in [46], used Wolbachia as a control agent for Dengue fever and analysed the model via control theory. Along with this, in [27], the authors discussed the birth, death rate impulsive model to control mosquito-borne diseases using Wolbachia via Stroboscopic map method. Furthermore, via finding reproduction number of a mathematical model which depicts the virus transmission via human sexual contact was analysed in [2]. The integer order mathematical model which describes the interplay among the wild and wolbachia infected mosquitoes was analysed in [40]. In that work, the author divide the mosquito population into two groups one is aquatic and another one is adult. In [11], the author used Wolbachia as a biological control and created a delayed mathematical model, by using positive systems theory and spectrum analysis the author proved the stability results of the proposed model. By practical results of [1], we can release the wolbachia infected eggs, larvae, pupae and wolbachia infected adult mosquitoes. So to obtain a optimal control, we have to release the wolbachia in all stages. Due to these conditions only, we have created a mathematical model which depicts the full life cycle of Aedes aegypti mosquitoes. Motivated by the above arguments, the main aim of this paper is to introduce the LMI approach to fractional order systems with CF operator and find the application for the proposed methods.

The essential theme of this paper lies in the following aspects:

(i) This paper is mainly concentrated on two concepts. One is to create an appropriate mathematical model to control the mosquito borne diseases and another one is to derive and check the essential properties of Caputo–Fabrizio operator. Some important theorems and lemmas are extended to Caputo–Fabrizio operator.
(ii) New fractional order mathematical model which depicts the interplay among the wild mosquitoes and wolbachia infected mosquitoes using CF operator is proposed. And examined the existence and uniqueness of the solution of the created mathematical model.
(iii) There is no literature which considers the biological control in all stages. By world mosquito program, we can release wolbachia infected eggs and larvae in the form of 'Zancu kits' and also we can release the adult male and female wolbachia infected mosquitoes into the wild mosquitoes. By considering these reasons we optimized the control by releasing wolbachia infection in all stages.
(iv) The α-exponential stability result of the created mathematical model is examined via LMI approach.
(v) Finally, by using real world data, we checked the stability results by MATLAB LMI tool box.

This paper is organized as follows: In Section 2, we provide some basic definitions and notations which are used later in this paper. And also in this section we have extended some important theorems and lemmas to CF operator. In Section 3, the fractional order mathematical model of wild mosquitoes and interplay among the wild and wolbachia infected mosquitoes is proposed and the existence and uniqueness of the solution is also derived for the proposed model. In Sections 4 and 5, we present existence and uniqueness of the solution respectively. In Section 6, the α-exponential stability results are provided via LMI approach. The numerical examples are presented in Section 7. Finally, we concluded this work in Section 8.

2. Preliminaries

In this section, we provide some basic tools of Caputo–Fabrizio operator. From this, we have extended some properties, theorems and lemmas which were proposed by Podlubny [25] to Caputo–Fabrizio operator.

Definition 2.1. The Caputo–Fabrizio operator for the function $g \in \mathcal{H}^1(a,b)$, $0 < \alpha < 1$ is defined by Caputo & Fabrizio in [9] (2015), as

$$ {}_{a}^{CF}D_t^{\alpha} g(t) = \frac{M(\alpha)}{1-\alpha}\int_a^t g'(\tau) exp\left[\frac{-\alpha(t-\tau)}{1-\alpha}\right] \mathrm{d}\tau, \tag{1}$$

and for $g \notin \mathcal{H}^1(a,b)$, $\leq \alpha \leq 1$ as

$$ {}_{a}^{CF}D_t^{\alpha} g(t) = \frac{\alpha M(\alpha)}{1-\alpha}\int_a^t (g(t)-g(\tau)) exp\left[\frac{-\alpha(t-\tau)}{1-\alpha}\right] \mathrm{d}\tau, \tag{2}$$

where, $\mathcal{H}$ is a Sobolev space; $M(\alpha)$ is a normalization function with $M(0) = 1 = M(1)$. Normalization function means, to make the value of the function takes between 0 to 1 for that we can add or multiply by constants in that function. Here, the normalization function is not depending on τ.

Throughout this paper, ${}_{a}^{CF}D_t^{\alpha}$ denotes the Caputo–Fabrizio operator of order α with the initial condition a, and we use CF as an abbreviation of Caputo–Fabrizio operator.

In [29], $M(\alpha) = \frac{2\alpha}{2-\alpha}$, $0 \leq \alpha \leq 1$. By considering this, the author modified the CF operator as

$$ {}_{a}^{CF}D_t^{\alpha} g(t) = \frac{1}{1-\alpha}\int_a^t exp\left[\frac{-\alpha(t-\tau)}{1-\alpha}\right] \mathrm{d}\tau\,. \tag{3}$$

Definition 2.2. Nieto et al. [29], derived the integral of CF operator as,

$$ {}_{a}^{CF}I_t^{\alpha} g(t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}u(t) + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t g(s)\,\mathrm{d}s,\ t \geq 0, \tag{4}$$

and $0 < \alpha < 1$.

The general form of the Laplace transform of CF operator is defined by Caputo et al. in [9], as

$$ \mathfrak{L}\{{}_{0}^{CF}D_t^{\alpha+n} g(t)\} = \frac{p^{n+1}\mathfrak{L}g(t) - p^n g(0) - p^{n-1}g'(0) - g^n(0)}{p+\alpha(1-p)}. \tag{5}$$

The Mittag Leffler function with two parameters is defined as [25]

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)}, \tag{6}$$

where, $z\in\mathbb{C}$, $\mathbb{C}$ =set of all complex numbers, $\alpha>0$, and $\beta>0$. If $\beta=1$ then $E_\alpha(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)}$. If both $\alpha=1$ and $\beta=1$, then $E_{1,1}(z)=e^z$.

Next, we prove some important theorems, properties and lemmas which are newly introduce the concepts of Lyapunov and LMI into the Caputo–Fabrizio operator.

Property 2.3. *The CF operator is linear, that is, for constants p and q,*

$${}_0^{CF}D_t^\alpha\Big(pf(t)+qg(t)\Big)=p\,{}_0^{CF}D_t^\alpha f(t)+q\,{}_0^{CF}D_t^\alpha g(t). \tag{7}$$

Proof. For $0<\alpha<1$, by Definition 2.1

$${}_0^{CF}D_t^\alpha f(t)=\frac{M(\alpha)}{1-\alpha}\int_{t_0}^t f'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\text{ds}$$

$$\begin{aligned}{}_0^{CF}D_t^\alpha\Big(pf(t)+qg(t)\Big)&=\frac{M(\alpha)}{1-\alpha}\int_{t_0}^t\Big(pf(t)+qg(t)\Big)'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\text{ds}\\&=\frac{M(\alpha)}{1-\alpha}\int_{t_0}^t\Big(pf'(t)+qg'(t)\Big)(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\text{ds}\\&=\frac{M(\alpha)}{1-\alpha}\int_{t_0}^t pf'(t)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\text{ds}\\&\quad+\frac{M(\alpha)}{1-\alpha}\int_{t_0}^t qg'(t)(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\text{ds}\\&=p\frac{M(\alpha)}{1-\alpha}\int_{t_0}^t f'(t)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\text{ds}\\&\quad+q\frac{M(\alpha)}{1-\alpha}\int_{t_0}^t g'(t)(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\text{ds}\\&=p\,{}_0^{CF}D_t^\alpha f(t)+q\,{}_0^{CF}D_t^\alpha g(t).\end{aligned}$$

Hence, the linearity property is true for Caputo–Fabrizio operator. □

Lemma 2.4. *Let $z(t)\in\mathbb{R}$ ($\mathbb{R}$ = set of all real numbers) be a continuous and derivable function. Then for any time instant $t\geq t_0$ the following inequality holds,*

$$\frac{1}{2}{}_0^{CF}D_t^\alpha z^2(t)\leq z(t){}_0^{CF}D_t^\alpha z(t),\ for\ all\ \alpha\in(0,1). \tag{8}$$

Proof. It is equivalent to prove that

$$z(t){}_0^{CF}D_t^\alpha z(t)-\frac{1}{2}{}_0^{CF}D_t^\alpha z^2(t)\geq 0. \tag{9}$$

By using Definition 2.1, ${}_0^{CF}D_t^\alpha z(t)=\frac{M(\alpha)}{1-\alpha}\int_{t_0}^t z'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\text{ds}$ and

$$\begin{aligned}\frac{1}{2}{}_0^{CF}D_t^\alpha z^2(t)&=\frac{1}{2}\frac{M(\alpha)}{1-\alpha}\int_{t_0}^t 2z(s)z'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\text{ds}\\&=\frac{M(\alpha)}{1-\alpha}\int_{t_0}^t z(s)z'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\text{ds}\,.\end{aligned}$$

Substituting these expressions in (9), we have

$$\frac{M(\alpha)}{1-\alpha}\int_{t_0}^{t} z(t)z'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\mathrm{ds} - \frac{M(\alpha)}{1-\alpha}\int_{t_0}^{t} z(s)z'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\mathrm{ds} \geq 0, \tag{10}$$

$$\frac{M(\alpha)}{1-\alpha}\int_{t_0}^{t} (z(t)-z(s))z'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\mathrm{ds} \geq 0. \tag{11}$$

Let us define a new variable by $x(s) = z(t) - z(s) \Longrightarrow x'(s) = -\dot{z}(s)$. Therefore, (11) becomes,

$$\frac{M(\alpha)}{1-\alpha}\int_{t_0}^{t} -x(s)x'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\mathrm{ds} \geq 0$$

$$\frac{M(\alpha)}{1-\alpha}\int_{t_0}^{t} x(s)x'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\mathrm{ds} \leq 0. \tag{12}$$

By using the integration by parts, let $u = \frac{M(\alpha)}{1-\alpha}exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]$ this implies

$\mathrm{du} = \frac{M(\alpha)}{1-\alpha}exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\left(\frac{-\alpha}{1-\alpha}\right)(-1)\,\mathrm{ds}$, $\mathrm{dv} = x(s)x'(s)\,\mathrm{ds}$ and $v = \frac{1}{2}x^2(s)$. Then substitute in the following equation,

$$\int u\,\mathrm{d}\,v = uv - \int v\,\mathrm{d}\,u$$

$$\frac{M(\alpha)}{1-\alpha}\int_{t_0}^{0} x(s)x'(s)exp\left[\frac{-\alpha(t-s)}{1-\alpha}\right]\mathrm{ds} \leq 0$$

$$\left[\frac{M(\alpha)}{2(1-\alpha)}e^{\frac{-\alpha(t-s)}{1-\alpha}}x^2(s)\right]_{t_0}^{t} - \int_{t_0}^{t}\frac{1}{2}x^2(s)\frac{\alpha M(\alpha)}{(1-\alpha)^2}e^{\frac{-\alpha(t-s)}{1-\alpha}}x^2(s)\,\mathrm{ds} \leq 0$$

$$\left[\frac{M(\alpha)}{2(1-\alpha)}e^{\frac{-\alpha(t-s)}{1-\alpha}}x^2(s)\right]\Bigg|_{s=t} - \frac{M(\alpha)}{2(1-\alpha)}e^{\frac{-\alpha(t-t_0)}{1-\alpha}}x_0^2 - \int_{t_0}^{t}\frac{1}{2}x^2(s)\frac{\alpha M(\alpha)}{(1-\alpha)^2}e^{\frac{-\alpha(t-s)}{1-\alpha}}x^2(s)\,\mathrm{ds} \leq 0.$$

Now, we find the first term of the above equation

$$\begin{aligned}\lim_{s\to t}\frac{M(\alpha)}{2(1-\alpha)}e^{\frac{-\alpha(t-s)}{1-\alpha}}[x(t)-x(s)]^2 &= \frac{M(\alpha)}{2(1-\alpha)}\lim_{s\to t}e^{\frac{-\alpha(t-s)}{1-\alpha}}[x(t)-x(s)]^2\\ &= \frac{M(\alpha)}{2(1-\alpha)}e^{\frac{-\alpha(t-t)}{1-\alpha}}[x(t)-x(t)]^2\\ &= 0\end{aligned}$$

$$\Rightarrow -\frac{M(\alpha)}{2(1-\alpha)}e^{\frac{-\alpha(t-t_0)}{1-\alpha}}x_0^2 - \int_{t_0}^{t}\frac{1}{2}x^2(s)\frac{\alpha M(\alpha)}{(1-\alpha)^2}e^{\frac{-\alpha(t-s)}{1-\alpha}}x^2(s)\,\mathrm{ds} \leq 0$$

$$\Rightarrow \frac{M(\alpha)}{2(1-\alpha)}e^{\frac{-\alpha(t-t_0)}{1-\alpha}}x_0^2 + \int_{t_0}^{t}\frac{1}{2}x^2(s)\frac{\alpha M(\alpha)}{(1-\alpha)^2}e^{\frac{-\alpha(t-s)}{1-\alpha}}x^2(s)\,\mathrm{ds} \geq 0. \tag{13}$$

Therefore, Eq. (13), is true.

$$\frac{1}{2}{}_{0}^{CF}D_t^{\alpha}z^2(t) \leq z(t){}_{0}^{CF}D_t^{\alpha}z(t),\ for\ all\ \alpha \in (0,1). \quad \square$$

Lemma 2.5 (*[45]*). *Let $z(t) \in \mathbb{R}^n$ be a continuous and derivable function. Then for any time instant $t > t_0$, the following inequality holds*

$$\frac{1}{2}{}_{0}^{CF}D_t^{\alpha}z^{\top}(t)Pz(t) \leq z^{\top}(t)P{}_{0}^{CF}D_t^{\alpha}z(t),\ for\ all\ \alpha \in (0,1), \tag{14}$$

where $P \in \mathbb{R}^{n\times n}$ is a constant positive definite matrix.

Lemma 2.6. *Let $z(t) \in \mathbb{R}^n$ be a continuous and piecewise smooth function, and $z'(t)$ is piecewise continuous. Then for any time instant $t > 0$*

$$ {}_0^{CF}D_t^{\alpha}\frac{1}{2}z^2(t) \leq z(t){}_0^{CF}D_t^{\alpha}z(t) \ for\ all\ 0 < \alpha \leq 1. \tag{15}$$

Proof. The proof is obvious. □

Theorem 2.7. *Let us consider the equilibrium point of z as 0 for the system ${}_0^{CF}D_t^{\alpha}z(t) = g(t,z)$ and the domain D is a subset of $\mathbb{R}^n$, it contains the origin and $0 < \alpha < 1$. Now, let us consider the continuously differentiable function as $V(t,z) : [0,\infty) \times D \to \mathbb{R}$ which is also a locally Lipschitz with respect to z such that*

$$\beta_1\|z\|^a \leq V(t,z(t)) \leq \beta_2\|z\|^{ab} \tag{16}$$

$${}_0^{CF}D_t^{\alpha}V(t,z(t)) \leq -\beta_3\|z\|^{ab}, \tag{17}$$

where, β_1, β_2, β_3, a and b are positive constants. And $t \geq 0$. Then the equilibrium point is globally α-exponential stable.

Proof. Let us consider the following expressions from Eqs. (16) and (17),

$${}_0^{CF}D_t^{\alpha}V(t,z(t)) \leq -\beta_3\|z\|^{ab}$$

and

$$\beta_2\|z\|^{ab} \geq V(t,z(t))$$
$$-\|z\|^{ab} \leq \frac{-1}{\beta_2}V(t,z(t)).$$

This implies that,

$${}_0^{CF}D_t^{\alpha}V(t,z(t)) \leq \frac{-\beta_3}{\beta_2}V(t,z(t)).$$

Then there exists a non-negative function $S(t)$ such that

$${}_0^{CF}D_t^{\alpha}V(t,z(t)) + S(t) = \frac{-\beta_3}{\beta_2}V(t,z(t)). \tag{18}$$

By finding the Laplace transform of (18), we get the following

$$\mathfrak{L}\Big\{{}_0^{CF}D_t^{\alpha}V(t,z(t))\Big\} + \mathfrak{L}\Big\{S(t)\Big\} = \mathfrak{L}\Big\{\frac{-\beta_3}{\beta_2}V(t,z(t))\Big\}$$

$$\frac{p\mathfrak{L}\Big\{V(t,z(t))\Big\} - V(0)}{p+\alpha(1-p)} + S(P) = -\frac{\beta_3}{\beta_2}V(p)$$

$$\frac{pV(p) - V(0)}{p+\alpha(1-p)} + S(P) = -\frac{\beta_3}{\beta_2}V(p).$$

$$\beta_2 pV(p) + \beta_3 V(p)(p+\alpha(1-p)) - \beta_2 V(0) + \beta_2 S(p)(p+\alpha(1-p)) = 0.$$

$$\Big[(\beta_2+\beta_3-\alpha\beta_3)p + \alpha\beta_3\Big]V(p) = \beta_2 V(0) - \beta_2 S(p)(p+\alpha(1-p)).$$

For simplicity, we denote $\beta_2+\beta_3-\alpha\beta_3$ as γ.

$$\begin{aligned} V(p) &= \frac{\beta_2 V(0)}{\gamma p + \alpha\beta_3} - \frac{\beta_3 S(p)(p+\alpha(1-p))}{\gamma p+\alpha\beta_3} \\ &= \frac{\beta_2 V(0)}{\gamma\left[p+\frac{\alpha\beta_3}{\gamma}\right]} - \frac{\beta_3 S(p)(p+\alpha(1-p))}{\gamma\left[p+\frac{\alpha\beta_3}{\gamma}\right]} \\ &= \frac{\beta_2 V(0)}{\gamma}\left[\frac{1}{p+\frac{\alpha\beta_3}{\gamma}}\right] - \frac{\beta_2}{\gamma}\left[\frac{S(p)p + \alpha S(p)(1-p)}{p+\frac{\alpha\beta_3}{\gamma}}\right] \end{aligned}$$

$$= \frac{\beta_2 V(0)}{\gamma}\left[\frac{1}{p+\frac{\alpha\beta_3}{\gamma}}\right] - \frac{\beta_2 S(p)}{\gamma}\left[\frac{p}{p+\frac{\alpha\beta_3}{\gamma}}\right] - \frac{\alpha\beta_2 S(p)(1-p)}{\gamma[p+\frac{\alpha\beta_3}{\gamma}]}.$$

$$\begin{aligned}
V(p) &= \frac{\beta_2 V(0)}{\gamma}\left[\frac{1}{p+\frac{\alpha\beta_3}{\gamma}}\right] - \frac{\beta_2 S(p)}{\gamma}\left[\frac{p+\frac{\alpha\beta_3}{\gamma}-\frac{\alpha\beta_3}{\gamma}}{p+\frac{\alpha\beta_3}{\gamma}}\right] - \frac{\alpha\beta_2 S(p)}{\gamma}\left[\frac{1}{p+\frac{\alpha\beta_3}{\gamma}}\right] \\
&+\frac{\alpha\beta_2 S(p)}{\gamma}\left[\frac{p+\frac{\alpha\beta_3}{\gamma}-\frac{\alpha\beta_3}{\gamma}}{p+\frac{\alpha\beta_3}{\gamma}}\right] \\
&= \frac{\beta_2 V(0)}{\gamma}\left[\frac{1}{p+\frac{\alpha\beta_3}{\gamma}}\right] - \frac{\beta_2 S(p)}{\gamma}\left[1-\frac{\frac{\alpha\beta_3}{\gamma}}{p+\frac{\alpha\beta_3}{\gamma}}\right] - \frac{\alpha\beta_2 S(p)}{\gamma}\left[\frac{1}{p+\frac{\alpha\beta_3}{\gamma}}\right] \\
&+\frac{\alpha\beta_2 S(p)}{\gamma}\left[1-\frac{\frac{\alpha\beta_3}{\gamma}}{p+\frac{\alpha\beta_3}{\gamma}}\right] \\
&= \frac{\beta_2 V(0)}{\gamma}\left[\frac{1}{p+\frac{\alpha\beta_3}{\gamma}}\right] - \left[\frac{\beta_2 S(p)}{\gamma}-\frac{\alpha\beta_2 S(p)}{\gamma}\right]\left[1-\frac{\frac{\alpha\beta_3}{\gamma}}{p+\frac{\alpha\beta_3}{\gamma}}\right] - \frac{\alpha\beta_2 S(p)}{\gamma}\left[\frac{1}{p+\frac{\alpha\beta_3}{\gamma}}\right].
\end{aligned}$$

Let us take the inverse Laplace transformation for the last expression,

$$\begin{aligned}
\mathfrak{L}^{-1}\{V(p)\} = \mathfrak{L}^{-1}&\left\{\frac{\beta_2 V(0)}{\gamma} * \left[\frac{1}{p+\frac{\alpha\beta_3}{\gamma}}\right]\right\} - \mathfrak{L}^{-1}\left\{\left[\frac{\beta_2 S(p)}{\gamma}-\frac{\alpha\beta_2 S(p)}{\gamma}\right] * \left[1-\frac{\frac{\alpha\beta_3}{\gamma}}{p+\frac{\alpha\beta_3}{\gamma}}\right]\right\} \\
&-\mathfrak{L}^{-1}\left\{\frac{\alpha\beta_2 S(p)}{\gamma} * \left[\frac{1}{p+\frac{\alpha\beta_3}{\gamma}}\right]\right\},
\end{aligned}$$

where $'*'$ is the Convolution operator.

$$\begin{aligned}
V(t,z(t)) = &\frac{\beta_2 V(0)}{\gamma} E_{1,1}\left(-t\frac{\alpha\beta_3}{\gamma}\right) - \left[\frac{\beta_2 S(t)}{\gamma}-\frac{\alpha\beta_2 S(t)}{\gamma}\right]\left(\delta(t)-\frac{\alpha\beta_3}{\gamma}exp\left[-t\frac{\alpha\beta_3}{\gamma}\right]\right) \\
&-\frac{\alpha\beta_2 S(t)}{\gamma}exp\left[-t\frac{\alpha\beta_3}{\gamma}\right].
\end{aligned}$$

Then,

$$\begin{aligned}
V(t,z(t)) = &\frac{\beta_2 V(0)}{\gamma} exp\left[-t\frac{\alpha\beta_3}{\gamma}\right] - \left[(1-\alpha)\frac{\beta_2 S(t)}{\gamma}\right]\left(\delta(t)-\frac{\alpha\beta_3}{\gamma}exp\left[-t\frac{\alpha\beta_3}{\gamma}\right]\right) \\
&-\frac{\alpha\beta_2 S(t)}{\gamma}exp\left[-t\frac{\alpha\beta_3}{\gamma}\right].
\end{aligned}$$

Here, $S(t)$ and $exp(-t\frac{\alpha\beta_3}{\gamma})$ are non-negative functions. Now, $(1-\alpha)\frac{\beta_2 S(t)}{\gamma} > 0$ whenever, $1-\alpha > 0$. That is,

$$\alpha < 1. \tag{19}$$

And $\delta(t) - \frac{\alpha\beta_3}{\gamma}exp\left[-t\frac{\alpha\beta_3}{\gamma}\right]$ should be greater than 0. For the conditions $\alpha < 1$ and $\delta(t) > \frac{\alpha\beta_3}{\gamma}exp\left[-t\frac{\alpha\beta_3}{\gamma}\right]$, we get,

$$V(t) \leq \frac{\beta_2 V(0,z(0))}{\gamma} exp\left[-t\frac{\alpha\beta_3}{\gamma}\right]. \tag{20}$$

Substitute Eq. (20) in (16), we get

$$\|z(t)\| \leq \left\{\frac{\beta_2 V(0,z(0))}{\beta_1\gamma} exp\left[-t\frac{\alpha\beta_3}{\gamma}\right]\right\}^{\frac{1}{a}},$$

where, $\beta_1, \beta_2 > 0$, $V(0, z(0)) > 0$ for $z(0) \neq 0$. $c = \frac{\beta_2 V(0)}{\beta_1 \gamma} > 0$ whenever $\gamma > 0$. That is, $\beta_2 + \beta_3 - \alpha\beta_3 > 0$. Therefore,

$$\alpha < 1 + \frac{\beta_2}{\beta_3}. \tag{21}$$

Combining Eqs. (19) and (21) we get $\alpha < 1 < 1 + \frac{\beta_2}{\beta_3}$. Therefore it is enough to consider that $\alpha < 1$. Therefore,

$$\|z(t)\| \leq \left\{ c \; exp \left[-t \frac{\alpha\beta_3}{\gamma} \right] \right\}^{\frac{1}{a}},$$

where, $c = 0$ is true iff $z(0) = 0$. Also, $V(0, z(0)) = 0$ iff $z(0) = 0$ and V is locally Lipschitz with respect to z. This implies that, c is also locally Lipschitz with respect to z and $c(0) = 0$ iff $z(0) = 0$, This implies the α-exponential stability of the system ${}_0^{CF}D_t^{\alpha} z(t) = g(t, z)$. □

Remark 2.8. Put $a = 2$ and $b = 1$ in the above theorem, the derived result matches the result of [45].

3. System formulation

In this section, the novel fractional order mathematical model to picturize the life stages of Aedes Aegypti mosquitoes in both aerial and aquatic is proposed. The population dynamics of wild mosquitoes are structured as follows:

$$\begin{cases} \dfrac{\mathrm{d}\,W_e(t)}{\mathrm{d}\,t} = \Lambda_{w_e} W_e(t) \left[1 - \frac{W_e(t)}{\kappa} \right] - \lambda_{w_e} W_e(t) - \gamma_{w_e} W_e(t) \\ \dfrac{\mathrm{d}\,W_l(t)}{\mathrm{d}\,t} = \gamma_{w_e} W_e(t) - \lambda_{w_l} W_l(t) - \gamma_{w_l} W_l(t) \\ \dfrac{\mathrm{d}\,W_p(t)}{\mathrm{d}\,t} = \gamma_{w_l} W_l(t) - \lambda_{w_p} W_p(t) - \gamma_{w_p} W_p(t) \\ \dfrac{\mathrm{d}\,W_{f_i}(t)}{\mathrm{d}\,t} = \rho\gamma_{w_p} W_p(t) - \lambda_{w_{f_i}} W_{f_i}(t) - \gamma_{w_{f_i}} W_{f_i}(t) \\ \dfrac{\mathrm{d}\,W_{f_m}(t)}{\mathrm{d}\,t} = \gamma_{w_{f_i}} W_{f_i}(t) - \lambda_{w_{fm}} W_{f_m}(t) \\ \dfrac{\mathrm{d}\,W_a(t)}{\mathrm{d}\,t} = (1 - \rho)\gamma_{w_p} W_p(t) - \lambda_{w_a} W_a(t), \end{cases} \tag{22}$$

with initial conditions as, $W_e(0) = W_{e_0}$, $W_l(0) = W_{l_0}$, $W_p(0) = W_{p_0}$, $W_{f_i}(0) = W_{f_{i_0}}$, $W_{f_m}(0) = W_{f_{m_0}}$ and $W_a(0) = W_{a_0}$. Where W is used to denote the wild Aedes Aegypti mosquito population and W with the subscripts such as W_e, W_l, W_p, W_{f_i}, W_{f_m}, and W_a are population densities of eggs, larvae, pupae, female immature, female mature, and adult male mosquitoes respectively. The description of parameters are given in Table 1.

In particular, Aedes aegypti mosquito population is a main host for some major mosquito-borne diseases such as Dengue, Zika virus, Yellow fever, and Chikungunya. The spread dynamics of these viruses can be visualized by the following block diagram Fig. 1: In this environment, our main aim is to control the vector population that, do not transmit the virus to the uninfected human while taking a blood meal. There is a life shortening bacteria called Wolbachia which will be very useful to reach our aim. Wolbachia is a gram negative bacteria and it is first reported in the tissues of the mosquito culex pipients (Hertig and Wolbach, 1924). In recent results, they found that Yellow fever virus can also blocked by Wolbachia [1].

If the mosquito carry this bacteria, then the virus inside the mosquito cannot be transmitted to the uninfected human (see Fig. 2). It blocks the virus inside the mosquito at salivary gland. Here, the process of releasing wolbachia bacteria into mosquito population can be framed with the following stages:

(1) In laboratory the Wolbachia pipients are injected into eggs, larvae, and pupae of Aedes aegypti via micro injection.
(2) Cytoplasmic Incapability: The adult wolbachia infected mosquitoes which are reared at laboratory are released to the wild mosquito population of Aedes aegypti. Throughout this process there exist three types of possibilities which are

Table 1
Description of parameters.

Λ_{w_e}	The reproduction rate of wild mosquitoes
κ	The environmental carrying capacity
λ_{w_e}	The natural mortality rate of wild mosquito eggs
λ_{w_l}	The natural mortality death rate of wild mosquito larvae population
λ_{w_p}	The natural mortality death rate of wild mosquito pupae population
$\lambda_{w_{f_i}}$	The natural mortality death rate of immature female wild mosquito population
$\lambda_{w_{fm}}$	The natural mortality death rate of mature female wild mosquito population
λ_{w_a}	The natural mortality death rate of adult male wild mosquito population
γ_{w_e}	The corresponding part of the wild mosquito eggs population from which the next life stage(Larvae) merge at time t
γ_{w_l}	The corresponding part of the wild mosquito larvae population from which the next life stage(Pupae) merge at time t
γ_{w_p}	The corresponding part of the wild mosquito pupae population from which the next life stage(female immature and male) merge at time t
$\gamma_{w_{f_i}}$	The corresponding part of the wild mosquito female immature population from which the next life stage(mature female mosquitoes) merge at time t
ρ	The probability constant

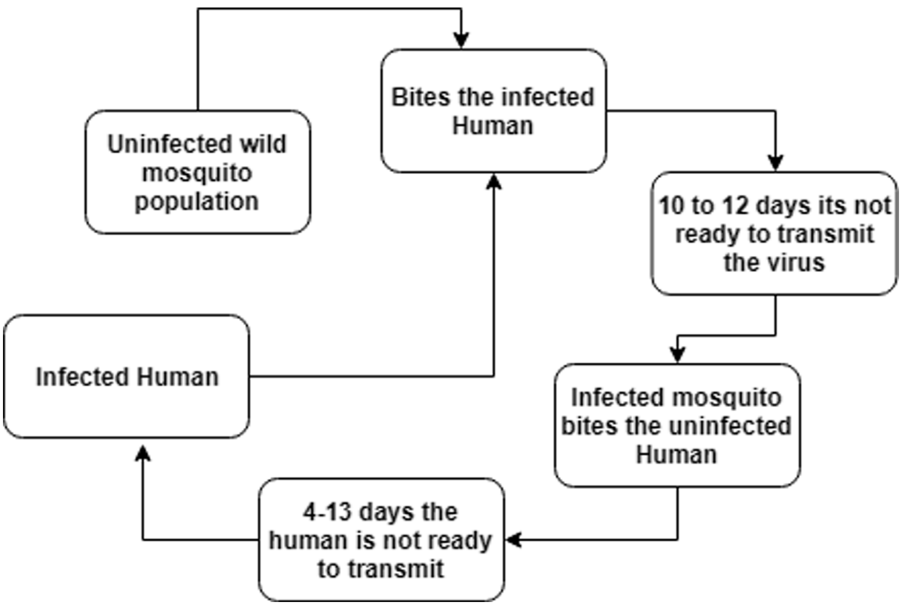

Fig. 1. Dynamics of virus infection before wolbachia.

(i) If the Wolbachia infected female mosquitoes mate with the Wolbachia infected male then the progeny should have the Wolbachia by birth which is compatible.
(ii) If the Wolbachia infected female cross with Wolbachia uninfected male then the progeny face the same problems as in (i).
(iii) If Wolbachia uninfected female cross with Wolbachia infected male then there is no viable progeny.

In eggs, larvae, and pupae population, we can micro inject the wolbachia and release this in patches. And the adult mosquitoes which are reared at lab can also be released into wild mosquito population. Our main aim is to increase

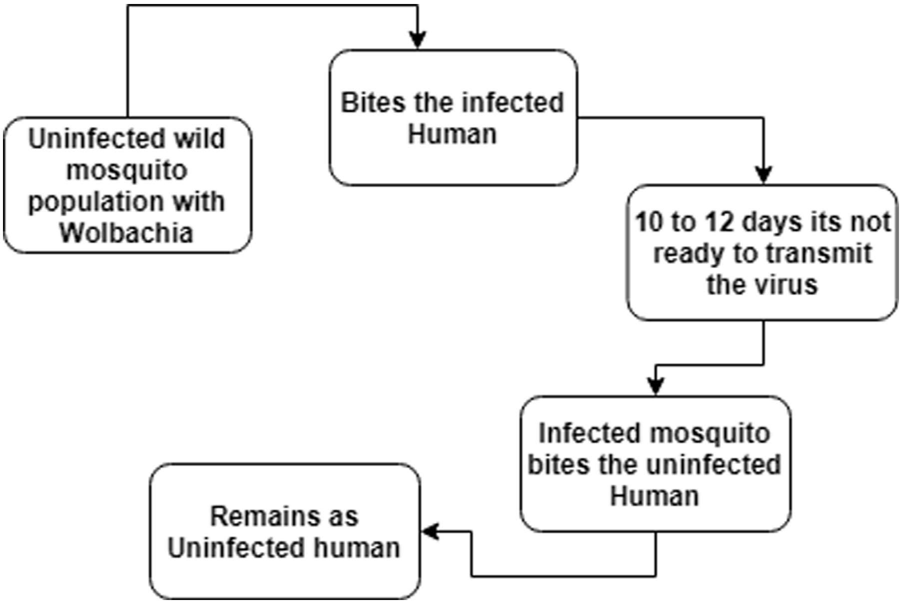

Fig. 2. Dynamics of virus infection after wolbachia.

the Wolbachia infection in the wild mosquito population. First, if we release the artificially Wolbachia injected mosquitoes into the wild mosquitoes then naturally wolbachia can spread into the wild mosquito population.

$$\begin{cases} \dfrac{\mathrm{d}\,W_e(t)}{\mathrm{d}\,t} = & \Lambda_{w_e} W_e(t)\left[1-\frac{W_e(t)}{\kappa}\right] - \lambda_{w_e} W_e(t) - \gamma_{w_e} W_e(t) + Q_1 \\ \dfrac{\mathrm{d}\,W_l(t)}{\mathrm{d}\,t} = & \gamma_{w_e} W_e(t) - \lambda_{w_l} W_l(t) - \gamma_{w_l} W_l(t) + Q_2 \\ \dfrac{\mathrm{d}\,W_p(t)}{\mathrm{d}\,t} = & \gamma_{w_l} W_l(t) - \lambda_{w_p} W_p(t) - \gamma_{w_p} W_p(t) + Q_3 \\ \dfrac{\mathrm{d}\,W_{f_i}(t)}{\mathrm{d}\,t} = & \rho\gamma_{w_p} W_p(t) - \lambda_{w_{f_i}} W_{f_i}(t) - \gamma_{w_{f_i}} W_{f_i}(t) \\ \dfrac{\mathrm{d}\,W_{f_m}(t)}{\mathrm{d}\,t} = & \gamma_{w_{f_i}} W_{f_i}(t) - \lambda_{w_{f_m}} W_{f_m}(t) + Q_4 \\ \dfrac{\mathrm{d}\,W_a(t)}{\mathrm{d}\,t} = & (1-\rho)\gamma_{w_p} W_p(t) - \lambda_{w_a} W_a(t) + Q_5, \end{cases} \tag{23}$$

Let $Q_1 = S_{i_e} I_e(t)$, $Q_2 = S_{i_l} I_l(t)$, $Q_3 = S_{i_p} I_p(t)$, $Q_4 = S_{i_{fm}} I_{fm}(t)$, and $Q_5 = S_{i_a} I_a(t)$ denoting the control inputs corresponding to the life stages of a mosquito are given into that corresponding compartments. Where, S_{i_e}, S_{i_l}, S_{i_p}, $S_{i_{fm}}$ and S_{i_a} are the survivability constant of the corresponding compartment. And $I_e(t)$, $I_l(t)$, $I_p(t)$, $I_{f_i}(t)$, $I_{f_m}(t)$ and $I_a(t)$ denote the Wolbachia infected population density of corresponding compartment. Let $N_1 = E + L + P + F_I + F_M + A$ be the total population. In addition to that, consider $N_2 = F_M + A$ be the total population which are ready to mate. (i.e.,) $N_2 = W_{f_m} + I_{f_m} + W_a + I_a$. Assume that the sex ratio is 1:1. Therefore, we get that $\frac{W_{fm}}{W_a} = \frac{I_{fm}}{I_a} = 1$. This implies that, the wolbachia infected eggs population is generated in two ways. They are,

(1) If the wolbachia infected female (I_{f_m}) mated with the non wolbachia male (W_a).
(2) If the wolbachia infected female (I_{f_m}) mated with the wolbachia infected male (I_a).

Therefore, the eggs with wolbachia infection are generated in the reproduction rate Λ_e is $\frac{\Lambda_e(I_{fm}W_a+I_{fm}I_a)}{N_2}$. By using $W_a + I_a = \frac{N_2}{2}$, we get that $\frac{\Lambda_e(I_{fm}W_a+I_{fm}I_a)}{N_2} = \frac{\Lambda_e I_{fm}}{2}$. Then, the population dynamics of the mosquitoes in that wolbachia released environment can be visualized by the following mathematical model:

Assume that, the reproduction rate, natural mortality rate, and the rate at which the current compartment moved into the next compartment all are same in both wild(W) and Wolbachia infected(I) mosquitoes.

$$\begin{cases} \frac{\mathrm{d}\left(W_e(t)+I_e(t)\right)}{\mathrm{d}t} = \Lambda_e W_e(t)\left[1-\frac{W_e(t)}{\kappa}\right] + S_{i_e} I_e(t) - \lambda_e\left(W_e(t)+I_e(t)\right) - \gamma_e\left(W_e(t)\right. \\ \qquad \left. +I_e(t)\right) + \Lambda_e I_e(t)\left[1-\frac{I_e(t)}{\kappa}\right] + \frac{\Lambda_e I_{fm}}{2} \\ \frac{\mathrm{d}\left(W_l(t)+I_l(t)\right)}{\mathrm{d}t} = \gamma_e\left(W_e(t)+I_e(t)\right) + S_{i_l} I_l(t) - \lambda_l\left(W_l(t)+I_l(t)\right) - \gamma_l\left(W_l(t)+I_l(t)\right) \\ \frac{\mathrm{d}\left(W_p(t)+I_p(t)\right)}{\mathrm{d}t} = \gamma_l\left(W_l(t)+I_l(t)\right) + S_{i_p} I_p(t) - \lambda_p\left(W_p(t)+I_p(t)\right) - \gamma_p\left(W_p(t)+I_p(t)\right) \\ \frac{\mathrm{d}\left(W_{f_i}(t)+I_{f_i}(t)\right)}{\mathrm{d}t} = \rho\gamma_p\left(W_p(t)+I_p(t)\right) - \lambda_{f_i}\left(W_{f_i}(t)+I_{f_i}(t)\right) - \gamma_{f_i}\left(W_{f_i}(t)+I_{f_i}(t)\right) \\ \frac{\mathrm{d}\left(W_{fm}(t)+I_{fm}(t)\right)}{\mathrm{d}t} = \gamma_{f_i}\left(W_{f_i}(t)+I_{f_i}(t)\right) + b_1\left(W_{fm}(t)+I_{fm}(t)\right)n_a + S_{i_{fm}} I_{fm}(t) \\ \qquad -\lambda_{fm}\left(W_{fm}(t)+I_{fm}(t)\right) \\ \frac{\mathrm{d}\left(W_a(t)+I_a(t)\right)}{\mathrm{d}t} = (1-\rho)\gamma_p\left(W_p(t)+I_p(t)\right) + b_2\left(W_a(t)+I_a(t)\right)n_{fm} + S_{i_a} I_a(t) \\ \qquad -\lambda_a\left(W_a(t)+I_a(t)\right). \end{cases} \tag{24}$$

Let $E(t) = \left(W_e(t)+I_e(t)\right)$, $L(t) = \left(W_l(t)+I_l(t)\right)$, $P(t) = \left(W_p(t)+I_p(t)\right)$, $F_I(t) = \left(W_{f_i}(t)+I_{f_i}(t)\right)$, $F_m(t) = \left(W_{fm}(t)+I_{fm}(t)\right)$, and $A(t) = \left(W_a(t)+I_a(t)\right)$.

Then, Eq. (24) becomes

$$\begin{cases} \frac{\mathrm{d}E(t)}{\mathrm{d}t} = \Lambda_e E(t) - \frac{\Lambda_e E^2(t)}{\kappa} + S_{I_e} I_e(t) + \frac{\Lambda_e I_{fm}}{2} - \lambda_e E(t) - \gamma_e E(t) \\ \frac{\mathrm{d}L(t)}{\mathrm{d}t} = \gamma_e E(t) - \lambda_l L(t) + S_{I_l} I_l(t) - \gamma_l L(t) \\ \frac{\mathrm{d}P(t)}{\mathrm{d}t} = \gamma_l L(t) - \lambda_p P(t) + S_{I_p} I_p(t) - \gamma_p P(t) \\ \frac{\mathrm{d}F_I(t)}{\mathrm{d}t} = \rho\gamma_p P(t) - \lambda_{f_i} F_I(t) - \gamma_{f_i} F_I(t) \\ \frac{\mathrm{d}F_m(t)}{\mathrm{d}t} = \gamma_{f_i} F_I(t) + b_1 F_m(t) n_a + S_{I_{fm}} I_{fm}(t) - \lambda_{fm} F_m(t) \\ \frac{\mathrm{d}A(t)}{\mathrm{d}t} = (1-\rho)\gamma_p P(t) + b_2 A(t) n_{fm} + S_{I_a} I_a(t) - \lambda_a A(t). \end{cases} \tag{25}$$

Now, to get the memory property, we replace the ordinary integer order derivative by Caputo–Fabrizio operator. Then Eq. (25) becomes

$$\begin{cases} {}_0^{CF}D_t^{\alpha} E(t) = \Lambda_e E(t) - \frac{\Lambda_e E^2(t)}{\kappa} + S_{I_e} I_e(t) + \frac{\Lambda_e I_{fm}}{2} - \lambda_e E(t) - \gamma_e E(t) \\ {}_0^{CF}D_t^{\alpha} L(t) = \gamma_e E(t) - \lambda_l L(t) + S_{I_l} I_l(t) - \gamma_l L(t) \\ {}_0^{CF}D_t^{\alpha} P(t) = \gamma_l L(t) - \lambda_p P(t) + S_{I_p} I_p(t) - \gamma_p P(t) \\ {}_0^{CF}D_t^{\alpha} F_I(t) = \rho\gamma_p P(t) - \lambda_{f_i} F_I(t) - \gamma_{f_i} F_I(t) \\ {}_0^{CF}D_t^{\alpha} F_M(t) = \gamma_{f_i} F_I(t) + b_1 F_m(t) n_a + S_{I_{fm}} I_{fm}(t) - \lambda_{fm} F_m(t) \\ {}_0^{CF}D_t^{\alpha} A(t) = (1-\rho)\gamma_p P(t) + b_2 A(t) n_{fm} + S_{I_a} I_a(t) - \lambda_a A(t). \end{cases} \tag{26}$$

4. Existence of a solution

In this section, we analysed the existence of solution for the proposed model (26). Let us find the fractional integral of (26) using Caputo–Fabrizio fractional integral operator

$$
\begin{aligned}
E(t)-E_0(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\Lambda_e E(t)-\frac{\Lambda_e E^2(t)}{\kappa}+S_{I_e}I_e(t)+\frac{\Lambda_e I_{fm}(t)}{2}-\lambda_e E(t)-\gamma_e E(t)\right\}\\
&+\frac{2\alpha}{(2-\alpha)M(\alpha)}\\
&\left\{\int_0^t\left(\Lambda_e E(s)-\frac{\Lambda_e E^2(s)}{\kappa}+S_{I_e}I_e(s)+\frac{\Lambda_e I_{fm}(s)}{2}-\lambda_e E(s)-\gamma_e E(s)\right)\mathrm{ds}\right\}\\
L(t)-L_0(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\gamma_e E(t)-\lambda_l L(t)+S_{I_l}I_l(t)-\gamma_l L(t)\right\}\\
&+\frac{2\alpha}{(2-\alpha)M(\alpha)}\left\{\int_0^t\left(\gamma_e E(s)-\lambda_l L(s)+S_{I_l}I_l(s)-\gamma_l L(s)\right)\mathrm{ds}\right\}\\
P(t)-P_0(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\gamma_l L(t)-\lambda_p P(t)+S_{I_p}I_p(t)-\gamma_p P(t)\right\}\\
&+\frac{2\alpha}{(2-\alpha)M(\alpha)}\left\{\int_0^t\left(\gamma_l L(s)-\lambda_p P(s)+S_{I_p}I_p(s)-\gamma_p P(s)\right)\mathrm{ds}\right\}\\
F_I(t)-F_{I_0}(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\rho\gamma_p P(t)-\lambda_{f_i}F_I(t)-\gamma_{f_i}F_I(t)\right\}\\
&+\frac{2\alpha}{(2-\alpha)M(\alpha)}\left\{\int_0^t\left(\rho\gamma_p P(s)-\lambda_{f_i}F_I(s)-\gamma_{f_i}F_I(s)\right)\mathrm{ds}\right\}\\
F_M(t)-F_{M_0}(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{\gamma_{f_i}F_I(s)+b_1F_m(t)n_a+S_{I_{fm}}I_{fm}(s)-\lambda_{fm}F_m(s)\right\}\\
&+\frac{2\alpha}{(2-\alpha)M(\alpha)}\left\{\int_0^t\left(\gamma_{f_i}F_I(s)+b_1F_m(t)n_a+S_{I_{fm}}I_{fm}(s)-\lambda_{fm}F_m(s)\right)\mathrm{ds}\right\}\\
A(t)-A_0(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left\{(1-\rho)\gamma_p P(t)+b_2A(t)n_{fm}+S_{I_a}I_a(t)-\lambda_a A(t)\right\}\\
&+\frac{2\alpha}{(2-\alpha)M(\alpha)}\left\{\int_0^t\left((1-\rho)\gamma_p P(s)+b_2A(t)n_{fm}+S_{I_a}I_a(s)-\lambda_a A(s)\right)\mathrm{ds}\right\}
\end{aligned}
$$

For simplicity, we choose our kernels as

$$
\begin{aligned}
g_1(t,E(t)) &= \Lambda_e E(t)-\frac{\lambda_e E^2(t)}{\kappa}+S_{I_e}I_e(t)+\frac{\Lambda_e I_{fm}}{2}-\lambda_e E(t)-\gamma_e E(t)\\
g_1(t,L(t)) &= \gamma_e E(t)-\lambda_l L(t)+S_{I_l}I_l(t)-\gamma_l L(t)\\
g_1(t,P(t)) &= \gamma_l L(t)-\lambda_p P(t)+S_{I_p}I_p(t)-\gamma_p P(t)\\
g_1(t,F_I(t)) &= \rho\gamma_p P(t)-\lambda_{f_i}F_I(t)-\gamma_{f_i}F_I(t)\\
g_1(t,F_M(t)) &= \gamma_{f_i}F_I(t)+b_1F_m(t)n_a+S_{I_{fm}}I_{fm}(t)-\lambda_{fm}F_m(t)\\
g_1(t,A(t)) &= (1-\rho)\gamma_p P(t)+b_2A(t)n_{fm}+S_{I_a}I_a(t)-\lambda_a A(t)
\end{aligned}
$$

First, we need to be able to identify an operator and then show that this operator is compact. So that, the operator $\nu:\mathcal{H}\to\mathcal{H}$. Then, we get

$$
\begin{aligned}
\nu E(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}g_1(t,E(t))+\frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t g_1(s,E(s))\,\mathrm{ds}\\
\nu L(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}g_1(t,L(t))+\frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t g_1(s,L(s))\,\mathrm{ds}\\
\nu P(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}g_1(t,P(t))+\frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t g_1(s,P(s))\,\mathrm{ds}
\end{aligned}
$$

$$\nu F_I(t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, F_I(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, F_I(s))\,\mathrm{ds}$$
$$\nu F_M(t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, F_M(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, F_M(s))\,\mathrm{ds}$$
$$\nu A(t) = \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, A(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, A(s))\,\mathrm{ds}$$

Lemma 4.1. *The mapping $\nu : \mathcal{H} \to \mathcal{H}$ is completely continuous.*

Proof. Let $B \subset \mathcal{H}$ be bounded. There exist some constants $l_i > 0$, (i=1,2, …,6) such that $\|E\| < l_1$, $\|L\| < l_2$, $\|P\| < l_3$, $\|F_I\| < l_4$, $\|F_M\| < l_5$ and $\|A\| < l_6$, where $X_1 = E$; $X_2 = L$; $X_3 = P$; $X_4 = F_I$; $X_5 = F_M$; and $X_6 = A$;

Let

$$M_i = \max_{0<t<1;0\le X_i \le l_i} g_1(t, X_i(t)),\ i = 1, 2, \dots, 6.$$

For every $X_i \in B$, we have

$$\begin{aligned}
|\nu X_i(t)| &= \left| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, X_i(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, X_i(s))\,\mathrm{ds} \right| \\
&\le \left| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, X_i(t)) \right| + \left| \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, X_i(s))\,\mathrm{ds} \right| \\
&\le \left| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \right| \ |g_1(t, X_i(t))| + \left| \frac{2\alpha}{(2-\alpha)M(\alpha)} \right| \ \left| \int_0^t g_1(s, X_i(s))\,\mathrm{ds} \right| \\
&\le \left| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \right| \ |g_1(t, X_i(t))| + \left| \frac{2\alpha}{(2-\alpha)M(\alpha)} \right| \ a_i\ \ |g_1(t, X_i(t))| \\
&\le \left| \frac{2-2\alpha+2\alpha a_i}{(2-\alpha)M(\alpha)} \right| \ |g_1(t, X_i(t))| \\
&\le \frac{2M_i}{(2-\alpha)M(\alpha)} (1-\alpha+\alpha a_i).
\end{aligned}$$

This implies that,

$$|\nu X_i(t)| \le \frac{2M_i}{(2-\alpha)M(\alpha)} (1-\alpha+\alpha a_i),\ i = 1, 2, \dots, 6.$$

Therefore, ν is bounded.

Now, in the following part we will consider $t_1 < t_2$ and $X_i \in B$, $i = 1, 2, \dots, 6$, and then for a given $\epsilon > 0$, if $|t_2 - t_1| < \delta$, we have

$$\begin{aligned}
\|\nu X_i(t_2) - \nu X_i(t_2)\| &= \Big| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \{g_1(t_2, X_i(t_2)) - g_1(t_1, X_i(t_1))\} \\
&\quad + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, X_i(s))\,\mathrm{ds} \Big| \\
&\le \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \ |g_1(t_2, X_i(t_2)) - g_1(t_1, X_i(t_1))| \\
&\quad + \left| \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^{t_2} g_1(s, X_i(s))\,\mathrm{ds} - \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^{t_1} g_1(s, X_i(s))\,\mathrm{ds} \right| \\
&\le \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \ |g_1(t_2, X_i(t_2)) - g_1(t_1, X_i(t_1))| \\
&\quad + \left| \frac{2\alpha}{(2-\alpha)M(\alpha)} \right| \ M_i \ \ |g_1(t_2, X_i(t_2)) - g_1(t_1, X_i(t_1))|
\end{aligned} \tag{27}$$

Hence, the mapping $\nu : H \to H$ is completely continuous. □

Theorem 4.2. *Let $f : [E_1, E_2] \times [0, \infty) \to [0, \infty)$, then $f(t, .)$ is non-decreasing for each t in $[E_1, E_2]$. Then there exist positive constants, E_1 and E_2, so that $b_n\gamma_1 \leq g_1(t, E_1)$, $b_n\gamma_2 \geq f_1(t, \gamma_2)$, $0 \leq \gamma_1(t) \leq \gamma_2(t)$, $E_1 \leq t \leq E_2$. Thus, the equation has a positive solution.*

Proof. We only need to consider the fixed point for the operators of f_1. Here we considered that $\nu : \mathcal{H} \to \mathcal{H}$ is completely continuous. Let $E_1 \leq E_2$, $L_1 \leq L_2$, $P_1 \leq P_2$, $F_{I_1} \leq F_{I_2}$, $F_{M_1} \leq F_{M_2}$, and $A_1 \leq A_2$ and the chosen variables are arbitrary.

$$\begin{aligned}
\nu E_1(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, E_1(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, E_1(s))\,\mathrm{ds} \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, E_1(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, E_2(s))\,\mathrm{ds} \\
&\leq \nu E_2(t).
\end{aligned}$$

$$\begin{aligned}
\nu L_1(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, L_1(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, L_1(s))\,\mathrm{ds} \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, L_1(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, L_2(s))\,\mathrm{ds} \\
&\leq \nu L_2(t).
\end{aligned}$$

$$\begin{aligned}
\nu P_1(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, P_1(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, P_1(s))\,\mathrm{ds} \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, P_1(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, P_2(s))\,\mathrm{ds} \\
&\leq \nu P_2(t).
\end{aligned}$$

$$\begin{aligned}
\nu F_{I_1}(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, F_{I_1}(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, F_{I_1}(s))\,\mathrm{ds} \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, F_{I_1}(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, F_{I_2}(s))\,\mathrm{ds} \\
&\leq \nu F_{I_2}(t).
\end{aligned}$$

$$\begin{aligned}
\nu F_{M_1}(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, F_{M_1}(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, F_{M_1}(s))\,\mathrm{ds} \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, F_{M_1}(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, F_{M_2}(s))\,\mathrm{ds} \\
&\leq \nu F_{M_2}(t).
\end{aligned}$$

and

$$\begin{aligned}
\nu A_1(t) &= \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, A_1(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, A_1(s))\,\mathrm{ds} \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} g_1(t, A_1(t)) + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t g_1(s, A_2(s))\,\mathrm{ds} \\
&\leq \nu A_2(t).
\end{aligned}$$

Hence, ν is non-decreasing operator, so that the operator $\nu : \langle\gamma_1, \gamma_2\rangle \to \langle\gamma_1, \gamma_2\rangle$ is compact and continuous via Lemma 4.1. This implies that the solution exists. □

5. Uniqueness of the solution

In this section, we analysed the Uniqueness of the solution for the proposed model (26).

Let us assume that, we can find six special coupled solutions (E_1, E_2), (L_1, L_2), (P_1, P_2), (F_{I_1}, F_{I_2}), (F_{M_1}, F_{M_2}), and (A_1, A_2). Then the uniqueness of the solution is presented as follows:

$$
\begin{aligned}
|\nu E_1(t) - \nu E_2(t)| &\leq \left| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\Big(g_1(t, E_1(t)) - g_1(t, E_2(t))\Big) \right. \\
&\quad \left. + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \Big(g_1(s, E_1(s)) - g_1(s, E_2(s))\Big)\,\mathrm{ds} \right| \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \left|\Big(g_1(t, E_1(t)) - g_1(t, E_2(t))\Big)\right| \\
&\quad + \frac{2\alpha}{(2-\alpha)M(\alpha)} \left| \int_0^t \Big(g_1(s, E_1(s)) - g_1(s, E_2(s))\Big)\,\mathrm{ds} \right| \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_1 |E_1(t) - E_2(t)| + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_1 |E_1(t) - E_2(t)| \\
&\leq \left\{ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_1 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_1 \right\} |E_1(t) - E_2(t)|.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
|\nu L_1(t) - \nu L_2(t)| &\leq \left| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\Big(g_1(t, L_1(t)) - g_1(t, L_2(t))\Big) \right. \\
&\quad \left. + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \Big(g_1(s, L_1(s)) - g_1(s, L_2(s))\Big)\,\mathrm{ds} \right| \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \left|\Big(g_1(t, L_1(t)) - g_1(t, L_2(t))\Big)\right| \\
&\quad + \frac{2\alpha}{(2-\alpha)M(\alpha)} \left| \int_0^t \Big(g_1(s, L_1(s)) - g_1(s, L_2(s))\Big)\,\mathrm{ds} \right| \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_2 |L_1(t) - L_2(t)| + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_2 |L_1(t) - L_2(t)| \\
&\leq \left\{ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_2 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_2 \right\} |L_1(t) - L_2(t)|.
\end{aligned}
$$

$$
\begin{aligned}
|\nu P_1(t) - \nu P_2(t)| &\leq \left| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\Big(g_1(t, P_1(t)) - g_1(t, P_2(t))\Big) \right. \\
&\quad \left. + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \Big(g_1(s, P_1(s)) - g_1(s, P_2(s))\Big)\,\mathrm{ds} \right| \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)} \left|\Big(g_1(t, P_1(t)) - g_1(t, P_2(t))\Big)\right| \\
&\quad + \frac{2\alpha}{(2-\alpha)M(\alpha)} \left| \int_0^t \Big(g_1(s, P_1(s)) - g_1(s, P_2(s))\Big)\,\mathrm{ds} \right| \\
&\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_3 |P_1(t) - P_2(t)| + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_3 |P_1(t) - P_2(t)| \\
&\leq \left\{ \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_3 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_3 \right\} |P_1(t) - P_2(t)|.
\end{aligned}
$$

$$
\begin{aligned}
|\nu F_{I_1}(t) - \nu F_{I_2}(t)| &\leq \left| \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\Big(g_1(t, F_{I_1}(t)) - g_1(t, F_{I_2}(t))\Big) \right. \\
&\quad \left. + \frac{2\alpha}{(2-\alpha)M(\alpha)} \int_0^t \Big(g_1(s, F_{I_1}(s)) - g_1(s, F_{I_2}(s))\Big)\,\mathrm{ds} \right|
\end{aligned}
$$

$$\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left|\Big(g_1(t, F_{I_1}(t)) - g_1(t, F_{I_2}(t))\Big)\right| + \frac{2\alpha}{(2-\alpha)M(\alpha)}\left|\int_0^t \Big(g_1(s, F_{I_1}(s)) - g_1(s, F_{I_2}(s))\Big)\,\mathrm{ds}\right|$$

$$\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_4|F_{I_1}(t) - F_{I_2}(t)| + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_4|F_{I_1}(t) - F_{I_2}(t)|$$

$$\leq \left\{\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_4 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_4\right\}|F_{I_1}(t) - F_{I_2}(t)|.$$

$$|\nu F_{M_1}(t) - \nu F_{M_2}(t)| \leq \left|\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\Big(g_1(t, F_{M_1}(t)) - g_1(t, F_{M_2}(t))\Big) + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \Big(g_1(s, F_{M_1}(s)) - g_1(s, F_{M_2}(s))\Big)\,\mathrm{ds}\right|$$

$$\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left|\Big(g_1(t, F_{M_1}(t)) - g_1(t, F_{M_2}(t))\Big)\right| + \frac{2\alpha}{(2-\alpha)M(\alpha)}\left|\int_0^t \Big(g_1(s, F_{M_1}(s)) - g_1(s, F_{M_2}(s))\Big)\,\mathrm{ds}\right|$$

$$\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_5|F_{M_1}(t) - F_{M_2}(t)| + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_5|F_{M_1}(t) - F_{M_2}(t)|$$

$$\leq \left\{\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_5 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_5\right\}|F_{M_1}(t) - F_{M_2}(t)|.$$

$$|\nu A_1(t) - \nu A_2(t)| \leq \left|\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\Big(g_1(t, A_1(t)) - g_1(t, A_2(t))\Big) + \frac{2\alpha}{(2-\alpha)M(\alpha)}\int_0^t \Big(g_1(s, A_1(s)) - g_1(s, A_2(s))\Big)\,\mathrm{ds}\right|$$

$$\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\left|\Big(g_1(t, A_1(t)) - g_1(t, A_2(t))\Big)\right| + \frac{2\alpha}{(2-\alpha)M(\alpha)}\left|\int_0^t \Big(g_1(s, A_1(s)) - g_1(s, A_2(s))\Big)\,\mathrm{ds}\right|$$

$$\leq \frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_6|A_1(t) - A_2(t)| + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_6|A_1(t) - A_2(t)|$$

$$\leq \left\{\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_6 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_6\right\}|A_1(t) - A_2(t)|.$$

Therefore, if the following conditions hold:

$$\left\{\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_1 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_1\right\} < 1$$

$$\left\{\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_2 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_2\right\} < 1$$

$$\left\{\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_3 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_3\right\} < 1$$

$$\left\{\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_4 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_4\right\} < 1$$

$$\left\{\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_5 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_5\right\} < 1$$

and

$$\left\{\frac{2(1-\alpha)}{(2-\alpha)M(\alpha)}\rho_6 + \frac{2\alpha}{(2-\alpha)M(\alpha)}\rho_6\right\} < 1.$$

then the mapping ν is a contraction, we can say that the model has a unique positive solution using fixed point theorem.

6. Stability results

In this section, the stability results for the proposed model (26) are analysed by using the results from Section 2. In order to prove the stability results, we need the following assumption.

Assumption 1. The function $f(q(t))$ is continuous and satisfying the Lipschitz condition $\|f(q(t))\| \leq \|Hq(t)\|$ where, $H \in \mathbb{R}^{n\times n}$ is a constant matrix.

Consider the system of equations (26), and let E^*, L^*, P^*, F_I^*, F_M^*, and A^* are equilibrium points of the corresponding stages $E(t)$, $L(t)$, $P(t)$, $F_I(t)$, $F_M(t)$, and $A(t)$.

Define $Z = [E(t), L(t), P(t), F_I(t), F_M(t), A(t)]^\top$ and $Z = [E^*, L^*, P^*, F_I^*, F_M^*, A^*]^\top$. By the definition of equilibrium point

$$\begin{cases} \Lambda_e E^* - \frac{\lambda_e E^{2^*}}{\kappa} + S_{I_e} I_e + \frac{\Lambda_e I_{fm}}{2} - \lambda_e E^* - \gamma_e E^* & = 0 \\ \gamma_e E^* - \lambda_l L^* + S_{I_l} I_l - \gamma_l L^* & = 0 \\ \gamma_l L^* - \lambda_p P^* + S_{I_p} I_p - \gamma_p P^* & = 0 \\ \rho\gamma_p P^* - \lambda_{f_i} F_I^* - \gamma_{f_i} F_I^* & = 0 \\ \gamma_{f_i} F_I^* + b_1 F_m^* n_a + S_{I_{fm}} I_{fm} - \lambda_{fm} F_m^* & = 0 \\ (1-\rho)\gamma_p P^* + b_2 A^* n_{fm} + S_{I_a} I_a - \lambda_a A^* & = 0. \end{cases} \tag{28}$$

To construct a vector function, we define a new variable

$$q = Z - Z^*$$

and the control is defined by

$$u(t) = [I_e(t), I_l(t), I_p(t), I_{f_i}(t), I_{fm}(t), I_a(t)]^\top.$$

Then,

$${}_0^{CF}D_a^\alpha q(t) = {}_0^{CF}D_a^\alpha (Z - Z^*). \tag{29}$$

By using Property 2.3, one can get

$${}_0^{CF}D_a^\alpha q(t) = {}_0^{CF}D_a^\alpha Z(t) - {}_0^{CF}D_a^\alpha Z^*(t).$$

This implies that, we get the following error system

$${}_0^{CF}D_a^\alpha q(t) = Wq(t) + f(q(t)) + Cu(t) \tag{30}$$

where, $W = \begin{bmatrix} \Lambda_e - \lambda_e - \gamma_e & 0 & 0 & 0 & 0 & 0 \\ \gamma_e & -\lambda_l - \gamma_l & 0 & 0 & 0 & 0 \\ 0 & \gamma_l & -\lambda_p - \gamma_p & 0 & 0 & 0 \\ 0 & 0 & \rho\gamma_p & -\lambda_{f_i} - \gamma_{f_i} & 0 & 0 \\ 0 & 0 & 0 & \gamma_{f_i} & b_1 n_a - \lambda_{fm} & 0 \\ 0 & 0 & (1-\rho)\gamma_p & 0 & 0 & b_2 n_{fm} - \lambda_a \end{bmatrix}$;

$$f(q(t)) = \begin{bmatrix} -\frac{\Lambda_e q_1^2}{\kappa} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \; C = \begin{bmatrix} S_{I_e} & 0 & 0 & 0 & \frac{\Lambda_e}{2} & 0 \\ 0 & S_{I_l} & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{I_p} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{I_{fm}} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{I_a} \end{bmatrix}.$$

In the next theorem, we investigate the α-exponential stability results for system (26), via LMI approach.

Theorem 6.1. *If there exist a positive definite matrix P, real matrix Z and a positive scalar ω_1 satisfying the following LMI:*

$$\bar{\Omega} = \begin{bmatrix} 2PW + 2CZ + \omega_1 H^\top H & P \\ \star & -\omega_1 I \end{bmatrix} < 0. \tag{31}$$

Furthermore, the control gain matrix is designed as $K = P^{-1}Z$. Then, the system (30) is α-exponentially stable.

Proof. Let us consider the following Lyapunov function

$$V(t, q(t)) = q^\top(t)Pq(t). \tag{32}$$

By taking the Caputo–Fabrizio fractional operator for (32), and by using Lemma 2.4, we sustain

$$\begin{aligned} {}_0^{CF}D_t^\alpha V(t, q(t)) &= {}_0^{CF}D_t^\alpha q(t)Pq^\top(t) \\ &\leq 2q^\top(t)P{}_0^{CF}D_t^\alpha q(t) \\ &= 2q^\top(t)P\left[Wq(t) + f(q(t)) + Cu(t)\right] \\ &= 2q^\top(t)PWq(t) + 2q^\top(t)Pf(q(t)) + 2q^\top(t)PCu(t) \end{aligned} \tag{33}$$

Put $u(t) = Kq(t)$, $2q^\top(t)PWq(t) \leq q^\top(t)(PW + W^\top P)q(t)$. By Assumption 1, we get $2q^\top(t)Pf(q(t)) \leq q^\top(t)[\omega_1^{-1}PP^\top + \omega_1 H^\top H]q(t)$. By substituting these expressions into (33), we get

$${}_0^{CF}D_t^\alpha V(t, q(t)) \leq q^\top(t)(PW + W^\top P^\top)q(t) + q^\top(t)2KPCq(t) + q^\top(t)[\omega_1^{-1}PP^\top + \omega_1 H^\top H]q(t)$$

Put $KP = Z$,

$$\begin{aligned} {}_0^{CF}D_t^\alpha V(t, q(t)) &\leq q^\top(t)[2PW + \omega_1 H^\top H + \omega_1^{-1}PP^\top]q(t) + 2q^\top(t)[CZ]q(t) \\ &= q^\top(t)[2PW + \omega_1 H^\top H + \omega_1^{-1}PP^\top + 2CZ]q(t) \end{aligned}$$

Therefore,

$${}_0^{CF}D_t^\alpha V(t, q(t)) \leq q(t)^\top(t)\Omega q(t)$$

where,

$$\bar{\Omega} = \begin{bmatrix} 2PW + 2CZ + \omega_1 H^\top H & P \\ \star & -\omega_1 I \end{bmatrix} < 0. \tag{34}$$

$$\begin{aligned} {}_0^{CF}D_t^\alpha V(t, q(t)) &\leq \lambda_{max}(\bar{\Omega})q^\top(t)q(t) \\ &= \lambda_{max}(\bar{\Omega})\|q(t)\|^2 \end{aligned}$$

According to Theorem 2.7, the system (30) is α-exponentially stable. Hence the proof is completed. □

7. Numerical example

In this section, we apply the real world data into our proposed model (26) and check the stability properties using the derived results.

Let us consider the data: the reproduction rate of wild mosquitoes $\Lambda_{w_e} = 0.95$ is reduced after the release of Wolbachia infected mosquitoes to $\Lambda_e = 0.56$. Furthermore, $K = 1$, $\rho = 0.5$ and the natural mortality death rates are $\lambda_{w_e} = 0.1285$; $\lambda_{w_l} = 0.1285$; $\lambda_{w_p} = 0.1285$; $\lambda_{w_{f_i}} = 0.0714$; $\lambda_{w_{fm}} = 0.0714$; $\lambda_{w_a} = 0.0714$. Maturation rates of Wild mosquitoes are $\gamma_{w_e} = 0.1499$; $\gamma_{w_l} = 0.1499$; $\gamma_{w_p} = 0.1499$; $\gamma_{w_{f_i}} = 0.1499$. The survivability rates (Fitted) of Wolbachia infected mosquito population are $S_{i_e} = 0.1$; $S_{i_l} = 0.23$; $S_{i_p} = 0.56$; $S_{i_{fm}} = 0.89$; $S_{i_a} = 0.56$; with the following initial release rate (Fitted) of Wolbachia infected mosquito population $I_e = 0.01$; $I_l = 0.10$; $I_p = 0.02$; $I_{fm} = 0.03$; $I_a = 0.019$; and $b_1 = 0.012$; $b_2 = 0.367$; $n_a = 0.036$; $n_{fm} = 0.002$ for instance, refer Table 2,

Table 2
List of parameters.

The reproduction rate of both wolbachia and non-wolbachia mosquitoes	$\Lambda_{w_e} = 0.95$	[34,47]
The death rate of the aquatic stages like egg, larvae, pupae of both wolbachia and non-wolbachia mosquitoes	$\lambda_{w_e} = \lambda_{w_l} = \lambda_{w_p} = \frac{1}{7.78}/day$	[50]
The death rate of the adult stages like male, female mature and female immature of both wolbachia and non-wolbachia mosquitoes	$\lambda_{w_{f_i}} = \lambda_{w_{fm}} = \lambda_{w_a} = \frac{1}{14}$	[50]
The maturation rate of the aquatic stages like egg, larvae, pupae and female immature of both wolbachia and non-wolbachia mosquitoes	$\gamma_{w_e} = \gamma_{w_l} = \gamma_{w_p} = \frac{1}{6.67}/day$	[50]
The transmission rate in which the current compartment moved into the next compartment	0.9	[34,47]
K= environmental carrying capacity	1	[34]

For the above values, we get the following parameters which satisfies the derived LMI (31).

$$W = \begin{bmatrix} -0.3189 & 0 & 0 & 0 & 0 & 0 \\ 0.1499 & -0.4349 & 0 & 0 & 0 & 0 \\ 0 & 0.1499 & -0.4349 & 0 & 0 & 0 \\ 0 & 0 & 0.0899 & -0.3213 & 0 & 0 \\ 0 & 0 & 0 & 0.1499 & -0.2710 & 0 \\ 0 & 0 & 0.0600 & 0 & 0 & -0.2707 \end{bmatrix};$$

Via MATLAB, we have plotted the solution of the system of equations (22) and (26) at various orders. Figs. 3, 5, 7, 9 and 11, are the trajectories of the solutions of the system of equations (22) which describes the wild mosquito population dynamics before the release of Wolbachia infected mosquitoes at $\alpha = 0.18, 0.28, 0.38, 0.41$ and $\alpha = 1$ respectively. Along with this, Figs. 4, 6, 8, 10 and 12 all are the trajectories of the system of equations (26) which describe the dynamics of wild mosquito population after the release of Wolbachia infected mosquitoes at $\alpha = 0.18, 0.28, 0.38, 0.41$ and $\alpha = 1$ respectively. We can observe from the Figures that, the population is stable and under control after the release of Wolbachia infected mosquitoes. From Fig. 13 , one can observe that the dynamics of wild mosquito population model are identical at $\alpha = 1$ at integer order, $\alpha = 0.98$ at Caputo derivative and $\alpha = 0.28$ at Caputo–Fabrizio derivative. Similarly, the dynamics of wild mosquitoes after the release of Wolbachia infected mosquitoes are identical at $\alpha = 1$ at integer order, $\alpha = 0.98$ at Caputo derivative and $\alpha = 0.28$ at Caputo–Fabrizio derivative (see Fig. 14). From this we can observe that Caputo–Fabrizio operator has higher rate of convergence than that of Caputo derivative and integer order system. $C = \begin{bmatrix} 0.1000 & 0 & 0 & 0 & 0.0008 & 0 \\ 0 & 0.5123 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5600 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.8900 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5600 \end{bmatrix};$

and $H = \begin{bmatrix} 0.0400 & -0.0940 & 0.6090 & 0.0100 & 0 & 0 \\ 0 & 0.0980 & 0 & 0.0001 & 0 & 0 \\ 0.0003 & 0 & 0.0900 & 0.0353 & 0 & 0 \\ 0.6900 & 0.0058 & 0 & 0.2500 & 0 & 0 \\ 0.0600 & 0.0030 & 0 & 0 & 0.0010 & 0 \\ 0.0010 & 1.9210 & 0.0700 & 0.0300 & 0.1530 & 0.0300 \end{bmatrix}$; the state feedback control gain matrix is obtained as: $K = \begin{bmatrix} 2.0084 & 0.1184 & -0.0000 & -0.0000 & -0.3315 & -0.0471 \\ 0.0645 & -0.0521 & -0.0000 & -0.0000 & 0.0083 & 0.0029 \\ 0.0110 & -0.0002 & 0.0000 & 0.0000 & -0.0015 & -0.0005 \\ 0.4653 & 0.0269 & -0.0000 & -0.0000 & -0.0910 & -0.0104 \\ -0.0226 & 0.0066 & 0.0000 & 0.0000 & -0.1705 & -0.0081 \\ -0.0050 & 0.0035 & 0.0000 & 0.0000 & -0.0128 & -0.2526 \end{bmatrix}.$

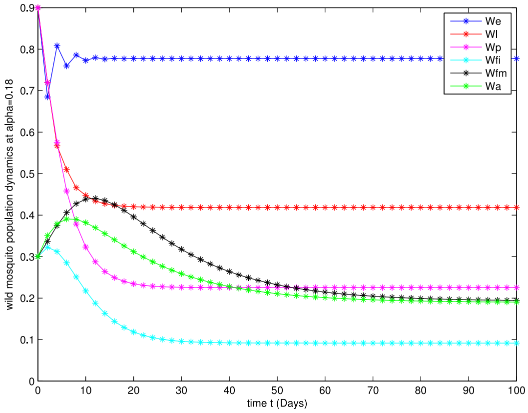

Fig. 3. Population dynamics of wild mosquito at $\alpha = 0.18$.

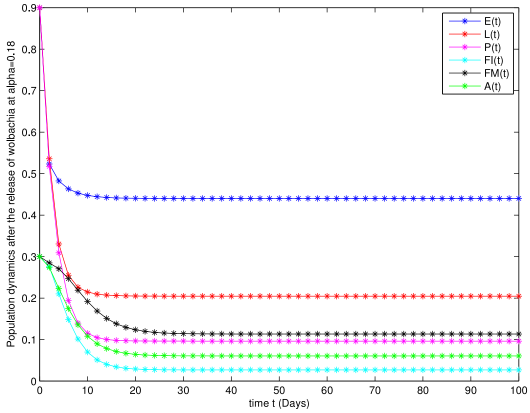

Fig. 4. Population dynamics after the release of Wolbachia at $\alpha = 0.18$.

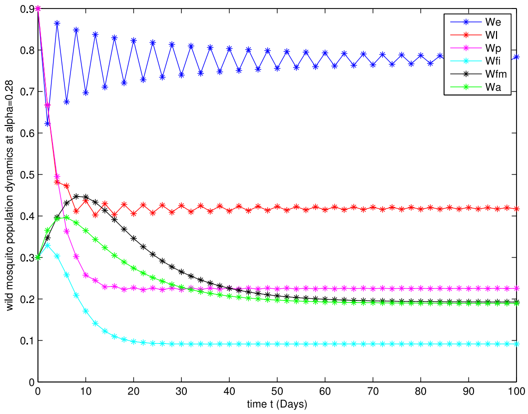

Fig. 5. Population dynamics of wild mosquito at $\alpha = 0.28$.

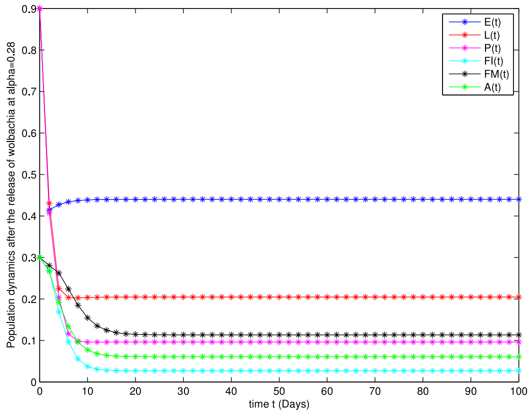

Fig. 6. Population dynamics after the release of Wolbachia at $\alpha = 0.28$.

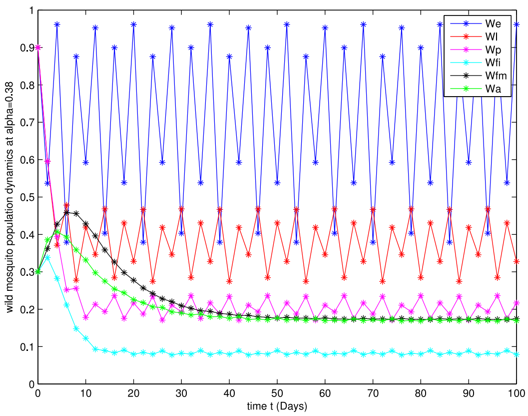

Fig. 7. Population dynamics of wild mosquito at $\alpha = 0.38$.

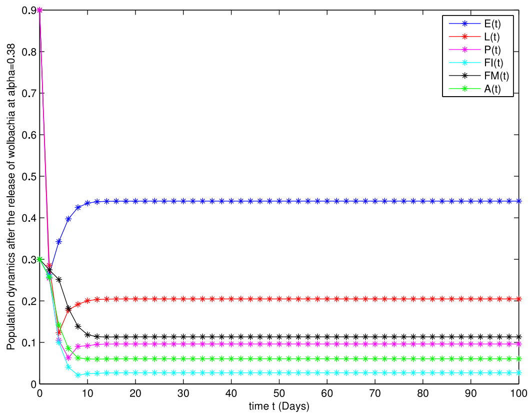

Fig. 8. Population dynamics after the release of Wolbachia at $\alpha = 0.38$.

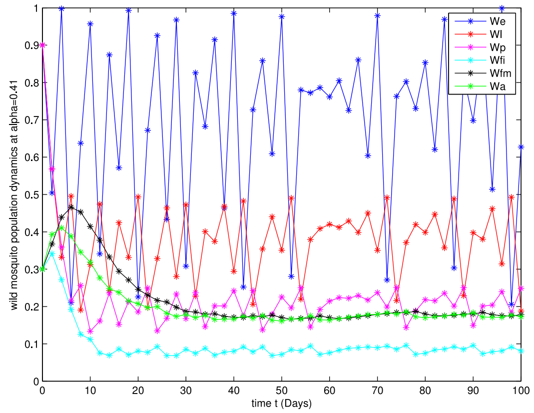

Fig. 9. Population dynamics of wild mosquito at $\alpha = 0.41$.

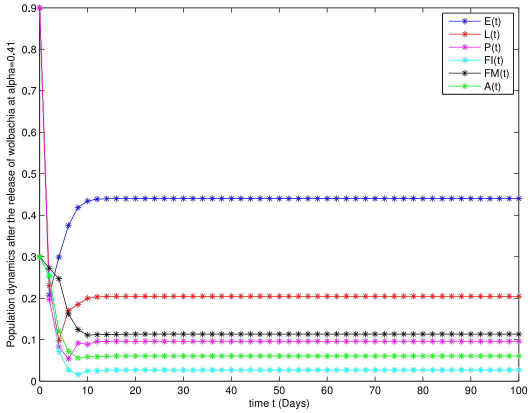

Fig. 10. Population dynamics after the release of Wolbachia at $\alpha = 0.41$.

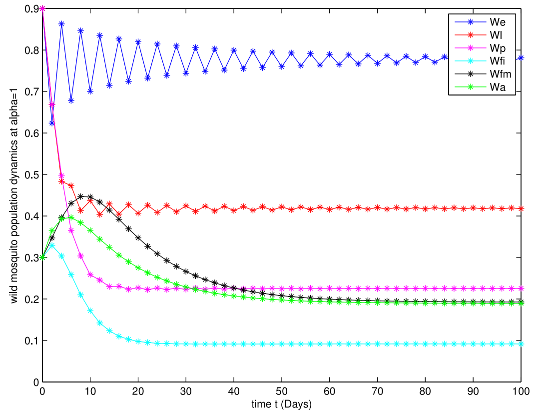

Fig. 11. Population dynamics of wild mosquito at $\alpha = 1$.

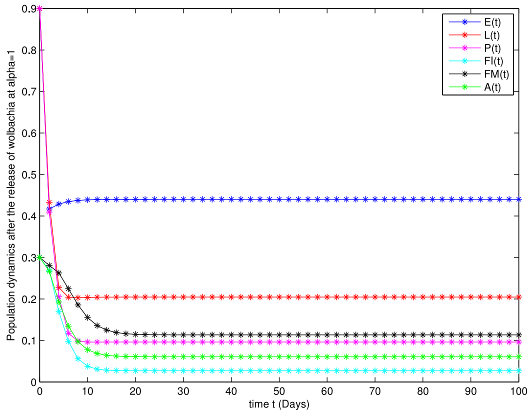

Fig. 12. Population dynamics after the release of Wolbachia at $\alpha = 1$.

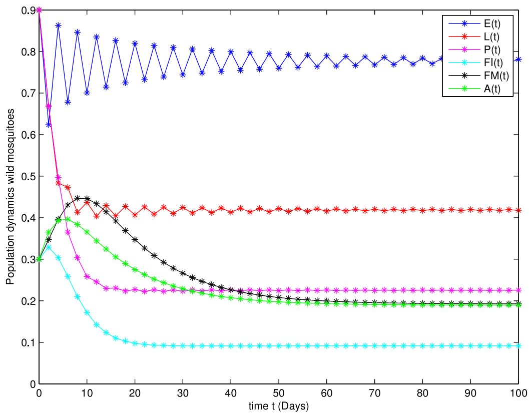

Fig. 13. Population dynamics of wild mosquito at $\alpha = 1$ in integer order, $\alpha = 0.98$ in Caputo derivative and $\alpha = 0.28$ in Caputo–Fabrizio operator.

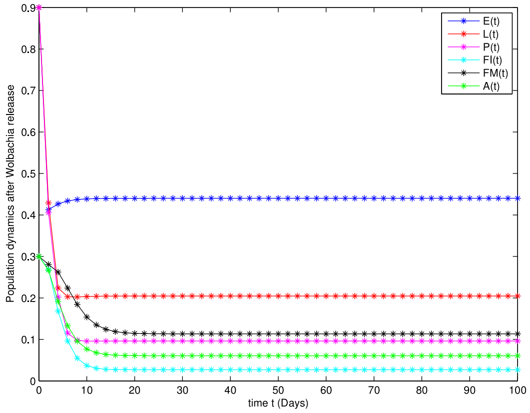

Fig. 14. Population dynamics after the release of Wolbachia at $\alpha = 1$ in an integer order, $\alpha = 0.98$ in Caputo derivative and $\alpha = 0.28$ in Caputo–Fabrizio operator.

These are the values obtained from LMI using the data provided in Table 2 and these figures depict the effectiveness of the proposed theoretical results.

8. Conclusion

In this paper, we have proved that the fractional order system with Caputo–Fabrizio derivative can obtain only global exponential stability results and not for Mittag-Leffler stability. For the first time the proven theoretical results were justified with a real life model to control the mosquito borne diseases using Wolbachia as a biological control. Moreover, by the release of Wolbachia infected mosquitoes into the wild one, we attained the optimal control of mosquito borne diseases. Furthermore, by using Caputo–Fabrizio operator, we proved the α-exponential stability for the considered population system. Finally, a numerical example was drawn to justify the usefulness of the obtained main results. In future, the LMI based stability results can be extended to fractional order delay differential equations and Impulsive differential equation based on CF operator and we aimed to find the application problems for CF operator.

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