MODELLING AND SIMULATION OF A QUAD-ROTOR HELICOPTER

A. Bousbaine*,M. H. Wu*, G. T. Poyi*

**University of Derby, School of Technology, Derby, UK, A.Bousbaine@derby.ac.uk*

**University of Derby, School of Technology, Derby, UK, M.H.Wu@derby.ac.uk*

**University of Derby, School of Technology, Derby, UK, G.T.Poyi@derby.ac.uk*

Keywords: Unmanned Aerial Vehicle, Quad-rotor, Dynamic Modelling, Simulink, Helicopter.

Abstract

Small size quad-rotor helicopters are often used due to the simplicity of their construction and maintenance, their ability to hover and also to take-off and land vertically. The first step in control development is an adequate dynamic system modelling, which should involve a faithful mathematical representation of the mechanical system. This paper presents a detailed dynamic analytical model of the quad-rotor helicopter using the linear Taylor series approximation method. The developed analytical model was simulated in the MatLab/Simulink environment and the dynamic behaviour of the quad-rotor assessed due to voltage changes. The model is further calibrated and linearized for use on any quad-rotor helicopter.

1 Introduction

Helicopters are generally known to be dynamically unstable vehicles; hence the need to achieve stability using suitable control methods. The changing helicopter parameters and complex weather conditions make the vehicle unstable, although the unstable dynamics are good for providing it with the required agility.

In the recent past, Unmanned Aerial Vehicles (UAVs) have captured enormous commercial potential by attracting the attention of many people including laymen, potential appliers, vehicle professionals and researchers. Research and development in this field has gained increasing significance, due to the emergence of a large number of potential civil applications. UAVs are important when it comes to performing a desired task in a dangerous and/or inaccessible environment. In fact, several industries including automotive, medical, manufacturing, aerospace, require robots to replace men in dangerous, boring or onerous situations. A wide area of this research is dedicated to aerial platform [3].

Primarily enabled by advancements in Artificial Intelligence, Computer Science, Automatic Control, Robotics, Communications and Sensor Technologies, UAVs are expected to become a major part of the aviation industry over the next few decades. Furthermore, the ever-increasing performance of Micro Electro-Mechanical Systems (MEMS) Inertial Measurement Units (IMUs) and low cost GPS have given them roles of enabling technologies for new autonomous vehicle applications [5].

A quad-rotor, one such UAV with four fixed pitch rotors is highly manoeuvrable. It has the ability to hover and to take off, fly and land in small areas and also has a simple control mechanism. However, it is a complex unstable vehicle and can be difficult to fly with the absence of modern embedded control systems.

2 The quad-rotor model and system

Design and analysis of control systems are usually started by carefully considering mathematical models of physical systems. The model is very important because it gives a description of how the system responds to the inputs given to it. An adequate dynamic system modelling should involve a faithful mathematical representation of the whole system.

The dynamic behaviour of the quad-rotor is usually described by ordinary differential equations. Such equations can be linearized and the Laplace transform can be used to simplify the method of solution to reduce computation time. This section describes the derivation of the model of the quad-rotor helicopter from [2, 3]. It is possible to define and predict the positions that the helicopter will reach by investigating the speeds of the four motors, using the equations obtained.

2.1 Quad-rotor basic concepts

The quad-rotor is a helicopter with four lift-generating propellers mounted on four motors. It is very well modelled with four rotors in a cross configuration style. In most cases, each propeller is directly connected to a brushless DC motor, with all the propellers having fixed and parallel axes of rotation. Furthermore, it has fixed-pitch blades and the air flow from each of the blades points downwards to generate an upward lift. These considerations show that the structure is rigid and control of the vehicle can be achieved by varying the propeller speeds by very small amounts because of the high sensitivity of the vehicle to rotor speed changes.

Two of the motors (front and rear) rotate counter-clockwise, while the other two (left and right) rotate clockwise. This configuration of opposite pairs' directions completely eliminates the need for a tail rotor, which is needed for stability in the conventional helicopter structure. Figure 1 shows the model in stable hover, where all the motors rotate at the same speed, so that all the propellers generate equal lift and all the tilt angles are zero.

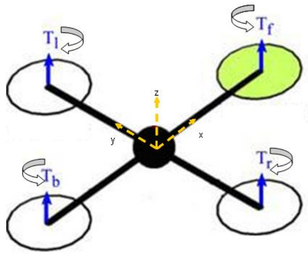


Figure 1: Simplified quad-rotor vehicle in a stable hover.

There are four basic movements, which allow the helicopter to reach a certain altitude and attitude and are;

a) Throttle

This command is provided by simultaneously increasing (or decreasing) all propeller speeds by the same amount and at the same rate. This generates a collective vertical force from the four propellers, with respect to the body-fixed frame. In consequence, the quad-rotor is raised or lowered to a certain altitude.

b) Roll

The roll command is provided by simultaneously increasing (or decreasing) the left propeller speed and by decreasing (or increasing) the right propeller speed at the same rate. It creates a torque with respect to the x axis and this makes the quad-rotor to tilt about the same axis, thereby creating a roll angle. The total vertical thrust is maintained as in hovering, thus this command leads only to a roll angular acceleration.

c) Pitch

The pitch and roll commands are very similar. It is provided by simultaneously increasing (or decreasing) the rear propeller speed and by decreasing (or increasing) the front propeller speed at the same rate. This creates a torque with respect to the y axis which makes the quad-rotor to tilt about the same axis, thereby creating a pitch angle (known as a nose-up or nose-down in a conventional aircraft). Again, there is no loss in the total vertical thrust; hence this command leads only to a pitch angular acceleration.

d) Yaw

This command is provided by simultaneously increasing (or decreasing) the front-rear propellers' speed and by decreasing (or increasing) that of the left-right duo. This creates a torque imbalance with respect to the z axis, which makes the quad-rotor turn about the same axis. The yaw movement is generated because of the fact that the left-right propellers rotate clockwise while the front-rear pair rotates counter clockwise. Hence, when the total torque is unbalanced, the helicopter turns on itself around z. As obtained in the other movements, the total vertical thrust is still maintained as in hovering; hence this command leads only to a yaw angular acceleration.

2.2 Dynamics of the quad-rotor system

There are two coordinate systems to be considered, figure 2:

- The earth inertial frame (E-frame)
- The body-fixed frame of the vehicle (B-frame)

These are related through three successive rotations:

- Roll: Rotation of φ around the x-axis;
- Pitch: Rotation of θ around the y-axis;
- Yaw: Rotation of ψ around the z-axis.

The following assumptions have been made in this approach:

- The origin of the body-fixed frame coincides with the centre of mass (COM) of the body of the vehicle.
- The axes of the B-frame coincide with the body principal axes of inertia.
- The drag torque is proportional to the propeller speed.

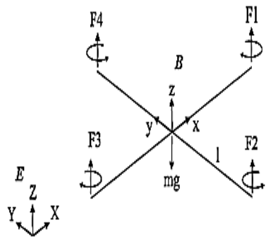


Figure 2: Quad-rotor configuration frame system

It is much easier to formulate differential equations that describe motion in the body-fixed frame of the vehicle, rather than the earth inertial frame because of the reasons listed below [3]:

- The inertia matrix is time-invariant.
- Advantage of body symmetry can be taken to simplify the equations.
- Measurements taken on-board are easily converted to body-fixed frame.
- Control forces are almost always given in body-fixed frame.

Equation (1) has been derived from the rotation sub-matrices about x, y and z respectively and given by

$$R(\varphi,\theta,\psi) = R(x,\varphi)R(y,\theta)R(z,\psi)$$

$$R_\theta = \begin{bmatrix} c\theta c\psi & c\psi s\theta s\varphi - s\psi c\varphi & s\varphi s\psi + c\varphi c\psi s\theta \\ s\psi c\theta & c\psi c\varphi + s\psi s\theta s\varphi & s\psi s\theta c\varphi - c\psi s\varphi \\ -s\theta & c\theta s\varphi & c\varphi c\theta \end{bmatrix} \quad (1)$$

(where c=cos and s=sin).

The equations of motion can be written using the force moment balance and given by;

$$\ddot{\theta} I_{xx} = (-F_1 - F_2 + F_3 + F_4) l$$
$$\ddot{\varphi} I_{yy} = (-F_1 + F_2 + F_3 - F_4) l$$
$$\ddot{\psi} I_{zz} = (\tau_{m1} - \tau_{m2} + \tau_{m3} - \tau_{m4})$$
$$\ddot{x} = \frac{1}{m} \sum_1^4 F_i [\sin\varphi \sin\psi + \cos\varphi \cos\psi \sin\theta]$$
$$\ddot{y} = \frac{1}{m} \sum_1^4 F_i [\sin\psi \sin\theta \cos\varphi - \cos\psi \sin\varphi]$$
$$\ddot{z} = \frac{1}{m} \sum_1^4 F_i [\cos\varphi \cos\theta] - g$$

(2)

Where F_i is the thrust force generated by motor *i*, *l* is the length of the quad-rotor arm, τ is the torque produced by each motor, I_i's are the moments of inertia with respect to the axes and m the mass of the helicopter.

2.2.1 Motor dynamics

The basic electrical circuit is shown in figure 3.

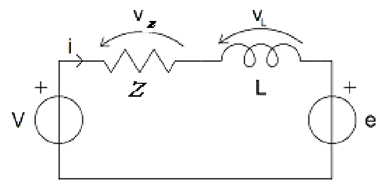


Figure 3: Electrical equivalent circuit of the motor

By applying Kirchhoff's voltage law, it follows that.

$$v = v_Z + v_L + e \quad (3)$$

Where v_Z is the voltage across the resistor Z and v_L is the voltage across the inductor L. The equation above can be rewritten as in the next equation.

$$v = Zi + L\frac{\partial i}{\partial t} + K_e\Omega \quad (4)$$

i is the motor current, K_e is called the motor constant and Ω is the motor angular speed. The first term has been changed using Ohm's law $v_Z = Zi$, while the second one using the inductor differential equation $v_L = L\frac{\partial i}{\partial t}$. The contribution of the inductor part is important to determine the characteristic of the DC-motor driver. However it is often neglected in the mechanics computation because of the reasons that follow [2, 3]:

- A greater percentage of the motors used in robotics show very small inductance because of design optimization.
- The response time of the electrical part is always much faster than the mechanical part, hence the speed of the overall system will be determined by the slowest contribution.
- It is much easier to solve a first order differential equation rather than a second order one.

Therefore the equation above can be simplified according to the one below.

$$v = Zi + K_e\Omega \quad (5)$$

The dynamics of the motor is described by the following equation.

$$J\dot{\Omega} = \tau_m - \tau_d \quad (6)$$

Where J is the total motor moment of inertia, $\dot{\Omega}$ is the motor angular acceleration, τ_m is the motor torque and τ_d is the load torque.

2.2.2 Voltage and angular velocity of propeller

Since the voltage inputs V to motors affect the rotational speed Ω of propellers.

$$V - e = iZ \quad (7)$$

The dynamics of the motor is described by the following equation

$$\tau_m - \tau_d = J\dot{\Omega} \quad (8)$$

Equation (8) states that when the motor torque τ_m and the drag torque τ_d are not equal there is an acceleration (or deceleration). This variation of speed depends also on the rotor inertia J the smaller the value of J, the higher the acceleration. The back electromotive-force voltage is proportional to motor speed

$$e = K_e\Omega \quad (9)$$

The motor torque is proportional to the field current

$$\tau_m = K_q i \quad (10)$$

On substituting, we get

$$V = \tau_d \frac{Z}{K_q} + \frac{ZJ\dot{\Omega}}{K_q} + K_e\Omega \quad (11)$$

As stated earlier, the drag torque is proportional to the square of propeller's speed

$$\tau_d = D\Omega^2 \quad (12)$$

The relationship between angular velocity and voltage as found in [51] and [5] can thus be obtained as

$$V = \frac{ZD\Omega^2}{K_q} + \frac{ZJ\dot{\Omega}}{K_q} + K_e\Omega \quad (13)$$

2.2.3 Voltage and thrust

Voltage is the input of the quad-rotor plant and each rotor produces a thrust force as it turns. The motor torque is known to be proportional to the field current

$$\tau_m = K_q i \quad (14)$$

$$\frac{\tau_m}{K_q} = i \quad (15)$$

The electrical power according to Joule's law is

$$P = IV = \frac{\tau_m}{K_q} V \quad (16)$$

And the mechanical power output is given as

$$P_m = \eta P = \eta \frac{\tau_m}{K_q} V \quad (17)$$

with η as the motor efficiency.

The propeller's figure of merit f is defined as the ratio of the induced power in air P_h to the mechanical power P_m [2].

$$f = \frac{P_h}{P_m} \quad (18)$$

Where P_h is given by

$$P_h = \eta f \frac{\tau_m}{K_q} V \quad (19)$$

The ideal power is the product of the thrust force and the speed at which it is applied. At hover P_h is

$$P_h = F v_h \quad (20)$$

Where v_h is the air velocity

$$\eta f \frac{\tau_m}{K_q} V = F v_h \quad (21)$$

By using the momentum theory

$$v_h = \sqrt{\frac{F}{2\rho A}} \quad (22)$$

Then

$$\eta f \frac{\tau_m}{K_q} V = F \sqrt{\frac{F}{2\rho A}} \quad (23)$$

The torque is proportional to the trust with constant ratio K_t depends on blade geometry.

$$\tau_m = K_t F \quad (24)$$

Then

$$\eta f \frac{K_t F}{K_q} V = F \sqrt{\frac{F}{2\rho A}} \quad (25)$$

Then the relationship between thrust and voltage thus established as

$$F = 2\rho A \left[\frac{f\eta K_t}{K_q}\right]^2 V^2 \quad (26)$$

a) Roll moment

The roll motion is about x-axis and the rolling moment caused by the actions of motors 2 and 4 can be defined as:

$$\ddot{\varphi} I_{yy} = (-F_1 + F_2 + F_3 - F_4) l \quad (27)$$

Since F_1 and F_3 remain unchanged,

$$\ddot{\varphi} I_{yy} = (F_2 - F_4) l \quad (28)$$

Substituting equation () for F, we have

$$\ddot{\varphi} I_{yy} = \left[(2\rho A \left[\frac{f\eta K_t}{K_q}\right]^2 V_2^2) - (2\rho A \left[\frac{f\eta K_t}{K_q}\right]^2 V_4^2) \right] l \quad (29)$$

The final equation for roll motion is obtained as

$$\ddot{\varphi} = \frac{2\rho A l}{I_{yy}} \left[\frac{f\eta K_t}{K_q}\right]^2 (V_2^2 - V_4^2) \quad (30)$$

b) Pitch Moment

The pitch motion is about the y-axis and the pitching moment caused by the actions of motors 1 and 3 can be defined as:

$$\ddot{\theta} I_{xx} = (-F_1 - F_2 + F_3 + F_4) l \quad (31)$$

Just like that of the roll motion, the final equation for pitch motion is obtained as

$$\ddot{\theta} = \frac{2\rho A l}{I_{xx}} \left[\frac{f\eta K_t}{K_q}\right]^2 (V_3^2 - V_1^2) \quad (32)$$

c) Yaw moment

The yaw moment is caused by a counter-torque imbalance. Torque about z-axis is generally defined as

$$\ddot{\psi} I_{zz} = (\tau_{m1} - \tau_{m2} + \tau_{m3} - \tau_{m4}) \quad (33)$$

From Equation (8) each motor supplies machine torque τ_m which is balanced by the drag torque so the net torque on propeller is:

$$\tau_m = J\dot{\Omega} + \tau_d \quad (34)$$

The yaw torque along z-axis given by:

$$\tau_{zz} = I_{zz}\ddot{\psi} = \tau_{m1} + \tau_{m3} - \tau_{m2} - \tau_{m4} \quad (35)$$

$$I_{zz}\ddot{\psi} = J(\dot{\Omega}_1 + \dot{\Omega}_3 - \dot{\Omega}_2 - \dot{\Omega}_4) + (\tau_{d1} + \tau_{d3} - \tau_{d2} - \tau_{d4}) \quad (36)$$

Since it has been assumed in equation (12) that the drag torque is proportional to the square of propeller's speed,

$$I_{zz}\ddot{\psi} = J(\dot{\Omega}_1 + \dot{\Omega}_3 - \dot{\Omega}_2 - \dot{\Omega}_4) + D(\Omega_1^2 + \Omega_3^2 - \Omega_2^2 - \Omega_4^2) \quad (37)$$

$$\ddot{\psi} = \frac{J}{I_{zz}}(\dot{\Omega}_1 + \dot{\Omega}_3 - \dot{\Omega}_2 - \dot{\Omega}_4) + \frac{D}{I_{zz}}(\Omega_1^2 + \Omega_3^2 - \Omega_2^2 - \Omega_4^2) \quad (38)$$

d) Forces along z-axis

From equation (2), the net force at the centre of mass of the vehicle is

$$\ddot{z} = \frac{1}{m}\sum_1^4 F_i[\cos\varphi\cos\theta] - g$$

On substituting equation (26) for F, we have

$$\ddot{z} = \frac{2\rho A}{m}\left[\frac{f\eta K_t}{K_q}\right]^2 (V_1^2 + V_2^2 + V_3^2 + V_4^2)(\cos\theta\cos\varphi) - g \quad (39)$$

e) Forces along x-axis

By actuators action, we have the force acting along the x-axis as

On substituting and simplifying further, we have

$$\ddot{x} = \frac{2\rho_a A}{m}\left[\frac{f\eta K_t}{K_q}\right]^2 (V_1^2 + V_2^2 + V_3^2 + V_4^2)[\sin\varphi\sin\psi + \cos\varphi\cos\psi\sin\theta] \quad (40)$$

f) Forces along y-axis

Similar to that of the x-axis, actuators action produces the following force, acting along the y-axis

$$\ddot{y} = \frac{2\rho_a A}{m}\left[\frac{f\eta K_t}{K_q}\right]^2 \left(V_1^2 + V_2^2 + V_3^2 + V_4^2\right)[\sin\psi \sin\theta \cos\varphi - \cos\psi \sin\varphi] \quad (41)$$

3 Simulation of quad-rotor dynamics

The set of differential equations describing the dynamics of quad-rotor that were obtained earlier have been modelled and simulated using the Matlab/Simulink software, figures 4 and 5. This is very essential because it will help to verify the correctness of the helicopter dynamic model and to test the control algorithms performance.

With the input voltages, V_1, V_2, V_3 and V_4 figure 4, the dynamic behaviour of the quad-rotor due to change in the excitation voltage is analysed, figure 6.

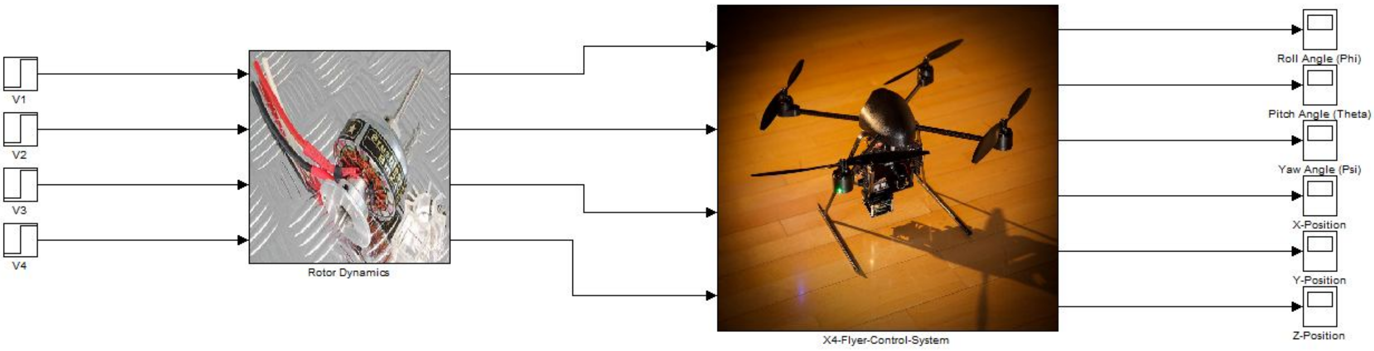


Figure 4: System structure

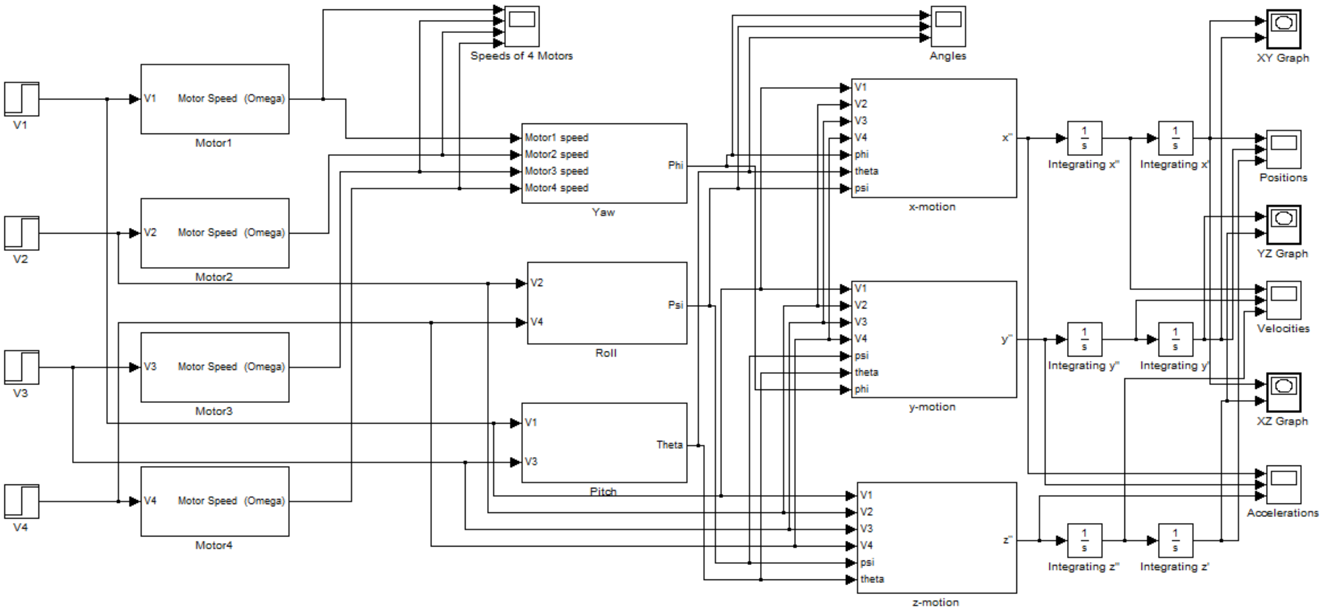
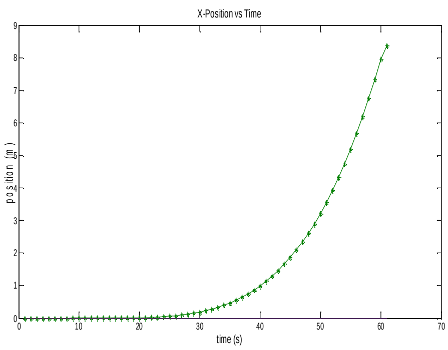
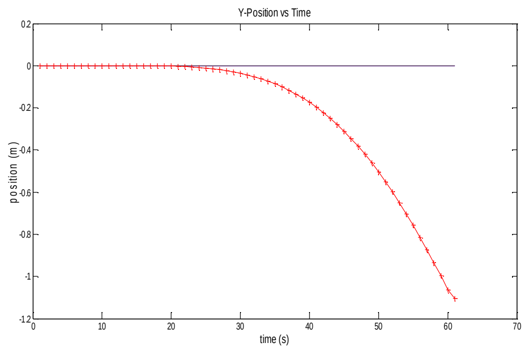


Figure 5: Dynamics implementation in Simulink





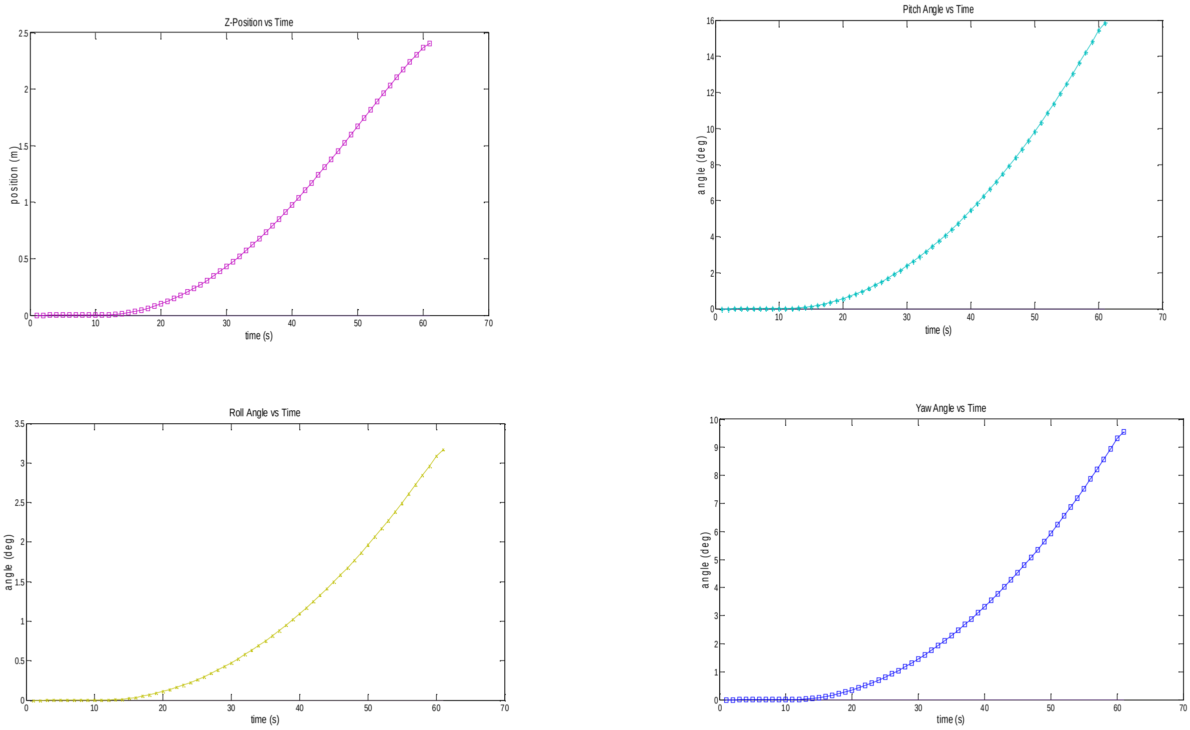


Figure 6: The dynamic behaviour of the quad-rotor due to change in the excitation voltage

.

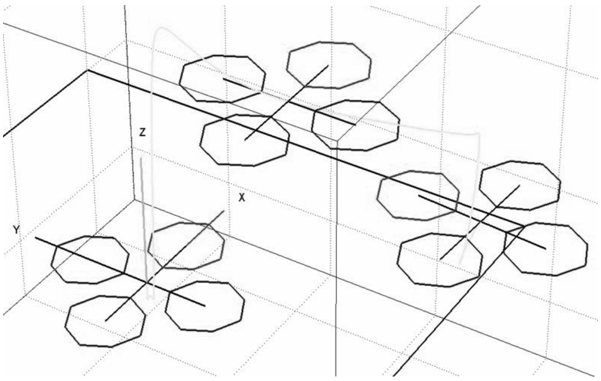


Figure 7: 3-D view of the Quad-rotor motion

The model as shown in figure 5 has a total of 10 sub-systems, with four of them simulating the rotor dynamics, another three simulating the angular accelerations and the last three simulating the linear accelerations.

5 Conclusions

Flying a quad-rotor is a difficult task because of its inherent sensitivity to changes in rotor speeds. With its aerodynamics being quite complex, the need for accurate modelling in order to achieve precise control cannot be over-emphasised. A good understanding of the helicopter's responses to speed changes of the four rotors can be very helpful in the development of a suitable controller. This paper presents a detailed dynamic analytical model of the quad-rotor helicopter using Matlab/ Simulink. The dynamic behaviour of the quad-rotor was assessed and its responses to voltage changes were critically analysed. The model is further calibrated and linearized using Taylor's Series for use on any quad-rotor helicopter. This will be the subject for future publications.

References

[1] E. Altug, J. P. Ostrowski, R. Mahony. "Control of a Quad-rotor Helicopter Using Visual Feedback", *International Conference on Robotics and Automation*, pp 72-77, (2002).

[2] M. Y. Amir, V. Abbas. "Modeling and Neural Control of Quad rotor Helicopter", *Yanbu Journal of Engineering and Science*, volume 2, pp 35-49, (2011).

[3] T. Bresciani. "Modelling, Identification and Control of a Quad rotor Helicopter", *MScThesis, Lund University*, (2008).

[4] M. Claudia, C. T. Luminita, K. K. Simon. "Modelling and Control of Autonomous Quad-rotor", *MSc Group Project, University of Aalborg, Denmark*, (2010).

[5] R. Czyba. "Attitude Stabilization of an Indoor Quad-rotor", (2009), http://www.emav09.org/EMAV-final-papers/paper_64.pdf.

[6] A. S. Sanca, P. J. Alsina, J. F. Cerqueira. "Dynamic Modelling of a Quad-rotor Aerial Vehicle with Nonlinear Inputs", *Robotic Symposium, 2008. LARS '08. IEEE Latin America*, pp 143-148, (2008).