Mental Models or Probabilistic Reasoning or Both: Reviewing the Evidence for and Implications of Dual-Strategy Models of Deductive Reasoning

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Abstract

The present paper presents an overview of contemporary reasoning research to examine the evidence for and implications of the Dual Strategy Model of Reasoning. The Dual Strategy Model of Reasoning proposes that there are two types of reasoning strategy applied in deductive reasoning – counterexample and statistical. The paper considers Mental Models Theory and The Probability Heuristics Model as candidate specifications for these respective strategies and hypotheses are proposed on this basis. The Dual Strategy Model is further considered in the context of Dual Process theory, the Dual Source Model and Meta-reasoning and implications of the synergy between these proposals are considered. We finally consider the Dual Strategy Model in the context of individual differences, and normative considerations before proposing novel hypotheses and further avenues of research which we argue require exploration in this context.

Keywords: Reasoning, Dual Process Theory, Dual-Strategy Theory, Meta-reasoning, Metacognition

Introduction

Deductive reasoning - the capacity to evaluate the logical validity of a conclusion based upon its premises - is a fundamental aspect of human cognition (e.g., Johnson-Laird & Byrne, 1991), and understanding its variability remains an essential problem for cognitive science to address. Historically there was a debate over the fundamental deductive mechanism employed when reasoning (Johnson-Laird & Byrne, 1991; Rips, 1994), but this has been superseded by the view that an array of strategies can be employed (Ford, 1995; Roberts, 1993; 2000; Verschueren, Schaeken, & d'Ydewalle, 2005a, 2005b). There has also been a switch to viewing reasoning processes through the lens of dual process theory whereby process are fast, effortless and heuristic (Type 1) or slow, effortful and analytic (Type 2). Over the last decade the ‘new paradigm’ in reasoning research has moved the emphasis away from normatively sanctioned logical benchmarks as a gold standard, and towards understanding and describing individual differences in interpretation, processing, metacognition and strategy (e.g., Elqayam & Evans, 2011; Roberts, Newstead, & Griggs, 2001; Stupple & Ball, 2014).

One innovative response to this development is the dual-strategy model of reasoning which proposes that individuals have access to both statistical and counterexample strategies (Markovits, Brisson, & de Chantal, 2015b; Verschueren, Schaeken, & d’Ydewalle, 2005a, 2005b). Thus individuals differ in their strategy preference and these strategies have differing processing demands and response outcomes. Counterexample strategies are typically slow and effortful and place higher demands on working memory, this contrasts with statistical strategies which are faster and entail a lower working memory demand (e.g. Markovits, Brisson, & de Chantal, 2015b). The model is broadly consistent with a dual-process framework, with each of the strategies featuring similar characteristics to Type 1 and Type 2 reasoning processes, however there are important contrasts between these theoretical proposals (Markovits, Brisson, & de Chantal, 2015b), as both statistical and counterexample strategies include Type 1 and Type 2 processes.

In the following literature review we present a brief overview of classic theoretical proposals that are candidates to underpin the different strategies (Mental Models Theory and the Probability Heuristic Model), before moving on to consider the evidence base for the theory. We further consider the Dual Strategy Model in the context of meta-reasoning, soft normativism and individual differences, and propose further avenues of research that we argue require exploration in this context.

Mental Models

Johnson-Laird (1983) proposed the Mental Models Theory (MMT) by adapting the proposal that 'small-scale models' of reality are constructed through perception (Craik, 1943), to the reasoning domain. This view was developed and refined into the MMT we know today. The contemporary MMT makes three main assumptions about model construction (Johnson-Laird, 2001; 2006): 1. Each model is representative of a possibility; 2. Models are iconic: the components and structure of the model correspond to the components and structure of the possibility; 3. Models represent what is true, but not what is false (Johnson-Laird & Savary, 1999). This process occurs in three stages. In the comprehension stage, reasoners use pragmatic understanding of language, and general knowledge, to interpret the premises and construct a model of them. During the description stage, a parsimonious model is formulated that integrates the premises and contains information not explicitly stated in them and is a putative conclusion to them. Finally, the validation phase is a search for counterexamples to falsify the putative conclusion. If no counterexamples are found the conclusion can be accepted. However, not all participants engage in the search for counterexamples.

MMT predicts that as the number of models required increases so does the difficulty in making an inference; and that multiple model problems take longer and exhibit more errors (Johnson-Laird, 2001). Errors and processing time predictions are related to the limitations of working memory, as the consideration of multiple models can overload its capacity (e.g., Halford, Wilson, & Phillips, 1998). Syllogistic Figure further influences the working memory demand whereby figures without contiguous middle terms require the reordering of premises or switching the order of terms within premise(s) through a conversion process (Johnson-Laird & Bara, 1984; Stupple & Ball, 2007). MMT further predicts that individuals spontaneously develop a variety of model building strategies for deductive inferences with increased experience, for example, considering the most informative premise first (Bucciarelli & Johnson-Laird, 1999).

If MMT is an accurate description of the counterexample strategy in dual-strategy theory, these predictions should hold for reasoners who prefer a counterexample-based approach. Indeed, it includes possibility of meta-strategies within this group whereby variations in counterexample search and model building strategies should vary based on reasoning experience and/or aptitude. There is also evidence that many reasoners do not move beyond their initial model to search for counterexamples when generating conclusions from premises rather than evaluating presented conclusions. These individuals may be better characterized as having a dichotomous interpretation of the truth value of the conclusion rather than employing a counterexample strategy, they could still show less influence from the statistical information than those who prefer a statistical strategy, but not go beyond their first mental model.

Probability Heuristic Model

Classic paradigms in the psychology of deduction were based on binary logic whereby all assertions can be allocated one of two values: true or false. However, a Bayesian view of cognition challenged this approach (e.g., Oaksford & Chater, 2007; Tenenbaum, Kemp, Griffiths, & Goodman, 2011), with probability theory or Bayes theorem considered as the normative standard against which reasoning should be judged - (e.g., Chater & Oaksford, 1999; Elqayam & Over, 2013), and the view that deductive reasoning was probabilistic rather than logical. When initiating the dual strategy perspective Verschueren, et al. (2005a; 2005b) proposed a minimalist view of the probabilistic/statistical strategy. However, in the present paper we explore the more highly specified Probability Heuristics Model (PHM), proposed by Chater and Oaksford (1999) as a putative account of the statistical strategy – particularly in the context of syllogistic reasoning.

PHM proposes that individuals employ heuristics to yield a probabilistically valid (p-valid) conclusion. The application of probabilistic heuristics has a number of assumptions. First, that different quantifiers vary in their informativeness such that some quantifiers are more informative than others. Second, some quantified assertions entail others (p – entailment). For example, the use of a 'particular', *Some* or *Some are not*, rather than a 'universal', All or No, implies that the universal statement is incorrect. In conjunction with these principles, reasoners apply probabilistic generation and testing heuristics to produce conclusions (see Chater & Oaksford, 1999, p.196-202). One example is the min-heuristic which prevents the generation of conclusions that are more informative than the least informative premise.

PHM proposes that these assumptions and heuristics combine to provide a complete description of the processes that underlie reasoning. Indeed, the model has been successfully applied to a variety of reasoning problems, previously explained by models subscribing to logic as the norm against which reasoning is measured, including conditional inference (e.g. Oaksford, Chater, & Larkin, 2000;), the Wason Selection task (e.g. Oaksford, Chater, & Grainger, 1999) and syllogistic reasoning problems (e.g. Chater & Oaksford, 1999). Similarly, work on syllogistic reasoning has also offered empirical support for the PHM demonstrating that the PHM can be extended to syllogisms featuring quantifiers such as 'Most' and 'Few' which have no logical conclusion (Chater & Oaksford, 1999). From the perspective of a Dual Strategy

Model the PHM allows a detailed specification of likelihood assessments derived from the properties of the quantifiers, which would occur in conjunction with any contextual or belief driven influences on conclusion plausibility.

The PHM is not without criticism: ‘No valid conclusion’ responses are not well explained by the model (Hattori, 2016); and there is some debate over the conclusions that should be produced (e.g., Elflein & Ragni, 2018; Hattori, 2016). Such issues could be avoided if the Probability Heuristic Model described a possible strategy rather than a fundamental reasoning mechanism. For example, no valid conclusion responses can be predicted by counterexample strategies, particularly if participants have interpreted the task as requiring a judgment of logical necessity (or if we allow the possibility of mid-task strategy switching among some reasoners). Finally, some authors suggest PHM as a dual process account with Type 1 heuristics to generate conclusions and test procedures to determine p-validity (e.g., Schroyens, Schaeken & Handley, 2003), as we will see later, this view aligns well with statistical strategies as outlined by Markovits and colleagues.

The Dual-Strategy Model

The dual-strategy model originated from work by Verschueren, Schaeken and d’Ydawelle (2005a; 2005b) and proposed that reasoners employ two qualitatively different strategies for deductive reasoning: statistical and counterexample. This model combines the explanations of counterexample models, derived from MMT (e.g., Johnson-Laird & Byrne, 1991) and probabilistic theories (e.g. Oaksford et al, 2009) and presents them as strategies rather than unitary frameworks. Counterexample strategies are associated with dichotomous assessments of validity and involve higher cognitive demand and increased working memory load – particularly where participants move beyond basic model description. In comparison, statistical strategies are faster, intuitive and require fewer cognitive resources –aligning with aspects of dual process theory but with notable caveats.

For instance, according to Markovits, Brisson and de Chantal (2016) the dual-strategy model does not, however, describe a pure heuristic process, rather, the model focuses on the way in which logical inferences are made. The essential distinction of the model is that underlying statistical information from knowledge about the premises can influence the way the information is processed. Statistical strategies are essentially Bayesian and generate estimates of presented/generated conclusions being true (Oaksford & Chater, 2007; Oaksford et al., 2000). This contrasts with counterexample strategies whereby the ease with which a counterexample can be generated is dependent on both the logical structure of the problem and the likelihood of the conclusion given our knowledge of reality – as the number of plausible counterexamples increases so does the likelihood of a rejection. (e.g., Johnson-Laird, 2001).

Support for the dual-strategy model of reasoning derives from a series of studies conducted by Markovits and colleagues (e.g. Markovits, Brisson, & de Chantal, 2015a, 2015b, 2016; Markovits, Brisson, de Chantal, & Singmann, 2018; Markovits, Brisson, de Chantal, & Thompson, 2017; Markovits, Brunet, Thompson, & Brisson, 2013; Markovits, Lortie Forgues, & Brunet, 2012) which provide strong evidence for the distinction between counterexample and statistical strategies. Markovits et al. (2012) developed a method to distinguish between these reasoning strategies, by presenting problems accompanied by statistical information that described the likelihood of the putative conclusion. Individuals who rejected low probability conclusions more frequently were identified as adopting a statistical strategy. In contrast, individuals who were not influenced by the presence of statistical information were considered to adopt a counterexample strategy. There does however, remain the question as to whether these strategic preferences exist on a continuum or as a dichotomy.

Under time constraints, individuals preferentially adopt a statistical reasoning strategy (Markovits et al., 2013), but when reasoning without time constraints, reasoners preferred a counterexample strategy. This supports the argument that statistical strategies are less cognitively demanding than counterexample strategies, consistent with the prediction of the dual-strategy model, and indeed dual-process frameworks more generally. However, the task characteristics that encourage this preference warrant further consideration and replication with alternative paradigms and instruction sets. Response-time effects have shown that deductive inferences are typically faster when reasoners adopt a statistical strategy (Markovits et al., 2016). These experiments demonstrate some clear commonalities with standard dual process theories and it was a logical step for Markovits and colleagues to apply this approach to belief bias.

The dual-strategy model predicts a greater tendency to base responses on conclusion believability when a statistical strategy is used, because beliefs can drive likelihood assessments of conclusion. Across three different forms of reasoning, effects of conclusion belief were observed to be stronger for statistical strategies, compared to counterexample strategies (Markovits et al., 2017). Belief bias was stronger with probabilistic strategies, but was not eliminated among those using counterexample strategies. Ball and Stupple (2016) described three categories of dual process theory of belief bias: Default Interventionist, Parallel and Hybrid accounts - it is also not clear-cut which category of belief bias theory best explains the data. Within the Default-Interventionist account a dominant, default intuition is initially generated by

Type 1 processing, which may be overridden by Type 2 processing. This intervention can occur when there is a lack of confidence in the default conclusion and cognitive resources are sufficient. Default responses generated by Type 1 processes are often convincing, and initiating Type 2 processing requires significant cognitive effort (Stanovich, 2009; Thompson, 2009) so Type 1 can win-out. Default interventionist explanations typically assume a mental models as representations (e.g., Evans, 2000; Stupple, Ball, Evans & Kamal-Smith 2011), but it would be possible to develop a probabilistic default interventionist account where defaults were derived from intuitive belief, but could be overridden by test procedures determining p-validity.

In contrast, parallel-processing models propose that both Type 1 and Type 2 occur simultaneously (e.g. Sloman, 1996) with Type 1 outcomes being suppressed when conflict generates meta-cognitive uncertainty. These models could also have more probabilistic specifications. Finally, hybrid dual-process theories offer a further alternative (Bago & De Neys, 2017; Handley & Trippas, 2015), whereby serial and parallel mechanisms that include intuitive heuristics and logical or probabilistic intuitions deliver Type 1 outputs that can coincide or conflict. When these processes deliver conflicting outcomes analytic processing is triggered to reduce metacognitive uncertainty by resolving the conflict. Hybrid accounts require little modification to include Type 1 processes based on beliefs/context and probabilistic heuristics applied to quantifier combinations. Evidence for multiple sources of intuition feeding into the reasoning process is now well established (e.g., De Neys, 2012; Handley & Trippas, 2015; Trippas, Handley, Verde, & Morsanyi, 2016) and these factors would be expected with either strategy, that belief based intuition has more impact in probabilistic strategies.

Hybrid models of belief bias are consistent with the proposals from Trippas, Thompson, and Handley (2017) who show evidence of slow belief responses and fast logic responses. They also concur with the dual-source model which proposes that, individuals must combine prior knowledge of information surrounding the subject matter and information concerning logical form (Singmann, Klauer, & Over, 2014; Singman, Klauer, & Beller, 2016). Hybrid and dual-source models are in broad agreement that there are intuitions based on knowledge/belief /context and upon the logical form/probabilistic heuristics/pragmatics that influence reasoning processes and outcomes – the precise specifications, such as the extent to which these processes conform to the characteristics of Type 1, how conflict is resolved and how they generalise across different tasks with differing complexities remains an open question for the field.

Markovits et al. (2018) combined the dual-source model, with the dual-strategy model, and demonstrated that logical form influenced deductive inference equally for both counterexample and statistical reasoners. Logical form was also observed to influence probabilistic inferences that used explicit statistical information although these findings were less clear-cut. This nuanced finding requires further unpacking in future studies to explore whether intuitive influences derived from logic or belief have the same underlying mechanism irrespective of the strategy adopted, or whether task interpretation and intuitions differ between strategy groups. We concur with Markovits et al. that understanding the effects of logical form, reasoning strategy and the form of inference is a minimum requirement for a complete theoretical account of deductive inference. However, we would go further and suggest that a comprehensive deductive reasoning theory should also predict individual differences based on working memory capacity, cognitive disposition, perceived normative standards that the participants work towards, and the interpretation of the quantifiers and connectives. These individual differences impact upon the metacognitive processes and meta-reasoning that are central to strategy selection (and perseverance with or abandonment of that strategy), as well as the degree of confidence required to endorse a solution (and the calibration between this and response accuracy).

A metacognitive account of reasoning process is a vital component in any reasoning theory, as the act of monitoring and controlling reasoning processes, and allocating cognitive resources are central to the completion or otherwise of the task (Thompson & Ackerman, 2017a). Ackerman and Thompson (2017a) presented a framework for meta-reasoning research inspired by metacognitive approaches in learning and memory research (Nelson & Narens, 1990; Bjork, Dunlosky, & Kornell, 2013). Much metacognition research focused on learning and memory and their associated monitoring and control processes, but the metacognitive processes that underlie reasoning (or meta-reasoning) and problem solving (Ackerman & Thompson, 2017a, 2017b; Bjork, Dunlosky, & Kornell, 2013), are an increasing priority.

One fruitful approach to examining meta-reasoning processes is that applied by Thompson and colleagues (e.g., Thompson, Prowse-Turner, & Pennycook, 2011) which has focused on feeling of rightness (FOR) using a two-response methodology. The two-response methodology requires participants to quickly provide a first response, then rate the FOR about this response before being offered the opportunity to revise it. This methodology was applied to the min-heuristic demonstrating that min-conclusions resulted in stronger FOR and were also processed more quickly, than non-min-conclusions. Additionally, min-conclusions were less frequently reassessed and fewer changes in responses were given to these conclusions. These data support at least this component of PHM as a candidate for Type 1 elements of statistical reasoning strategies.

However, as FOR are the result of heuristic cues, the amount of time allocated to reanalysing the initial response may not be indicative of problem difficulty or cognitive load (Ackerman & Thompson, 2017b). Indeed, as answer fluency is related to heuristic cues, incorrect answers can be given with high levels of confidence (Thompson et al., 2011; Thompson & Johnson, 2014). Familiarity with problem content also has a similar effect of producing high levels of confidence in incorrect answers (Markovits, Thompson, & Brisson, 2015; Shynkaruk & Thompson, 2006). However, it is also possible in some cases that FOR can indeed reflect problem difficulty, such as in the presence of conflicting answers (De Neys, Cromheeke, & Osman, 2011; Thompson et al., 2011).

Despite the developing evidence for the metacognitive processes involved in the initiation and termination of analytic thinking, more work is required to explore the metacognitive processes that underlie strategy selection. Beilock and deCaro, (2007) investigated the costs of selecting strategies, and found that the least demanding strategy is the most likely to be selected. Further research on the monitoring and control processes that influence strategy selection is also needed. Reder and Ritter (1992) demonstrated that strategy selection may be impaired by monitoring processes that are based on misleading information. The processes associated with strategy selection may be based on heuristic cues and thus the reliability of such cues should influence the quality of strategy selection (Ackerman & Thompson, 2017b). Determining the extent to which strategy selection is volitional or implicit is a further consideration – particularly as Roberts (2000) has argued that participants are capable of switching strategy between tasks or even across trials using the same tasks. Some participants in the Dual strategy paradigm demonstrated strong preferences for statistical or counterexample approaches, but others were less clear cut – these individuals may indeed be strategy switchers.

In a novel study Bajšanski, Žauhar, and Valerjev (2018) applied metacognitive methods to syllogistic reasoning tasks and examined the extent to which consensuality effects generalise from general knowledge tasks to reasoning. They demonstrated that while confidence and accuracy were not strongly correlated, participants were more confident when generating the most common answers to the reasoning problems. This methodology is an excellent approach to identifying commonalities between reasoners and can help to identify the cues that influence reasoning processes and outcomes. This approach may be particularly valuable in the context of the Dual Strategy theory as the different outcomes for statistical and counterexample reasoners would suggest a lack of commonality between these groups. Applying this method to the different groups may identify different patterns of consensuality and help to triangulate the different properties of the strategies.

Metacognitive processes have been shown to play a vital role in initiating (e.g. Markovits et al., 2015; Shynkaruk & Thompson, 2006; Thompson et al., 2011; Thompson, Evans, & Campbell, 2014) and terminating (e.g. Ackerman, 2014) analytic thinking, as well as strategy selection (e.g. Broder & Newell, 2008; Reder & Ritter, 1992; Rieskamp & Hoffrage, 2008; Markovits et al., 2013). As such, developing understanding of these processes is essential to expanding our understanding of reasoning, and potentially, improving performance (Ackerman & Thompson, 2017b).

Future directions and conclusions

It is our view that the has immense promise in explaining the extreme variability in reasoning performance. Perhaps the great strength of the approach is its compatibility with other perspectives, with success and potential to draw upon classic theories proposed as fundamental deductive mechanisms, dual process theory and dual source models, as well as the important development in meta-reasoning. We would contend however, that while the integration of such a range of paradigms makes for an exciting prospect we would caution that this will generate as many new questions as it does answers. In this final section we outline a modest range of proposals to further extend the approach.

The Dual Strategy Model has a good evidence base thus far, but the detailed specification of the mechanisms in the two strategies involved requires further empirical demonstration to determine where the limits of the generalisability lie. We argue that there is much potential in the use of MMT and PHM as candidate specifications for the two strategies. The fact that these accounts are so highly specified, with a substantial range of well-established effects is double-edged as it allows for strong predictions but also reduces the parsimony of the account.

One empirically testable contrast between statistical and counterexample strategies based upon the contrast between MMT and PHM is the role of syllogistic figure. A counter example strategy based on MMT principles would entail a cognitive load associated with integrating the premises to form a mode in the description stage when the middle terms of the problem are not contiguous (e.g., figure BACB should be more demanding than ABBC, cf. Stupple & Ball, 2007). In contrast to MMT the PHM makes no strong predictions about syllogistic figure and a more general statistical strategy would not necessarily require a premise integration process and as such would not show this differential cognitive load.

At the core of the Dual Strategy Model is the assumption of individual differences and that these differences can be fundamental to the way that individuals approach their reasoning. However, there is an extensive literature on individual differences in reasoning that has not yet been applied to the model (e.g., Stanovich & West, 2000). An examination of the individual differences in cognitive ability, disposition, interpretation, response time and motivation of reasoners who apply different strategies is likely to be prudent. It would be predicted – in line with dual process predictions (e.g., Stupple et al. 2011; Stupple, Pitchford, Ball, Hunt, & Steel 2017; Stupple & Ball, 2014) that participants who respond faster (using a statistical strategy) may be more likely to be cognitive misers with lower working memory spans (Stupple, Gale & Richmond, 2013). It is a further possibility that statistical strategies represent a failure or absence of 'decoupling' the problem from the mental simulation of its solution (Toplak & Stanovich 2012). In the present context this would entail suspending pragmatic, contextual and belief-driven elements of a problem to enable hypothetical thinking about its abstract properties - in line with view of Type 2 thinking at its purest.

We would argue that adopting a 'soft normativist' approach (e.g., Stupple & Ball, 2014) would facilitate this – soft normativism allows for normative evaluations of reasoning performance alongside the pursuit of descriptive research goals. Thus, the unnecessary constraints of rigid normativism and the slippery slope of strong relativism in judging the outcomes of our reasoning processes are avoided. For example, the use of Bayesian and Logic based normative benchmarks may be instructive as to the strategies being employed because different participants may view these different standards as appropriate to the task at hand. Indeed, the recruitment of participants untrained in logic raises the possibility that task interpretation involves an informal or naïve reflective equilibrium whereby participants attempt to give the most rational response available in line with normative standards they are aware of or reasoning strategies they deem appropriate to reach their goal (see Stupple & Ball, 2014 for further description of this view). Where a participant judges the reasoning task as one of determining the plausibility of the conclusion rather than its logical necessity (a judgement for which they may lack the requisite mindware) a probabilistic strategy could be considered a reasonable, rational approach.

In conclusion, the recent developments in the reasoning literature: the Dual Strategy Model, dual-source (e.g., Singmann et al., 2014) and Hybrid models (Bago & De Neys, 2017) and the increasing focus on Meta-reasoning (Thompson & Ackerman, 2017a) offer considerable optimism for the reasoning domain. Dual-process theories require more nuanced specification to address the critics of the approach (e.g., Keren & Schul, 2009; Kruglanski & Gigerenzer, 2011) and these new approaches – dual strategy, dual-source and meta-reasoning provide the means to answer this challenge within a dual process framework.

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